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Fluxes of cosmic rays: a delicately balanced stationary state

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Abstract

The analysis of cosmic rays fluxes as a function of energy reveals a *knee* slightly below 10^{16} eV and an *ankle* close to 10^{19} eV. Their physical origins remain up to now quite enigmatic; in particular, no elementary process is known which occurs at energies close to 10^{16} eV. We propose a phenomenological approach along the lines of nonextensive statistical mechanics, a formalism which contains Boltzmann–Gibbs statistical mechanics as a particular case. The knee then appears as a crossover between two fractal-like thermal regimes, the crossover being caused by process occurring at energies ten million times lower than that of the knee, in the region of the quark hadron transition ($\simeq 10^9$ eV). This opens the door to an unexpected standpoint for further clarifying the phenomenon.

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Cosmic rays fascinate since long. They provide galactic, extragalactic and cosmological information, related to recent or very old events concerning various sources, going back to the early times of the universe [1]. They reflect all types of elementary process and interactions, and are associated with phenomena of very different space and time scales. The complete physical scenario still remains quite enigmatic, although a variety of specific mechanisms for acceleration and propagation have been advanced along the years for various energy regions. The most known of these mechanisms is the Fermi one [2], which addresses acceleration in magnetized turbulent plasma, but many others have been advanced [3–10] in relation with the *knee* and energies below it; for the energies beyond these, for the *ankle*, as well as for general reviews, see [11–13]).

Through various types of detectors, the flux of cosmic rays at the top of the Earth atmosphere has been measured [14] and varies from 10^4 down to 10^{-29} $[\mathrm{m}^2\,\mathrm{sr}\,\mathrm{s}\,\mathrm{GeV}]^{-1}$ for energies increasing from 10^8 up to near 10^{21} eV: see Fig. 1. This distribution (which spans 13 decades in energy and 33 decades in flux!) is *not* exponential, hence it does not correspond to Boltzmann–Gibbs (BG) statistical mechanics thermal equilibrium. Consistently, even at a phenomenological level, i.e., without specifying any concrete model or mechanism, this problem represents a challenge. This is the one we address here. We shall use a point of view based on a current generalization of

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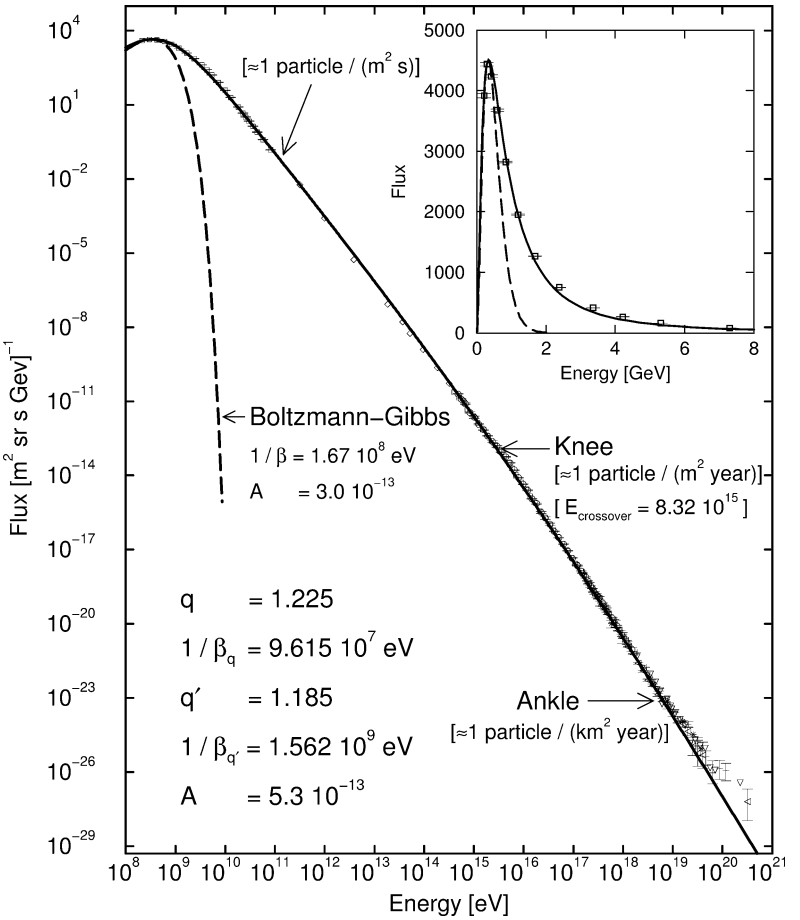

Fig. 1. Energy dependence of the fluxes of cosmic rays. Experimental error bars are indicated whenever available. The continuous curve is the one we obtain within the present phenomenological approach. The dashed curve is an optimized BG one (even at relatively low energies it fails by very many decades). The knee corresponds to $E_{\text{crossover}}$. Inset: Linear-linear representation of the low energy fluxes.

Boltzmann–Gibbs statistical mechanics, referred to as nonextensive statistical mechanics, we shall briefly describe later on. The first step will be to remark that the fluxes of cosmic rays in general, and the studies of the "knee" and the "ankle" in particular, involve phenomena such as turbulence (see, for instance, [15]), anomalous diffusion and fractality (see, for instance, [16]), self-organized criticality (see, for instance, [17]), long-range interactions (classical and quantum gravitation), among other complex phenomena (such as, for example, possible non-Markovianity [18]). It is precisely such phenomena that constitute the scope of nonextensive statistical mechanics; for turbulence and related matters see, for instance, [19–21]; for anomalous diffusion and fractality see, for instance, [22–28]; for self-organized criticality see, for instance, [29]; for long-range interactions see, for instance, [30–32].

Let us now first briefly review the usual, BG thermostatistics. If we optimize under appropriate constraints the BG $S = -k\sum_i p_i \ln p_i$ ($k \equiv$ Boltzmann constant; $\{p_i\} \equiv$ microscopic probabilities) we obtain the celebrated equilibrium distribution $p_i = \frac{e^{-\beta E_i}}{Z} \propto e^{-\beta E_i}$ ($\beta \equiv 1/kT$, $E_i \equiv$ energy of the ith state; $Z \equiv \sum_j e^{-\beta E_j} \equiv$ partition function). Excepting for the trivial normalizing factor $1/Z$, this distribution can alternatively be obtained as the solution of the linear differential equation

$$dp_i/dE_i = -\beta p_i. \tag{1}$$

In order to deal with a variety of thermodynamically anomalous systems, a more general formalism, nonextensive statistical mechanics, was introduced in 1988 [33–36]. It is based on the generalized entropic form $S_q = k(1 - \sum_i p_i^q)/(q-1)$ ($q \in \mathbb{R}$ and $S_1 = S$). Its optimization under appropriate constraints yields [37] a power-law, $p_i \propto [1-(1-q)\beta_q E_i]^{\frac{1}{1-q}} \equiv e_q^{-\beta_q E_i}$ (definition), which recovers the BG weight for $q = 1$ ($\beta_1 \equiv \beta$). As usual, $kT_q \equiv 1/\beta_q$ characterizes the conveniently averaged energy. This anomalous equilibrium-like distribution can be alternatively obtained (excepting for the normalizing factor) by solving the nonlinear differential equation

$$dp_i/dE_i = -\beta_q p_i^q. \tag{2}$$

This generalized weight naturally emerges in ubiquitous problems such as fully developed turbulence [19–21] (which is relevant for the Fermi mechanism, and most probably for others as well), electron–positron annihilation [18], motion of *Hydra viridissima* [38], long-range many-body Hamiltonians [31], among many others.

We now use the above differential equation path in order to further generalize the anomalous equilibrium distribution, in such a way as to have a crossover from anomalous ($q \neq 1$) to normal ($q = 1$) thermostatistics, while increasing the energy. We consider then the differential equation

$$dp_i/dE_i = -\beta_1 p_i - (\beta_q - \beta_1) p_i^q, \tag{3}$$

whose solution is $p_i \propto [1 - \frac{\beta_q}{\beta_1} + \frac{\beta_q}{\beta_1} e^{(q-1)\beta_1 E_i}]^{-\frac{1}{q-1}}$. The crossover typically occurs for $q > 1$ and $\beta_1 \ll \beta_q$, the distribution being anomalous at low energies and BG at high energies. It is undoubtedly interesting to notice that this differential equation precisely coincides, for $q = 2$, with the heuristic one that in 1900 led Planck to the discovery of the blackbody radiation law and ultimately to quantum mechanics [39].

Finally, by doing one more step along the same direction, we can further generalize the differential equation, now becoming

$$dp_i/dE_i = -\beta_{q'} p_i^{q'} - (\beta_q - \beta_{q'}) p_i^q. \tag{4}$$

This manner of writing the coefficient of the p_i^q term has the advantage of recovering the simplest generalization of the BG distribution by considering $\beta_{q'} = \beta_q$ or $\beta_{q'} = 0$ or even $q' = q$. For $1 < q' < q$ and $\beta_{q'} \ll \beta_q$, a crossover occurs at

$$E_{\text{crossover}} = \big[(q-1)\beta_q\big]^{\frac{q'-1}{q-q'}} \big/ \big[\big(q'-1\big)\beta_{q'}\big]^{\frac{q-1}{q-q'}}. \tag{5}$$

For $E \ll E_{\text{crossover}}$ we have an anomalous distribution characterized by (q, β_q) (namely, $p_i \propto e_q^{-\beta_q E_i}$), whereas for $E \gg E_{\text{crossover}}$ we have a different anomalous distribution characterized by $(q', \beta_{q'})$ (namely, $p_i \propto e_{q'}^{-\beta_{q'} E_i}$). The exact solution of the above differential equation (the most general one considered here) is given by $p_i \propto f(E_i)$ where $f^{-1}(x)$ is an explicit monotonic function of x involving hypergeometric functions (see Ref. [40] for details). Interestingly enough, this precise solution arrives in the discussion of the re-association of CO molecules in myoglobin [40], where time plays a role very analogous to the one played by energy in our cosmic rays problem. This time-energy analogy is not surprising after all if we take into account that, in the history of the universe after the Big Bang, the time scale reflects the energy scale, as discussed in detail in Ref. [41].

The flux $\Phi(E)$ can be obtained straightforwardly from $p_i \propto f(E_i)$ by calculating the density of states $\omega(E)$. In the ultrarelativistic limit $E \propto |\mathbf{p}|$ ($\mathbf{p}$ being the momentum), which we adopt here for simplicity given the high values of the involved energies, the density of states of an ideal gas in three dimensions is given by $\omega(E) \propto E^2$, hence, $\Phi(E) = AE^2 f(E)$, where A is a normalizing factor (and where red shift effects have been neglected). With this expression we fit the observational data and obtain the results displayed in Fig. 1. As we can see, the agreement is quite remarkable.

Our summarizing comments are:

(i) The high quality agreement over so many decades, including crossovers between different regimes, suggests that the phenomenological approach is correct, and specific models clarifying the various physical mechanisms that are involved should essentially satisfy it;

(ii) The deep explanation of the knee might well be found at energies extremely lower (ten million times lower, in fact), basically at energies related to

the characteristic temperatures obtained from the fitting, namely 9.615×10^7 eV (energy comparable to the pion mass) and 1.562×10^9 eV (energy corresponding to the quark–hadron transition [41], as well as to the proton mass), and given the fact that q and q' differ by only 3% (which in log-log representation asymptotically determines two almost parallel straight lines which very slowly approach to each other and intersect at $E = E_{\text{crossover}}$). The existence of two thermostatistical regimes, respectively related to (q, β_q) and $(q', \beta_{q'})$, could correspond to two different mechanisms of acceleration/propagation, for instance, related to galactic and extra-galactic contributions. It is worthy stressing at this point that the location of the knee emerges here through Eq. (5), i.e., using only the basic four phenomenological parameters $(q, \beta_q, q', \beta_{q'})$, and not by introducing an extra parameter on top of the previous ones (whose role would be to fix the knee at the right value);

(iii) Since the entropic index q is known to reflect (multi) fractality [42,43], the present results strongly suggest that either the generation or the transport (or both) of cosmic rays occur in scale invariant media, which is consisten with Ref. [16];

(iv) With the better observational statistics (i.e., with refined precision) at these very high energies, expected from the Pierre Auger Observatory (or from similar projects), it might happen that the ankle disappears; if it does not, then it is probably associated with strongly nonstationary phenomena (perhaps related to memory effects from early cosmic stages), certainly out from the present thermodynamical description—such study would eventually clarify the physical meaning, if any, of the GZK feature [44–48] (see also [49]);

(v) The Kaskade collaboration [9] has shown the relevance of the various constituents which compose cosmic rays. In order to take this into account, the present approach could be further improved by using the grand-canonical instead of the canonical ensemble used here;

(vi) The average energy $\langle E \rangle \equiv \int_0^\infty dE\, E\, \Phi(E) / \int_0^\infty dE\, \Phi(E)$ has been calculated, within the present approach, to be $\langle E \rangle \simeq 2.489\,\text{GeV}$ (2.882 GeV if we only use the q-distribution). Any connection of this value with other cosmological or astrophysical quantities is of course very welcome.

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