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PHYSICS LETTERS A

Physics Letters A 315 (2003) 213–218

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Generation of the reciprocal-binomial state for optical fields

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Received 15 April 2003; received in revised form 26 June 2003; accepted 27 June 2003

Communicated by P.R. Holland

Abstract

We compare the efficiencies of two interesting schemes to generate truncated states of the light field in running modes, namely the "quantum scissors" and the "beam-splitter array" schemes. The latter is applied to create the reciprocal-binomial state as a travelling wave, required to implement recent experimental proposals of phase-distribution determination and of quantum lithography.

PACS: 42.50.Dv

Keywords: Quantum states engineering; Reciprocal binomial state

Generation of states of the quantized light field (or atomic oscillators), named "quantum states engineering" (QSE), turned out to be a very important topic of Quantum Optics [1] and Atomic Physics [2] in recent years. The issue has interesting potential applications, as in teletransport [3], quantum computers [4], quantum cryptography [5], quantum lithography [6], etc. Besides these practical applications it is also relevant in fundamental aspects of quantum mechanics, as generation of entangled states [7] and Schrödinger's cat states [8], and investigation of decoherence of mesoscopic superpositions [9]. Recently, great advances have been achieved in engineering quantum states of the electromagnetic field both in cavities and as travelling waves; the latter are essential for transmitting information. It is worth mentioning that even apparently exotic states may become very important in the determination of certain properties of a given system. To give some examples, we cite the *reciprocal binomial state* (RBS) for running modes, decisive for the experimental determination of the phase-distribution $P(\theta)$ of an arbitrary state [10,11] and quantum lithography [12]. A similar role is played by the "polynomial state", crucial for the experimental determination of the Husimi Q-function [13].

In this Letter we concentrate in QSE for running modes of the radiation field. We compare the efficiencies of two procedures recently presented in the literature, namely the "quantum scissors" (QS) [14] and the "beam-splitter array" (BSA) [15] schemes, to gener-

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0375-9601/$ – see front matter © 2003 Published by Elsevier B.V.
doi:10.1016/S0375-9601(03)01049-1

ate quantum states of travelling fields. We then use the BSA scheme to propose a way of producing the RBS in a travelling-field mode. Generation of the RBS for stationary waves, trapped inside a high-Q cavity, was discussed in Ref. [13].

The QS method appeared [14], initially, as a proposal for preparation of an arbitrary running-wave superposition of the vacuum and the one-photon state, $|\Psi_1\rangle = C_0|0\rangle + C_1|1\rangle$. In this scheme, a travelling field would be available for further applications, as auxiliary to determine properties of other field states describing a system. The scheme is able to achieve the above mentioned superposition by a physical truncation of the photon number superposition making up a coherent state. The proposal requires no additional extension of current experiments and is reasonably insensitive to photodetection efficiency for the fields most likely to be used in practice. Since the scheme produces a truncation of the Hilbert space, it has been called "quantum scissors" device. As mentioned in [14], states with higher photon number might be constructed by superposing fields prepared as superpositions of zero and one photon number states.

The QS scheme was extended [16] to the case of generating superpositions of the first N number states, $|\Psi_N\rangle = \sum_{n=0}^{N} C_n|n\rangle$. To recover the situation considered in [16], we assume that the quantum state to be generated is a finite superposition of equally weighted Fock states: $|\Psi_N\rangle_a \sim |0\rangle + |1\rangle + \cdots + |N\rangle$. We will assume the scheme sketched in Fig. 1, with the input state entering the beam splitter BS1 given by $|\Psi_{\text{in}}\rangle_{ab} = |1\rangle_a|N-1\rangle_b$. In this case, we have the output of BS1, $|\Psi_{\text{out}}\rangle_{ab} = \widehat{R}_{ab}|\Psi_{\text{in}}\rangle_{ab}$, where $\widehat{R}_{ab}$ is the unitary operator

$$\widehat{R}_{ab} = \exp\left[i\theta_1(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger)\right] \tag{1}$$

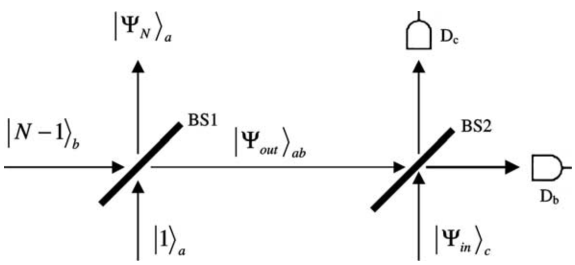

Fig. 1. Experimental setup of the generalized QS scheme to generate the state $|0\rangle + |1\rangle + \cdots + |N\rangle$ by projection synthesis.

and $\theta_j = \tan^{-1}(r_j/t_j)$, with $t_j = \cos(\theta_j)$, $r_j = \sin(\theta_j)$ standing, respectively, for the reflection and transmission coefficients of beam splitters BSj, $j = 1, 2$. Next, the input of BS2, $|\Psi_{\text{in}}\rangle_{bc} = |\Psi_{\text{out}}\rangle_{ab}|\Psi_{\text{in}}\rangle_c$, with $|\Psi_{\text{in}}\rangle_c = \sum_{n=0}^{\infty}\gamma_n|n\rangle$; consequently, the output state emerging from the BS2 reads $|\Psi_{\text{out}}\rangle_{abc} = \widehat{R}_{bc}(|\Psi_{\text{out}}\rangle_{ab}|\Psi_{\text{in}}\rangle_c)$. By measuring the field mode **b** in the state $|1\rangle_b$ and the field mode **c** in the state $|N-1\rangle_c$, we synthesize the projection of the field mode **a** in the desired superposition.

Once we have specified the truncated state to be prepared, this implies a system of N equations. The solution of such a system can be guaranteed if the number of equations is not greater than the number of free variables. Therefore, we stress that for the present generalized quantum scissors, θ_1 and θ_2 are free parameters to be adjusted for the achievement of the desired state. When $N > 3$ we must introduce $N - 3$ new parameters to permit solubility of a system of coupled equations. This goal is attained by substituting the auxiliary coherent state field $|\Psi_{\text{in}}\rangle_c$, entering the BS2 by a (convenient) discrete superposition of coherent states. So, we may write $|\Psi_{\text{in}}\rangle_c = \mathcal{N}\sum_n \gamma_n|n\rangle_c$ with

$$\gamma_n = \exp\left(-\frac{|\alpha|^2}{2}\right)\frac{|\alpha|^n}{\sqrt{n!}}\lambda_N, \tag{2}$$

where

$$\lambda_N = \begin{cases} 1, & \text{if } N \leqslant 3, \\ \sum_{m=4}^{N}\cos(n\phi_m), & \text{if } N > 3 \end{cases} \tag{3}$$

and the ϕ_m are the additional parameters to be determined for generation of the desired state.

In this scheme the probability P to produce a state $|\Psi_N\rangle_a$ is determined by the requirement that the detectors D_b and D_c register 1 and $N-1$ photons, respectively. So, it is given by

$$P = \left|{}_b\langle 1|_c\langle N-1||\Psi_{\text{out}}\rangle_{abc}\right|^2, \tag{4}$$

with

$$\begin{aligned} {}_b\langle 1|_c\langle N-1||\Psi_{\text{out}}\rangle_{abc} &= (it_1r_2)^N\sum_{n=0}^{N-1}\binom{N}{k} \\ &\quad\times\left(\frac{t_2r_1}{t_1r_2}\right)^n\left[A\gamma_{n+1}|n+1\rangle_a + B\,\gamma_n|n\rangle_a\right], \end{aligned} \tag{5}$$

where

$$A = n + 1 - (N - n - 1)\left(\frac{t_2}{r_2}\right)^2, \tag{6}$$

$$B = -n\frac{r_1 r_2}{t_1 t_2} + (N - n)\frac{t_2 r_1}{t_1 r_2}, \tag{7}$$

r_i, t_i being the reflection and the transmission coefficients of ith beam splitter.

The second representative scheme of QSE to generate states of a light field in a running-wave, involving an array of beam splitters (the BSA scheme), was introduced in Ref. [15]. In this method, a state $|\Psi\rangle$ is approximated by (for N sufficiently large) the truncated state

$$|\Psi\rangle \simeq \sum_{n=0}^{N} \Psi_n |n\rangle = \sum_{n=0}^{N} \frac{\Psi_n (\hat{a}^\dagger)^n}{\sqrt{n!}} |0\rangle, \tag{8}$$

with given Ψ_n. For convenience, this state can also be rewritten in the form

$$|\Psi\rangle = \frac{\Psi_N}{\sqrt{N!}} \prod_{i=1}^{N} (\hat{a}^\dagger - \beta_i^*)|0\rangle, \tag{9}$$

where the β_i^* are the roots of the polynomial equation

$$\sum_{n=0}^{N} \frac{\Psi_n}{\sqrt{n!}} (\beta^*)^n = 0. \tag{10}$$

Next, Eq. (9) is interesting for manipulation in the set of beam-splitter of Fig. 2. Using the well-known relation

$$\hat{a}^\dagger - \beta^* = \widehat{D}(\beta)\hat{a}^\dagger \widehat{D}^\dagger(\beta), \tag{11}$$

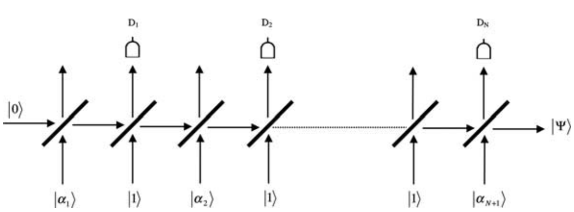

Fig. 2. Experimental setup of the BSA scheme. The first beam splitter, with a high transmittance $\widetilde{T} \sim 1$, produces the state $D(\alpha_1)|0\rangle$. The second one, where a photon is added, has transmittance T and has a photon detector aligned with its vertical output. This set is repeated N times.

where $\widehat{D}(\beta)$ stands for the displacement operator, the substitution of (11) in (9) gives

$$|\Psi\rangle = \frac{\Psi_N}{\sqrt{N!}} \prod_{i=1}^{N} \left[\widehat{D}(\beta_N)\hat{a}^\dagger \widehat{D}^\dagger(\beta_N)\right]|0\rangle. \tag{12}$$

Now, assuming zero detection in the detectors $D_1, D_2, \ldots, D_N$ and $|\Psi_1\rangle = \widehat{D}(\alpha_1)|0\rangle$, we obtain, step-by-step: $|\Psi_2\rangle = \hat{a}^\dagger T^{\hat{n}}|\Psi_1\rangle = \hat{a}^\dagger T^{\hat{n}}\widehat{D}(\alpha_1)|0\rangle$, $|\Psi_3\rangle = \widehat{D}(\alpha_2)(\hat{a}^\dagger T^{\hat{n}}\widehat{D}(\alpha_1)|0\rangle)$ etc., and

$$|\Psi_N\rangle \sim \left[\widehat{D}(\alpha_N)(\hat{a}^\dagger T^{\hat{n}}\widehat{D}(\alpha_{N-1})\right]\cdots[\widehat{D}(\alpha_1)]|0\rangle. \tag{13}$$

Of course, Eq. (13) differs from Eq. (12). However, they can be connected using successive action of each pair of neighboring beam-splitters, as follows

$$\begin{aligned}&\left[\widehat{D}^\dagger(\alpha)T^{\hat{n}}\hat{D}(\alpha)\right]\hat{a}^\dagger\\ &\quad = T\widehat{D}^\dagger(\overline{T}^*\alpha)\hat{a}^\dagger\widehat{D}(\overline{T}^*\alpha)\left[\widehat{D}^\dagger(\alpha)T^{\hat{n}}\widehat{D}(\alpha)\right]\end{aligned} \tag{14}$$

where $\overline{T} = 1 - T^{-1}$ (T and R denote the transmittance and the reflectance of the beam splitters where a photon is added). Next, after using Eq. (14) in Eq. (13) and comparing the result with Eq. (12), one shows that they become identical when

$$\alpha_1 = -\sum_{l=1}^{N} T^{-l}\alpha_{l+1}, \tag{15}$$

$$\alpha_k = T^{*N-k+1}(\beta_{k-1} - \beta_k), \quad k = 2, 3, 4, \ldots, N, \tag{16}$$

yielding the experimental parameters α_k.

The probability $P(\Psi)$ to produce our desired state $|\Psi\rangle$ is given by [15]

$$P(\Psi) = \left\|\widehat{Y}\widehat{D}(\alpha_N)\widehat{Y}\widehat{D}(\alpha_{N-1})\cdots\widehat{Y}\widehat{D}(\alpha_1)|0\rangle\right\|^2, \tag{17}$$

where $\widehat{Y} = R\hat{a}^\dagger T^{\hat{n}}$. That is, probability is given by the square of the norm of the state produced when no photon is registered in each of the N conditional output measurements. After some algebra on Eq. (17) we obtain

$$\begin{aligned}P(\Psi) = {}&|R|^{2N}|T|^{N(N-1)}\left\|\prod_{m=1}^{N}\left(\hat{a}^\dagger + b_{mN}^*\right)|\xi_N\rangle\right\|^2\\ &\times \exp\left(-|R|^2\sum_{m=1}^{N}\left|\sum_{j=1}^{m}T^{m-1}\alpha_j\right|^2\right),\end{aligned} \tag{18}$$

where $b_{1N}=0$, $b_{mN}=-\sum_{j=0}^{m-2}T^{*-j-1}\alpha_{N-j}$, $m=2,3,\ldots,N$, $\xi_N=\sum_{j=1}^{k}T^{k+1-j}\,\alpha_j$ and

$$\left\|\prod_{n=1}^{N}\left(\hat{a}^\dagger+b_{mN}^*\right)|\xi_N\rangle\right\|^2=\sum_{m,l=0}^{N}B_{N,m}(0)\,B_{N,l}^*(0)\times\langle\xi_N|\hat{a}^{N-m}\,\hat{a}^{\dagger(N-l)}|\xi_N\rangle \tag{19}$$

with

$$B_{N,\,p}(0)=\left[\frac{1}{(N-p)!}\frac{d^{N-p}}{dx^{N-p}}\left(\prod_{i=1}^{N}(x+b_{iN})\right)\right]_{x=0}. \tag{20}$$

We now compare the efficiencies of the QS and the BSA schemes. In the procedure of Ref. [14] the probability P for $N=1$ is obtained as $P_1=\frac{1}{4}\sum_{n=0}^{1}|C_n|^2$, which results $P_1=1/2e\simeq 18\%$, for $\alpha=1$ and assuming an equally weighted superposition ($C_0=C_1=1/\sqrt{2}$). In the extended scheme of [16] we obtain for $\alpha=1$ and equally weighted superposition: $P_2\cong 9\%$ for $N=2$; $P_3\cong 4.9\%$ for $N=3$; and $P_4\cong 6\%$ for $N=4$. However, these probabilities are only apparent. One should correct them by taking into account the probability of having the required input state, namely: for the "quantum scissors" we might also consider the probability to have the required input state $|N-1\rangle$ in the mode $\underline{\mathbf{b}}$ and the input state $|\alpha\rangle$ (or a superposition $\sum_i|\alpha_i\rangle$) in the mode $\underline{\mathbf{c}}$. Now, the state $|N-1\rangle$ is available with probability $P\cong 20\%$ [17], whereas the state $|\alpha\rangle$ is available with maximum efficiency $P=100\%$ [18]. For $N>3$ superposition of coherent states are required in the mode $\underline{\mathbf{c}}$ [16]. In this case, the probability depends on the phase ϕ between the coherent components [18]. For equally weighted superpositions as employed in [16], $\phi\simeq 8.8$; hence we obtain, using [18], $P\simeq 50\%$. Next, multiplying the previous values by the foregoing corrections we obtain the resulting probabilities, which are shown in the second column of Table 1.

Next, consider the scheme, proposed in Ref. [15], using the beam-splitter array depicted in Fig. 2. The detectors are also assumed ideals. Note that for $N=1$ the difference between the arrangements of QS and BSA procedures relies on the detection methods. In the BSA scheme, additional beam splitters are required when $N\geqslant 2$. Although no applications for equally weighted superpositions were considered in [15], a straightforward application of its scheme furnishes the efficiencies for generation of the equally weighted superpositions: $|\Psi_N\rangle\sim\sum_0^N|n\rangle$, for $N=2$, $N=3$ and $N=4$. In this case we obtain, taking $\alpha=1$, the values of the probabilities P' presented in the third column of Table 1.

Table 1
Probabilities for generating the state $\sum_0^N|n\rangle$ with QS (P_N) and BSA (P'_N) schemes

N	P_N (%)	P'_N (%)
1	18	40
2	1.8	14
3	0.9	5
4	0.6	1.5

We now address the question of generating the RBS in travelling modes. The RBS is a truncated state defined by [19]

$$|\mathrm{RBS}\rangle=C\sum_{k=0}^{N}\binom{N}{k}^{1/2}\exp\left[ik\left(\phi-\frac{\pi}{2}\right)\right]|k\rangle, \tag{21}$$

where $\binom{N}{k}$ stands for the binomial coefficient and C is a normalization constant. We shall investigate its generation using the BSA scheme. As one can see from Table 1, this scheme gives better probabilities for generation of equal weighted superpositions of the first number states, suggesting that the same might happen for the generation of other truncated states. In this case, the coefficients Ψ_n in Eq. (8) are identified with the coefficients $C\binom{N}{k}^{1/2}\exp[ik(\phi-\frac{\pi}{2})]$ appearing in Eq. (21), the characteristic equation (10) is solved for β_i and the experimental parameters α_k are determined.

The probability for generate the RBS depends on the transmittance T of the beam splitters. To be specific, consider the RBS with $N=5$ and $\phi=\pi$. For this case, the profile of the probability as a function of T is shown in Fig. 3; we see that the maximum probability, $P\simeq 0.25\%$, occurs when $T\simeq 0.87$. Taking this value for T, we find the values of the parameters corresponding to the generation of the RBS (with $N=5$ and $\phi=\pi$) presented in Table 2.

The relevance of producing a RBS in travelling modes appears in direct connection with some recent developments. For example, the determination of the phase distribution $P(\theta)$ of an arbitrary field state, using a simple experimental scheme [10], requires

Table 2
Values of parameters $\beta_k = |\beta_k| \exp(i\varphi_k)$ and $\alpha_k = |\alpha_k| \exp(i\theta_k)$ to generate the RBS (for $N = 5$ and $\phi = \pi$) with the BSA scheme

k	$\lvert\beta_k\rvert$	φ_k	$\lvert\alpha_k\rvert$	θ_k
1	1.00	−3.11	0.03	1.25
2	1.00	−1.93	0.64	2.19
3	1.00	1.54	1.30	−1.76
4	1.00	−0.78	1.39	1.95
5	1.00	0.36	0.94	−1.78
6			1.00	0.36

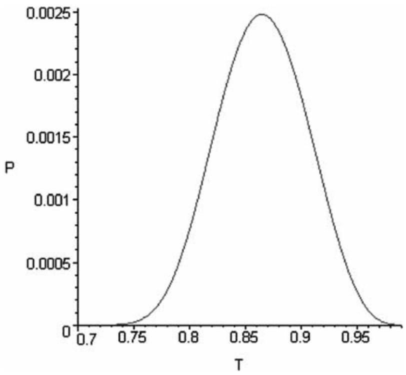

Fig. 3. Probability distribution, as a function of the transmittance T of the beam splitters, to creating the RBS with the BSA scheme.

the application of a travelling wave prepared in the RBS. The same requirement was found to be crucial for implementing quantum lithography in the scheme introduced in Ref. [12].

In both QS and BSA procedures described above, and in the comparison between them, we have assumed that beam splitters and photon detectors were ideal. While good beam splitters are available in laboratories the same does not occur for photon detectors. So, to improve our comparison concerning with these schemes, we will consider only the influence of non-ideal detectors which is dominant upon that coming from non-ideal beam-splitters.

Non-unit efficiency of a photodetector leads to a photon-count which is related to the ideal (efficiency $\eta = 1$) pre-measured photon distribution by a Bernoulli transformation [20]. Accordingly, the probability $p_n(\eta)$ to detect n-photons using a non-ideal photodetector ($\eta < 1$) is given in terms of the probability $p_m(\eta = 1)$ (using an ideal one) by

$$p_n(\eta) = \sum_{m=n}^{\infty} \binom{m}{n} \eta^n (1-\eta)^{m-n} p_m(1), \tag{22}$$

which reduces, for the particular case of zero-count, to

$$p_0(\eta) = \sum_{m=0}^{\infty} (1-\eta)^m p_m(1). \tag{23}$$

We see from Eq. (23) that $p_0(\eta) > p_0(1)$ which naively implies the surprising conclusion that the efficiency of a generation scheme based on 0-photon detection (like the BSA procedure) would be improved by using non-ideal detectors. Although this is not truly the case, the fact that such a relation between the photon-count and the photon distributions does not hold for all n suggests that the QS scheme (which involves detection of 1 and $N-1$ photons) is still worse than the BSA procedure when non-unit efficiency photodetectors are considered. This reinforces the previous conclusion on the comparison of these schemes. If we concentrate on the BSA scheme, considering a sequence of independent photodetectors, in general grounds the corrected probability $P_{|\Psi\rangle}(\eta)$ would be given by

$$P_{|\Psi\rangle}(\eta) = \left(\prod_{i=1}^{N} \frac{p_0^{(i)}(\eta_i)}{p_0^{(i)}(1)} \right) P_{|\Psi\rangle}(\mathbf{1}), \tag{24}$$

where $p_0^{(i)}(\eta_i)$ and $p_0^{(i)}(1)$ stand for the photon-count and the photon incoming distribution relative to the ith photodetector, respectively, and η is an abbreviation for the Ntuple $\eta_1, \ldots, \eta_N$.

However, the improvement caused by non-ideal photodetectors is only apparent, as expected. Actually such detectors lead to a mixed output state, instead of our desired pure state $|\Psi\rangle = |\text{RBS}\rangle$, this "decoherence" effect caused by losses being inherent to all generation schemes based on photon detection. Hence, instead of generating a wanted pure state, which is auxiliary in the measurement of a property of another field state, one ends up with the generation of a mixed state, this mixing increasing when the efficiency η becomes worse. Fortunately, there is a solution to the problem of detection loss provided by the inverse Bernoulli convolution (IBC), allowing one to reconstruct the pure state from the mixed state. Such reconstruction of pure states from a mixed output state via the IBC has been considered for some special examples, as Fock and Schrödinger's cat states mixtures [21,22]. In this procedure a pure state, represented by the density operator $\hat{\rho}_p = |\Psi\rangle\langle\Psi|$, is reobtained from the smeared data contained in the (mixed) density operator $\hat{\rho}_{\text{out}}(k,\eta)$ as follows

$$|\Psi\rangle\langle\Psi| = \frac{1}{p_0(\mathbf{1})} \sum_{k_1,\ldots,k_N} \big\{ b_{0;k_1}\big(\eta_1^{-1}\big) \cdots b_{0;k_N}\big(\eta_N^{-1}\big) \times p_{\mathbf{k}}(\eta)\hat{\rho}_{\text{out}}(\mathbf{k},\eta) \big\}, \quad (25)$$

where the functions $b_{0;k_i}$ are defined by

$$b_{l;m}(z) = \binom{m}{l} z^l (1-z)^{m-l} \quad (26)$$

and $\mathbf{k} = (k_1,\ldots,k_N)$ gives the number of counts obtained in the detectors.

Now, an explanation about the foregoing procedure is necessary: the IBC is no more than a mathematical approach, hence not able to experimentally correcting the mixing introduced by non-ideal detectors; reconstruction is made upon computer-data, not experimentally acting upon the running field states themselves. When applying a mixed output state in the measurement of certain property of another field state one finally ends up with some measurement data, a set of number inside a computer and not a quantum state, these numbers being the final outcome of the entire procedure. When these final data reproduce property of an arbitrary field state then it does not matter how this property has been obtained. In resume, the reconstruction of the auxiliary field state does correspond to the reconstruction of the measured property of another field state itself.

Acknowledgements

We wish to thank Dr. J. Clausen for very enlightening discussions concerning with the reconstruction method. We would also like to thank CAPES, CNPq and PRONEX (Brazilian agencies) for partial support.

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