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A new stabilized least-squares imaging condition

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Abstract
The classical deconvolution imaging condition consists of dividing the upgoing wave field by the downgoing wave field and summing over all frequencies and sources. The least-squares imaging condition consists of summing the cross-correlation of the upgoing and downgoing wave fields over all frequencies and sources, and dividing the result by the total energy of the downgoing wave field. This procedure is more stable than using the classical imaging condition, but it still requires stabilization in zones where the energy of the downgoing wave field is small. To stabilize the least-squares imaging condition, the energy of the downgoing wave field is replaced by its average value computed in a horizontal plane in poorly illuminated regions. Applications to the Marmousi and Sigsbee2A data sets show that the stabilized least-squares imaging condition produces better images than the least-squares and cross-correlation imaging conditions.

Keywords: One-way wave equation migration, imaging condition, seismic migration, acoustic wave, least-square imaging condition, deconvolution imaging condition

Introduction

The classic Claerbout's (1971) imaging condition is a deconvolution imaging condition where the ratio of the upgoing and downgoing wave fields is used to get a direct estimate of the reflection coefficient. Different stabilization techniques have been proposed to avoid the division by zero or near to zero values of the downgoing wave field (Valenciano and Biondi (2003), Guitton *et al* (2006), Vivas and Pestana (2007)).

The reflection coefficient can be estimated by a local least-squares procedure. The resulting imaging condition has previously been given by Shin *et al* (2001) and Plessix and Mulder (2004). In the appendix, it is shown that division and least-squares both produce unbiased estimates of reflectivity, but that the least-squares estimate always has less variance. In fact, a least-squares estimate is a minimum-variance estimate under fairly general conditions (Tarantola 1987).

A least-squares imaging condition, which uses all the shots simultaneously, computes the migration imaging weight as the summation of energy of the downgoing wave field by frequency and shot, also called the total illumination function (Plessix and Mulder 2004). Theoretically, the least-squares imaging condition avoids instabilities, but the finite and nonuniform source and receiver coverage produces instabilities in zones of strong defocused energy (Kiyashchenko *et al* 2007).

It is possible to stabilize the least-squares imaging by adding a positive condition constant to the illumination function in order to avoid division by zero or a very small number. This corresponds to a damped least-squares procedure (Lines and Treitel 1984). Schleicher *et al* (2008) implemented this approach by computing a data-driven damping constant. However, they advocate to compute a stable image by dividing with a spatially smoothed value of the illumination function. Here we follow the approach of

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Vivas and Pestana (2007) and implemented a more local and less severe stabilized procedure. For each source, frequency and depth level, the average, over horizontal coordinates, of the downgoing energy is computed. Wherever the downgoing energy is below this value times a given constant, it is replaced by the average value times the constant. The stabilized value of the illumination function is obtained by a final sum over frequency and sources, for each spatial position.

We show the advantage of this new criterion for seismic imaging applied to the Marmousi and Sigsbee2A synthetic data sets. We compare the imaging results obtained by cross-correlation and least-squares without stabilization with the least-squares imaging condition using the stabilization criterion.

Deconvolution imaging conditions

Claerbout (1971) introduced seismic migration using upgoing and downgoing wave fields. The wave field from the source is downward propagated, and the recorded primary reflected wave field is downward retropropagated. At a specific depth point, a reflector exists where the first arrival of the downgoing wave is time-coincident with the upgoing wave. A signal-and-noise model describing this is, in the frequency domain,

$$U(\mathbf{x}^s; \mathbf{x}, \omega) = R(\mathbf{x}) D(\mathbf{x}^s; \mathbf{x}, \omega) + N(\mathbf{x}^s; \mathbf{x}, \omega). \tag{1}$$

Here $U(\mathbf{x}^s; \mathbf{x}, \omega)$ is the upgoing wave field at $\mathbf{x}$, retropropagated from the receiver wave field from the source at $\mathbf{x}^s$. $D(\mathbf{x}^s; \mathbf{x}, \omega)$ is the wave field at $\mathbf{x}$, forward propagated from the source at $\mathbf{x}^s$. $R(\mathbf{x})$ is the reflectivity at $\mathbf{x}$ and $N(\mathbf{x}^s; \mathbf{x}, \omega)$ is the noise term.

The deconvolution imaging condition for shot-profile migration uses the quotient of the downward continued wave field from each of the sources divided by the upgoing wave field retropropagated from the receivers to obtain an estimate of reflectivity. For several shot records

$$\begin{aligned} R(\mathbf{x})_D &= \frac{1}{N}\frac{1}{M}\sum_{\mathbf{x}^s}\sum_{\omega}\frac{U(\mathbf{x}^s; \mathbf{x}, \omega)}{D(\mathbf{x}^s; \mathbf{x}, \omega)} \\ &= \frac{1}{N}\frac{1}{M}\sum_{\mathbf{x}^s}\sum_{\omega}\frac{U(\mathbf{x}^s; \mathbf{x}, \omega) D^*(\mathbf{x}^s; \mathbf{x}, \omega)}{|D(\mathbf{x}^s; \mathbf{x}, \omega)|^2}, \end{aligned} \tag{2}$$

where N is the number of shots and M is the number of discrete frequencies in the sum, and * denotes the complex conjugate.

Equation (2) is unstable for small values of the energy of the downgoing wave field. To avoid division by zero, the imaging condition can be modified to

$$R(\mathbf{x})_{DS} = \frac{1}{N}\frac{1}{M}\sum_{\mathbf{x}^s}^{N}\sum_{\omega}^{M}\frac{U(\mathbf{x}^s; \mathbf{x}, \omega) D^*(\mathbf{x}^s; \mathbf{x}, \omega)}{|D(\mathbf{x}^s; \mathbf{x}, \omega)|^2 + V}, \tag{3}$$

where V is a constant or a slowly varying function of frequency and space (Claerbout 1971). A possible choice for function V is the average of the energy of the downgoing wave field multiplied by a damping parameter λ (Valenciano and Biondi 2003):

$$V(\mathbf{x}) = \frac{\lambda}{N \cdot M}\sum_{\mathbf{x}^s}\sum_{\omega}|D(\mathbf{x}^s; \mathbf{x}, \omega)|^2. \tag{4}$$

However, at the points where the energy of the downgoing wave field is small for all frequencies a very noisy image is obtained, independent of the value of λ.

Smoothing of the downgoing wave field in the transversal coordinate (Guitton *et al* 2006) is another stabilization criterion which has been used. In this way, the zeros in the energy of the downgoing wave field are filled with the energy of the neighbouring points. The imaging condition, equation (2), is applied in the following form:

$$R_{DG}(\mathbf{x}) = \frac{1}{N \cdot M}\sum_{\mathbf{x}^s}\sum_{\omega}\frac{U(\mathbf{x}^s; \mathbf{x}, \omega) D^*(\mathbf{x}^s; \mathbf{x}, \omega)}{\langle |D(\mathbf{x}^s; \mathbf{x}, \omega)|^2\rangle}, \tag{5}$$

where $\langle\cdot\rangle$ represents a spatial smoothing filter. For a long-length smoothing filter, the spectral density amplitude of the downgoing wave field is strongly affected and as a consequence, the reflector amplitudes are also affected.

In order to avoid division by a small number, an image is often formed by cross-correlating the two wave fields

$$R_C(\mathbf{x}) = \sum_{\mathbf{x}^s}\sum_{\omega} U(\mathbf{x}^s; \mathbf{x}, \omega) D^*(\mathbf{x}^s; \mathbf{x}, \omega). \tag{6}$$

This will, however, not give the correct amplitudes.

Least-squares imaging conditions

The signal model in equation (1) represents the integrand in the single-scattering Born integral (Chapman 2004). Lailly (1983) showed that the gradient of the single-scattering integral with respect to the medium parameter perturbation can be computed by reverse-time migration. In our formulation, this corresponds to the cross-correlation function in equation (6). In least-squares estimation of the medium parameter perturbations, the main problem is the computation and inversion of a very large Hessian matrix. Shin *et al* (2001) proposed to use an approximate Hessian consisting only of its diagonal terms. The resulting parameter estimate is given in their equation (7) which in our formulation becomes

$$R_{\mathrm{LS}}(\mathbf{x}) = \frac{\sum_{x^s}\sum_{\omega} U(\mathbf{x}^s; \mathbf{x}, \omega) D^*(\mathbf{x}^s; \mathbf{x}, \omega)}{\sum_{x^s}\sum_{\omega}|D(\mathbf{x}^s; \mathbf{x}, \omega)|^2} = \frac{R_C(\mathbf{x})}{I(\mathbf{x})}, \tag{7}$$

where $R_C(\mathbf{x})$ is the cross-correlation (6) between the wave fields, and

$$I(\mathbf{x}) = \sum_{\mathbf{x}^s}\sum_{\omega}|D(\mathbf{x}^s; \mathbf{x}, \omega)|^2 \tag{8}$$

is the illumination or total energy from the sources at the image point. In the appendix, we show that this is also a local least-squares estimate of the reflectivity function given in equation (1).

Although the least-squares imaging condition (7) is more stable than the deconvolution imaging condition in equation (2) , some stabilizing is needed in poorly illuminated zones. This may be done using a damped least-squares procedure (Lines and Treitel 1984) which gives

$$R_{\mathrm{LSD}}(\mathbf{x}) = \frac{R_C(\mathbf{x})}{I(\mathbf{x}) + V(\mathbf{x})}, \tag{9}$$

where again V is a constant or a slowly varying function of space (Schleicher *et al* 2008).

However, they advocate to use the imaging condition

$$R_{AS}(\mathbf{x}) = \frac{R_C(\mathbf{x})}{\langle I(\mathbf{x})\rangle}, \tag{10}$$

where $\langle\rangle$ denotes a spatial smoothing operation (not specified in their paper).

Here we use a data-adaptive approach by first computing, for each source and frequency, the average of the downgoing energy at a depth z:

$$|D(\mathbf{x}^s; z, \omega)|^2_{AV} = \frac{1}{N_x}\frac{1}{N_y}\sum_x\sum_y |D(\mathbf{x}^s; x, y, z, \omega)|^2. \tag{11}$$

Here N_x and N_y are the numbers of image points in the x- and y-direction, respectively.

Then we replace the downgoing energy flux in equation (7) by a stabilized value

$$|D(\mathbf{x}^s; \mathbf{x}, \omega)|^2_{ST} = \begin{cases} |D(\mathbf{x}^s; \mathbf{x}, \omega)|^2, & \text{if} \quad |D(\mathbf{x}^s; \mathbf{x}, \omega)|^2 > \epsilon |D(\mathbf{x}^s; z, \omega)|^2_{AV}, \\ \epsilon |D(\mathbf{x}^s; z, \omega)|^2_{AV}, & \text{if} \quad |D(\mathbf{x}^s; \mathbf{x}, \omega)|^2 \leqslant \epsilon |D(\mathbf{x}^s; z, \omega)|^2_{AV}. \end{cases} \tag{12}$$

This gives a stabilized value for the illumination function

$$I(\mathbf{x})_{ST} = \sum_{\mathbf{x}^s}\sum_{\omega} |D(\mathbf{x}^s; \mathbf{x}, \omega)|^2_{ST}. \tag{13}$$

The result is an adaptive stabilized least-squares imaging condition:

$$R_{LSA}(\mathbf{x}) = \frac{R_C(\mathbf{x})}{I_{ST}(\mathbf{x})}. \tag{14}$$

Migration algorithm

To test the imaging conditions described in this paper, we used the phase–shift plus interpolation (PSPI) migration method (Gazdag and Sguazzero 1984). This migration method is based on the one-way wave equation solution and has been used as a powerful tool for imaging complex structures with less computation cost than reverse time migration. The PSPI migration technique is applied in the mixed domain and we used ten reference velocities, which were selected on the basis of the maximum entropy criterion proposed by Bagaini and Pieroni (1995). All the results presented here for different imaging conditions were generated with the same PSPI method.

Numerical test

In practice, the application of different imaging conditions produces different migrated images. We have compared seismic images produced with three different imaging conditions. The cross-correlation imaging condition in equation (6), the least-squares imaging condition in equation (7) and the stabilized least-squares imaging condition using equation (14) with $\epsilon = 1.0$ in equation (12). Figures 1, 2 and 3 show the three images obtained for the Marmousi synthetic data set. Figures 1 and 2 correspond to the migrated images obtained through the correlation and least-squares imaging conditions, respectively. In figure 2, using the least-squares imaging condition a significant amount of high amplitude noise is seen. This is most serious in the shallow part of the image. Using the stabilized least-squares imaging condition, figure 3 shows a better depth image, and most of the instabilities due to poor illumination are avoided.

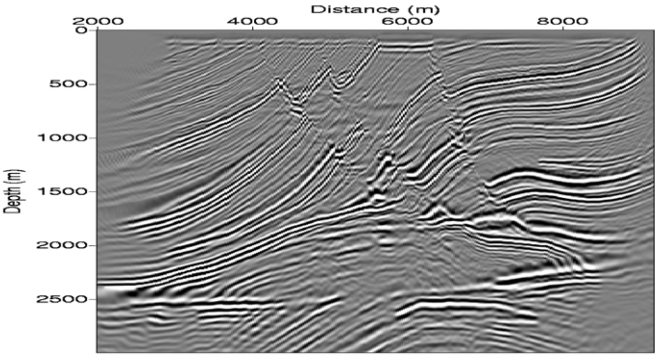

Figure 1. Migrated image of the Marmousi data set using the correlation imaging condition.

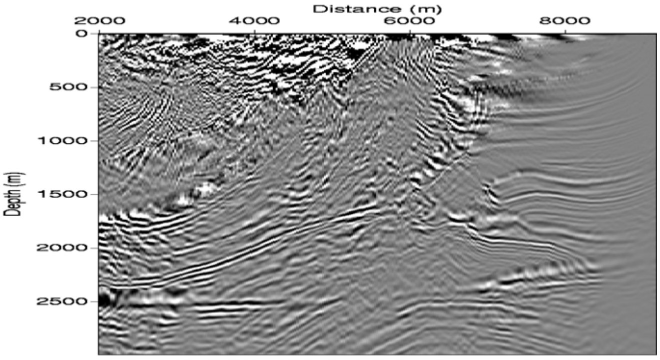

Figure 2. Migrated image of the Marmousi data set using the least-squares imaging condition.

Figures 4, 5 and 6 show the three images obtained for the Sigsbee2A synthetic data set distributed by SMAART JV. Figures 4 and 5 show the migrated images obtained through the correlation and least-squares imaging conditions, respectively. In figure 5, the amplitudes in the deepest reflectors below the salt body are better recovered than in the imaging condition using correlation (figure 4). However, instabilities show up in the regions of defocused energy and on the surface. These latter ones are associated with the poor source directivity of the one-way wave equations. Figure 6 shows the migrated image obtained using the stabilized least-squares imaging condition. The amplitudes of the deepest reflectors are improved in the poorly illuminated zones, and the noise presented in figure 5 is attenuated.

Conclusions

We have presented with success a criterion to avoid instability problems in the deconvolution-type imaging condition applied to the least-squares imaging condition. The result is a new data-adaptive stabilized least-square imaging condition where the energy of the downgoing wave field is replaced by its

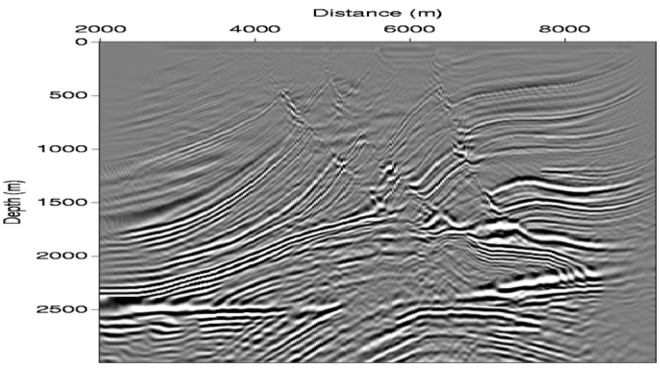

Figure 3. Migrated image of the Marmousi data set using the stabilized least-squares imaging condition.

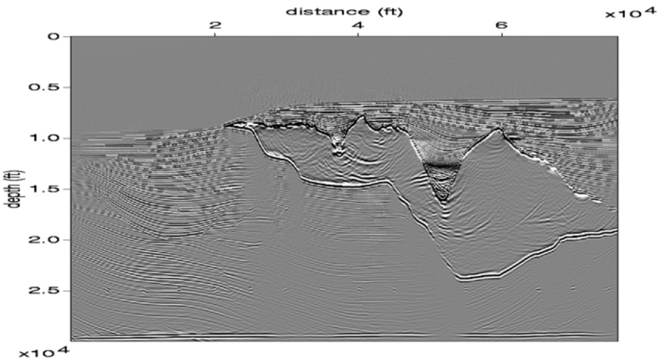

Figure 4. Migrated image of the Sigsbee2A data set using the correlation imaging condition.

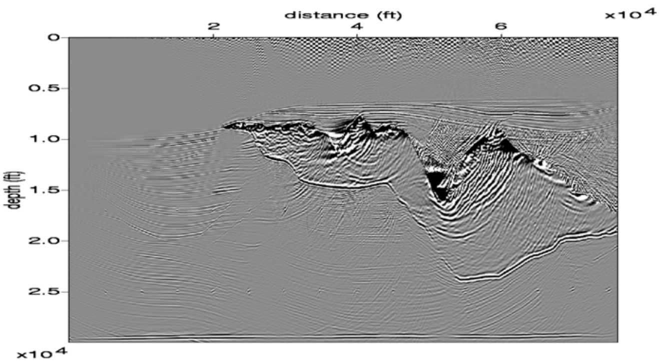

Figure 5. Migrated image of the Sigsbee2A data set using the least-squares imaging condition.

transverse average value in regions of poor illumination. It was compared to the cross-correlation imaging condition and the standard least-squares imaging condition for the Marmousi and Sigsbee2A synthetic data sets. The results show that the images produced with the new imaging condition have less noise and improved amplitudes as compared with the other imaging conditions.

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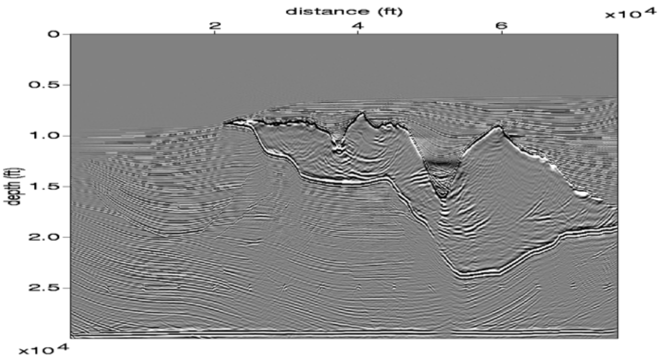

Figure 6. Migrated image of the Sigsbee2A data set using the stabilized least-squares imaging condition.

Appendix. Signal estimation: least-squares versus division

We consider the very simple signal-plus-noise method of discrete data

$$d_k = a_k R + n_k, \qquad k = 1, \ldots, K, \tag{A.1}$$

where the unknown parameter is R. The amplitude coefficients a_k are known, and the noise terms n_k are assumed to be independent with zero mean and variance σ^2. The least-squares estimate is

$$\hat{R}_{\mathrm{LS}} = \frac{\sum_k d_k a_k}{\sum_k a_k^2}, \tag{A.2}$$

where the summation (also in the following) is from $k = 1$ to $k = K$.

An alternative estimate, which is commonly used in migration, is obtained by division:

$$\hat{R}_D = \frac{1}{K} \sum_k \frac{d_k}{a_k}. \tag{A.3}$$

With the assumption about the noise, it is easily seen that both estimates are unbiased, so that

$$E\{\hat{R}_{\mathrm{LS}}\} = E\{\hat{R}_D\} = R, \tag{A.4}$$

where $E\{\}$ denotes the expectation operator. The variances of the two estimates in equations (A.2) and (A.3) are

$$\sigma_{\mathrm{LS}}^2 = E\{(\hat{R}_{\mathrm{LS}} - R)^2\} = \frac{\sigma^2}{\sum_k a_k^2} \tag{A.5}$$

and

$$\sigma_D^2 = E\{(\hat{R}_D - R)^2\} = \frac{\sigma^2}{K^2} \sum_k \frac{1}{a_k^2}. \tag{A.6}$$

We want to show that $\sigma_{\mathrm{LS}}^2 \leqslant \sigma_D^2$. This implies that

$$\frac{1}{\sum_k a_k^2} \leqslant \frac{1}{K^2} \sum_j \frac{1}{a_j^2} \tag{A.7}$$

or

$$\sum_k a_k^2 \sum_j \frac{1}{a_j^2} \geqslant K^2. \tag{A.8}$$

In the expression on the left-hand side, there are K^2 terms, both sums are from index 1 to K. For $k = j$ there are K terms equal to 1, and for $k \neq j$ there are two terms

$$\frac{a_k^2}{a_j^2} + \frac{a_j^2}{a_k^2} \geqslant 2 \tag{A.9}$$

since

$$\left(a_k^2 - a_j^2\right)^2 \geqslant 0. \tag{A.10}$$

Thus there are $K(K-1)/2$ terms greater than or equal to 2 and K terms equal to 1, so that the K^2 terms must be greater than or equal to K^2. It is seen that equality only occurs if $a_k = 1$ for $k = 1, \ldots, K$, and in that case the two estimates are identical and equal to the arithmetic mean of the data. It is well known that, under fairly general conditions, the least-squares estimate is also a minimum-variance estimate (Tarantola 1987).

A large signal (large value of a_k) is given a large weight in the least-squares estimate in equation (A.2), while it is given a small weight in the division estimate in equation (A.3). The opposite is the case for a small signal (small value of a_k). This will give a large contribution to the variance of the division estimate, because much noise is being added to this estimate from terms with small amplitudes. Therefore, the least-squares estimate is more stable than the estimate by division.

Note that in the Fourier domain, the data and the amplitudes are complex, and the least-squares estimate is

$$\hat{R}_{\mathrm{LS}} = \frac{\sum_k d_k a_k^*}{\sum_k |a_k|^2}, \tag{A.11}$$

where $*$ denotes the complex conjugate.

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