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Learning paths in complex networks

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Abstract – This letter addresses the issue of learning shortest paths in complex networks, which is of utmost importance in real-life navigation. The approach has been partially motivated by recent progress in characterizing navigation problems in networks, having as extreme situations the completely ignorant (random) walker and the rich directed walker, which can pay for information that will guide to the target node along the shortest path. A learning framework based on a first-visit Monte Carlo algorithm is implemented, together with four independent measures that characterize the learning process. The methodology is applied to a number of network classes, as well as to networks constructed from actual data. The results indicate that the navigation difficulty and learning velocity are strongly related to the network topology.

Introduction. – There has been great interest in the study of navigation in complex networks, and to relate the resulting dynamics to the network topological properties. The concept of navigation implies that an agent (or walker) has to move within the network from a source node s and reach the target node t. Two extreme navigation schemes have attracted the attention: navigation of *random walkers* and *directed walkers*. In the first one, the walker placed on a given node moves towards one of the neighboring nodes at each time step in a completely random way [1–3]. In an alternative but similar problem, the walker may use a weighted transition probability, which dynamically evolves with the network [4]. In the second approach, the walker selects the shortest path to the target, by asking (and paying for) information about the choice of the best motion at each node it visits. This strategy allows to reach the target using the minimal number of steps [5–8]. Situations lying between these extremes have also been considered. Directed navigation with limited information has been explored by assuming that the information the walker gets at each node is not complete [9]. The consequence of missing information is to substantially increase the traveled distance in comparison to the actual shortest path. Quite recently, one of us considered the issue of *optimal navigation* in complex networks [10], where the walker can either pay for (correct) information at each node it visits, or randomly follow one of the available paths. Since two constant costs are associated to the trajectory, the stepping and the information costs, the walker can make an optimal decision by minimizing the sum of the costs associated to the complete trajectory.

Although learning is one important phenomenon that arises in real-life navigation, none of the approaches presented above copes explicitly with it. In order to fill the gap, this letter addresses such issue by investigating how a walker learns paths in complex networks. During the learning process it acquires knowledge about all shortest paths and will be able to navigate optimally, without paying for any further information. The basic assumption is that learning paths arises from on-line experience, which is mimicked by a Monte Carlo reinforcement learning framework [11–13]. We show that the difficulty and velocity of learning paths in a given complex network is strongly associated to its topology.

Let us recall that one particular practical issue also considered in the literature of navigation is the problem of *searchability* of specific nodes in a given complex network [14–21]. This problem is inspired by a classical study [22], where individuals were asked to deliver letters

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to a specific target person. If the individual knew the target, the letter should be sent to this person. Otherwise, it should be addressed to a friend more likely to know the target. As we will see later on, this issue tackled here is related also to the difficulty of reaching certain network nodes.

Quantifying learning. – We start with a network G, with n nodes in a set $V(G)=\{1,2,\ldots,t,\ldots,n\}$, where t is a special node called target. Let $\mathcal{N}(i)$ denote the neighborhood of a node i, *i.e.*, the set of nodes j that are connected to node i and can be reached from i in one single step[1]. We define an episode η as either the complete trajectory from one given source s to the target t, or a trajectory that the walker followed in less than mn steps, where m is the a multiple of the number of nodes n. The $\eta-$episode is a basic definition for the current investigation, since the notion of on-line experience is based on the division of the problem in episodes. Furthermore, in order to ensure that past decisions will be available and the walker will learn by experience, we have limited the time duration of η by m times the number of nodes of the network. Episodes will be numbered sequentially, *i.e.*, $\eta=1,2,\ldots,N_\eta$.

Let $Q(s,a)$ be the expected number of steps required by the walker to go to the target t, when it departs from node s and steps towards $a\in\mathcal{N}(s)$. The essence of our method is to estimate $Q(s,a)$ for $s\in V(G)$ and $a\in\mathcal{N}(s)$ based on previous episodes using the so-called first-visit Monte Carlo algorithm (FVMCA) [11–13]. It amounts to average, over past η's, the number of steps that the walker used to go to target t, after the node s was visited and the action $a\in\mathcal{N}(s)$ was selected. From the definition of $Q(s,a)$, it is clear that the eligible best path $a^*(s)$ to be followed from node s is such that $Q(s,a^*(s))$ attains the minimum value when compared to the paths through other nodes in the neighborhood of s:

$$a^*(s)=\arg\min_a Q(s,a). \tag{1}$$

For a given η, consider that the walker is in node $s\in V(G)$ and wants to go to node t. If it is the first time that the walker is in this node, then the walker will randomly follow one of the paths available in this step. On the other hand, if it has been in this node before, with probability $(1-\epsilon+\epsilon/\text{degree}(s))$ the walker follows the direction that is the best direction given by a^* and with probability $\epsilon/\text{degree}(s)$ the walker follows randomly one of the other possible paths. One should note that it is essential for continuous learning that the walker does not always follow the supposed optimal path. If the walker always follows the supposed optimal path, there is no further learning. This policy, which allows the walker a global non-zero probability ϵ to choose any neighbor from a given node s, is called ϵ-soft policy. In the limit of $\epsilon\to 0$, the ϵ-greedy policy is reached. The key steps of the FVMCA, adapted to evaluate the discussed learning process, is presented in the algorithm (table 1). By using it, one can estimate the cost $J(s)$, as well as the optimal cost $J^*(s)$, to go to the target node t from node s based on the walker's current knowledge as

$$J(s)=(1-\epsilon)\min_a Q(s,a)+\frac{\epsilon}{\text{degree}(s)}\sum_a Q(s,a), \tag{2}$$

$$J^*(s)=Q(s,a^*(s)). \tag{3}$$

Although $J(s)$ reflects the actual cost to go from s to t when the walker follows FVMCA, we notice that $J^*(s)$ reflects more appropriated the current level of knowledge of the walker about the paths of the network. Furthermore, if the walker decides to stop learning and $Q(s,a)$ is estimated without learning, this is the cost that he/she will face to go to the target node t from node s.

Note that after the walker repeated FVMCA many times ($\eta\to N_\eta\gg 1$), $J^*(s)$ converges to a value very close to the shortest path length from node s to node t. This value is not exactly the shortest path length since $Q(s,a)$

Table 1: Key steps of the FVMCA adapted for the case where a walker wants to go to the target node t.

for all epsisode **do**
  $count=0$;
  Choose the source node s.
  while $s\neq t$ and $count\leqslant m\times n$ **do**
    $count=count+1$;
    if it is the first episode that the node s happens **then**
      the next node a is a neighbor randomly chosen;
      $a^\star(s)=a$;
    else
      With probability $(1-\epsilon+\epsilon/\text{degree}(s))$, the next node $a=a^\star(s)$;
      With probability $(\epsilon/\text{degree}(s))$, the next node a is a neighbor randomly chosen;
    end if
    $s=a$;
    for all $i\in V(G)$ and $j\in\mathcal{N}(i)$ such that the step from i to j happened in this episode **do**
      $\text{cost}(i,j)=\text{cost}(i,j)+1$;
    end for
  end while
  for all $i\in V(G)$ and $j\in\mathcal{N}(i)$ such that the step from i to j happened in this episode **do**
    $Q(i,j)=\text{average}(\text{cost}(i,j))$ over all episodes that the step from i to j happened;
  end for
  for all $i\in V(G)$ such that the node i happened in this episode **do**
    $a^\star(i)=\arg\min_a Q(i,a)$;
  end for
end for

Although the framework here is built for undirected and unweighed, it can be easily extended to the general case.

for $s \in V(G)$ and $a \in \mathcal{N}(s)$ was estimated using an ϵ-soft policy.

Several average values based on $J(s)$ are of relevance. So let $\bar{J}_N(s)$, $\forall s \in V(G)$ be the average of $J^*(s)$ over the set of targets t, which also depends on the value of η. It measures the current navigation difficulty ($\sim$ to the number of steps) to go from s to any other node j. Then let $\bar{J}_H(i)$, $\forall i \in V(G)$ be the average of $J^*(j)$, over set of starting points j with the condition that target $t = i$. Since it also depends on η, it measures the current ability that a node has to be hidden, which is $\sim$ the number of steps to go to t from any other node s (see [5,7] for a discussion of the so-called *hide* property). By combining $\bar{J}_N(s)$ and $\bar{J}_H(s)$, we obtain $\langle J \rangle$, the average of $J^*(s)$ taken over all sources s and targets t, the η-dependent overall network navigation difficulty. Keeping track of the η-evolution of these averages, it is possible to assess how difficult the optimal path learning is. In a similar way, it is possible to evaluate $\langle J \rangle_\ell$, the average of $J^*(s)/\ell(s,t)$ over all sources s and targets t, where $\ell(s,t)$ is the shortest path from node s to node t. An obvious advantage of $\langle J \rangle_\ell$ in comparison to $\langle J \rangle$ is that, independent on the network features, it is expected that this variable converges to a value close to 1.

One can measure the instantaneous velocity of learning using $V(\eta) = \langle J(\eta+1) \rangle - \langle J(\eta) \rangle$. Another way to compare velocity of learning among networks is to use the the instantaneous normalized velocity of learning given by $V_\ell(\eta) = (\langle J(\eta+1) \rangle - \langle J(\eta) \rangle)/(\langle J(\eta+1) \rangle - \bar{\ell})$, where $\bar{\ell}$ is the characteristic path length. $V_\ell(\eta)$ may be interpreted as the fraction of the total learning that the walker learns in a given episode. Furthermore, $0 \leqslant |V_\ell(\eta)| \leqslant 1$, since it is the ratio between the learning from one episode to the next one and the greatest possible learning.

By counting the number of times $N_s(i)$ that an episode η starting from a source node i was interrupted due to the fact that the number of steps *count* exceeds mn, one gets information about the difficulty to learn paths starting from i. Similarly, counting the number of times $N_t(i)$ that an episode to target i was interrupted by the same reason, one gets information about the difficulty to learn paths to reach target i. Finally, the overall difficulty to learn paths in a given network can be estimated by $N_T = \sum_i (N_s(i) + N_t(i))$.

In the rest of this paper, we analyze the results of FVMCA when applied to several complex networks. In all simulations of this paper, we have chosen $\epsilon = 0.2$, $m = 1000$ and $N_\eta = 500\,000$.

To find an optimal choice for ϵ, it is clear that the region of small values of ϵ must be investigated, as we have done in this paper. While larger values of ϵ imply in easier learning, smaller values of it imply in better convergence. We have varied $\epsilon \in [0.1, 0.3]$ and found that $\epsilon = 0.2$ is a good tradeoff between faster learning and better convergence. Finally, we note that the choice of m has impact only on the number of steps that a walker keeps trying to find a path from a source node s to a target node t and the number of events that generates N_t, N_s and N_T.

Results. – Let us now discuss the results we obtained when applying the proposed framework to several network classes.

Scale-free networks: We start with typical Barabasi-Albert (BA) scale-free networks [23], as well as for the corresponding maximally (Max) and minimally (Min) hierarchical versions [24,25]. These hierarchical versions were built by multiple preferential rewiring of pairs of edges [25], ensuring the full network connectivity. For building the maximally (minimally) hierarchical version, at each step one select two pairs of connected nodes and connects the node with the highest degree among these four nodes to the next highest degree node (the lowest degree) in this subset. The other two nodes are then linked. Figure 1 presents the relative difficulty of navigation, expressed by $\langle J \rangle_\ell$. In fig. 1(a), differences in the behavior of the learning process for BA, Max, and Min networks can be visualized, while fig. 1(b) illustrates the dependence of $\langle J \rangle_\ell$ for BA networks with different sizes. Since logarithmic scales are used in both axes of fig. 1, we note that the hierarchical structure causes a large difference in the learning process. Figures 1(a) and 1(b) make clear that: a) An increase in the network hierarchy is directly related both to the navigation facility and learning capacity; b) the path learning process runs faster in smaller networks. The dynamics of $\langle J \rangle$ (not shown) is exactly the same of $\langle J \rangle_\ell$.

Figure 2 provides us some clues about the learning velocity in the same networks used in fig. 1. Figure 2(a) shows that $|V^{\rm Max}| < |V^{\rm BA}| < |V^{\rm Min}|$. Since in the beginning of the learning process there is so much to be learned, it is expected that the absolute value of the velocities[2] decreases with η until it converges to zero. Figure 2(b) compares $V_\ell^{\rm BA}$ with $V_\ell^{\rm Max}$. For small values of η, one notes that $V_\ell^{\rm Max}/V_\ell^{\rm BA} > 1$, indicating that, in the first episodes of the learning process, the walker learns proportionally more paths in the Max version than in the original BA network. However, the value of $V_\ell^{\rm Max}/V_\ell^{\rm BA}$ decreases when η increases, as the number of paths to be learned in the Max version is smaller than the corresponding number in the original BA network. Although $|V^{\rm Max}| < |V^{\rm BA}|$, learning in the Max version proceeds at a faster pace than in BA network. Similar conclusions can be derived from the curves for $|V^{\rm Min}| < |V^{\rm BA}|$ and $|V^{\rm Max}| < |V^{\rm Min}|$, drawn in the same panel of fig. 2.

The value of N_T for the learning process of the same networks have been normalized by the number of episodes N_η, leading respectively to are respectively given by $N_T^{\rm BA}/N_\eta = 3.6 \times 10^{-5}$, $N_T^{\rm Max}/N_\eta = 1.4 \times 10^{-5}$ and $N_T^{\rm Min}/N_\eta = 1.0 \times 10^{-3}$. This pattern often happens for a large range of m. Furthermore, these values are in agreement with the results presented in fig. 1(a) regarding the learning path difficulties in the BA networks and other generalized versions of it. We have also found that N_s and N_t are highly irregular over the nodes for each

[2]Note that these velocities are by definition negative.

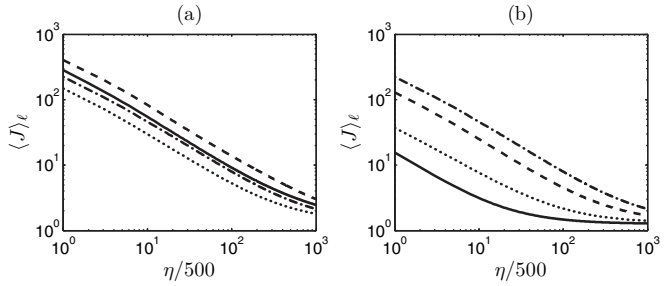

Fig. 1: (a) Behavior of average cost $\langle J\rangle_\ell$ as a function of η for representative BA (dash-dotted line), Max (dotted line), Min (dashed line) and ER (solid line) networks, when $n = 200$ nodes. (b) The same for BA networks with different values of $n = 50$ (solid line), 100 (dotted line), 150 (dashed line), 200 (dash-dotted line).

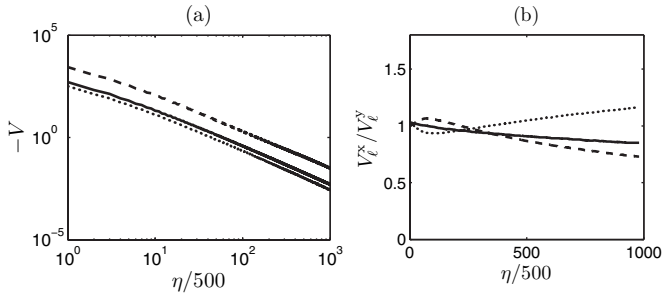

Fig. 2: (a) Behavior of V as a function of η for typical BA (solid line), Max (dotted line) and Min (dashed line) networks, when $n = 200$ nodes. (b) Behavior of the ratios V_ℓ^x/V_ℓ^y as a function of η for $x =$ Max and $y =$ BA (solid line), $x =$ Min and $y =$ BA (dotted line) and $x =$ Max and $y =$ Min (dashed line). In (b), the curves have been smoothed by a slide window averaging process of size 20.

one of the networks (not shown) implying that there are nodes that are quite easier to be reached, while others provide the best location to be hidden. In particular, larger degree nodes tend to present larger N_s and smaller N_t. This is intuitive since the walker in larger degree nodes has to choose among a larger number of paths to follow. On the other hand, it is easy to arrive at nodes with larger degrees since most paths connect other nodes to these nodes.

The study of the learning process in BA networks has shown that, in first place, there is clearly a relation between difficult of navigation in a network and difficult of learning paths in a network, *i.e.*, a walker learns paths faster in networks that are easier to be navigated. Next, it shows that the node degree is an important feature related to both navigation difficulty and hiding ability. Furthermore, besides the node degree, the learning also depends on the hierarchy of the network.

Random networks: We start this section by reporting the results for a random Erdős-Renyi (ER) network where the attachment probability p is such that leads to the same $\langle k\rangle$ of the BA networks presented in fig. 1. The resulting learning curve lies between those for BA and Min network

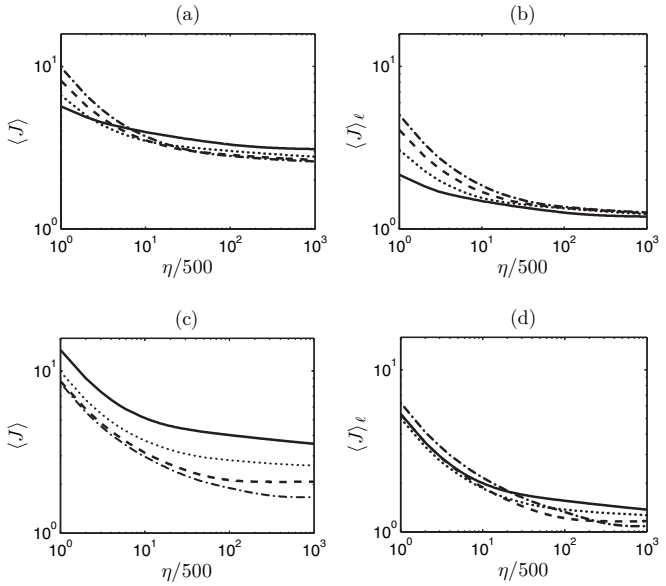

Fig. 3: Behavior of average cost $\langle J\rangle$ (a) and $\langle J\rangle_\ell$ (b) as a function of η, for representative ER networks with $p = 0.1$ and different values of $n = 50$ (solid line), 100 (dotted line), 150 (dashed line), 200 (dash-dotted line). Behavior of $\langle J\rangle$ (c) and $\langle J\rangle_\ell$ (d) as a function of η, for ER networks with $n = 200$ nodes and different values of the connection probability p: 0.05 (solid line), 0.1 (dotted line), 0.25 (dashed line), 0.5 (dash-dotted line).

in fig. 1(a), indicating that increased randomness with respect to BA makes navigation harder but, on the other hand, easier than in the more uniformly degree distributed Min.

Then, we considered several different values of p, so that we can explore learning features of ER networks with different size and average node degree. This is exemplarily shown in the different panels of fig. 3. In panel (a) we see that, for small values of η (for instance $\eta/500 = 1$, $\langle J\rangle_{n=50} < \langle J\rangle_{n=100} < \langle J\rangle_{n=150} < \langle J\rangle_{n=200}$. This happens because the upper bound of $\langle J\rangle$ is larger for the largest networks. However, since the number of connections of the largest networks is correspondingly larger, as well as the characteristic path length, $\langle J\rangle$ decreases at different pace for distinct values of n. It results that, the above relation is reversed after some learning (when η increases). On the other hand, such network size effect is excluded from the behavior of $\langle J\rangle_\ell$, as shown in fig. 3(b). In fact, the form of the curves is much similar to those in fig. 1(b).

The inverse effect is observed when we compare networks of the same size but with different number of connections. Figures 3(c) and (d) show the effect of varying the ER node attachment probability p in the values of $\langle J\rangle$ and $\langle J\rangle_\ell$. Now it is possible to observe that line crossings are observed for the latter measure, but not for the former. This can be explained by noting that when the number of connections is high, it implies that the shortest paths are smaller. Therefore, the crossing of the curves presented in fig. 3(d) is a consequence of the normalization procedure.

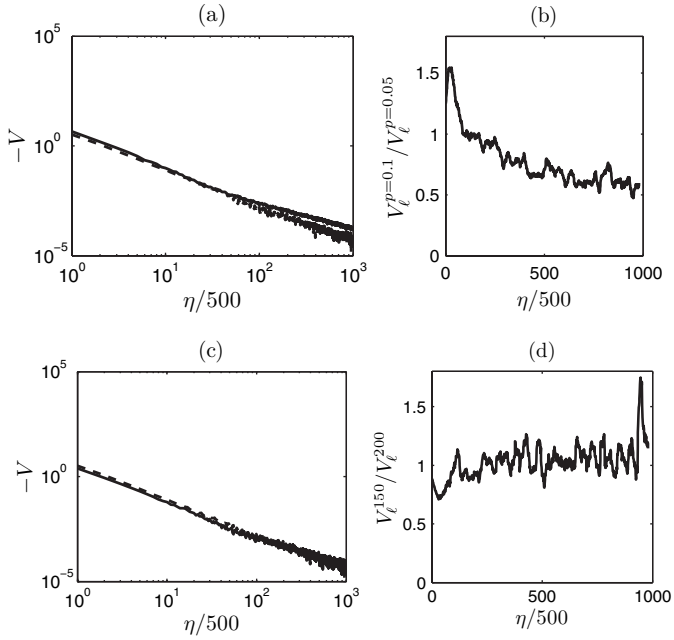

Fig. 4: (a) Behavior of V as a function of η for ER typical networks when $n=200$, and connection probability $p=0.05$ (solid line), and $p=0.1$ (dashed line). (b) Behavior of the ratio $V_\ell^{p=0.1}/V_\ell^{p=0.05}$ for the same data. (c) Behavior of V as a function of η for ER networks when $p=0.1$, and different values of $n=150$ (solid line) and $n=200$ (dashed line). (d) Behavior of the ratio $V_\ell^{n=150}/V_\ell^{n=200}$ for the same data. In (b) and (d), the curves have been smoothed by a slide window averaging process of size 20.

As in fig. 2, fig. 4 compares the learning velocities and relative velocities of some networks in fig. 3. We now observe that line crossing is observed for networks with the same value of n and different values of p. The fluctuations are also much larger than obtained for BA networks. Following the arguments of the discussion related to fig. 2, the main conclusion that can be drawn from fig. 2 is that the learning process in more connected networks is faster.

Apollonian networks: We now study how a walker learns paths in geometrically constructed Apollonian networks, which are simultaneously scale-free, small-world and Euclidian [26]. Although they share such important features with well studied network types as BA and Watts-Strogatz (WS) [27], they have own individual characteristics that put them apart from those quoted classes systematic application of multivariate statistical methods is applied [28]. Therefore, the analysis of a third network type helps clearing out the effect of network structure on the learning process. The qualitative behavior of $\langle J\rangle$ and $\langle J\rangle_\ell$ (not shown) over η is quite similar to those ones presented in fig. 1(b).

Figure 5 compares the learning velocities and the relative velocity of Apollonian networks with different sizes such as in fig. 2 and fig. 4. Figure 5(a) shows that larger networks have larger absolute learning velocities

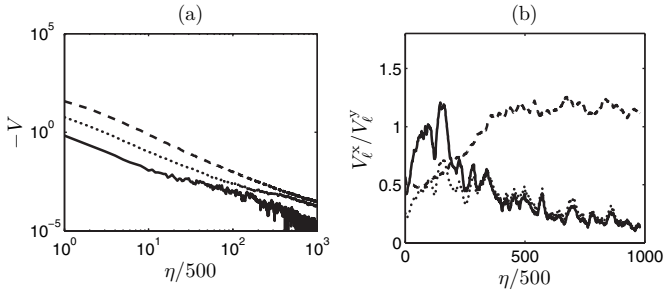

Fig. 5: (a) Behavior of V as a function of η for Apollonian networks with $n=43$ (solid line), $n=124$ (dotted line) and $n=367$ (dashed line). (b) Behavior of the ratios V_ℓ^x/V_ℓ^y as a function of η for $x=43$ and $y=124$ (solid line), $x=43$ and $y=367$ (dotted line) and $x=124$ and $y=367$ (dashed line). In (b), the curves have been smoothed by a slide window averaging process of size 20.

since the walker has more information about the paths to learn. Figure 5(b) shows the trend of V_ℓ^{43}/V_ℓ^{124} and V_ℓ^{43}/V_ℓ^{367} to zero meaning that the learning process is finishing in the smallest network. Different from the BA network (not shown), fig. 5(b) shows that in the beginning of the learning process, larger networks always have larger relative velocities.

Real networks: We have also applied this methodology to explore navigation in some real networks, namely (a) the US airlines connections network [29], (b) the Zachary Karate club social network [30], (c) a representative subgraphs with 200 nodes of the Swedish city Umeå mapped into an information network (see [6] and comment [24] at the reference list of [10]), and (d) the Boston underground transport system [31].

Figures 6(a) and (b) compare the measures $\langle J\rangle$ and V_ℓ for the real networks indicated above with with those of their random counterparts [32,33]. Much like the situation in fig. 1, there is not essential distinction between the behavior of $\langle J\rangle$ and $\langle J\rangle_\ell$. The results of fig. 6(a) show that the measures of the randomized network are noticeable smaller than those of the original networks for all but the Boston underground transport system. This is likely to happen due to the real (geometric) constraints that the real networks are subjected to as suggested by [6]. For the Boston underground transport system network, the random network achieves a small increase in the value of $\langle J\rangle$ for the whole considered η range. This indicates that is more difficult to find optimized paths in networks with some structure than in fully random ones. We conjecture that this particular result is a consequence of the rather unusual node distribution of this transport network.

As shown in fig. 6(b), although the absolute values of the learning velocities $|V|$ in most real networks are higher than their random counterparts, the learning process measured by $|V_\ell|$ proceeds at a faster pace in their random counterparts. This indicates that there is more to learn on such structured networks than in an random environment.

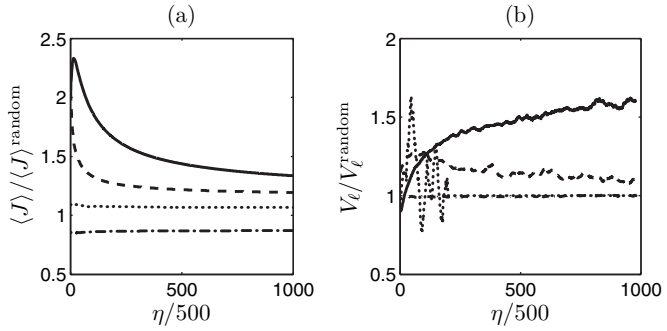

Fig. 6: While (a) shows the ratio between average costs $\langle J\rangle$ for the real network and $\langle J\rangle^{\text{random}}$ for the random counterpart, (b) shows the ratio between V_ℓ for the real network and V_ℓ^{random} for the random counterpart: US airlines connections network (solid line), the Zachary Karate club (dotted line), a representative subgraph with 200 nodes of the Swedish city Umeå (dashed line), Boston underground transportation system (dash-dotted line). In (a), the measures of the ER networks are smaller than those of real networks for all but the Boston underground transportation system network. In (b), the curves have been smoothed by a slide window averaging process of size 20. (b) confirms the odd behavior of the Boston transportation system network detected in (a). It shows that the randomized network version has slighter larger values of V_ℓ than the original one.

Final remarks. – This paper has introduced a framework to understand the issue of learning paths in complex networks based on the assumption that a walker learns paths by online experience. We fully characterized the learning process by a first-visit Monte Carlo algorithm, and proposed four different measures that allows to quantify the progress achieved by the the walker in the learning process. We have applied this framework to study random networks, scale-free networks, Apollonian networks and also four real networks. We have shown that the difficult of learning paths in complex networks is strongly related to the topology of the network.

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