

Large- N transition temperature for superconducting films in a magnetic field

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We consider the N -component Ginzburg-Landau model in large N limit, the system being embedded in an external constant magnetic field and confined between two parallel planes a distance L apart from one another. On physical grounds, this corresponds to a material in the form of a film in the presence of an external magnetic field. Using techniques from dimensional and ζ -function regularization, modified by the external field and the confinement conditions, we investigate the behavior of the system as a function of the film thickness L . This behavior suggests the existence of a minimal critical thickness below which superconductivity is suppressed.

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It is usually assumed that it is a good approximation to neglect magnetic thermal fluctuations in the Ginzburg-Landau (GL) model, when applied to study type II superconductors. This problem has been investigated by a number of authors, both in its single component and in its N -component versions. An account on the state of the subject can be found, for instance, in Refs. 1–6. In particular, in Ref. 5 a large- N theory of a second-order transition for arbitrary dimension D is presented and the fixed-point effective free energy describing the transition is found. Here we investigate a confined version of the model studied in Ref. 5. We consider the vector N -component Ginzburg-Landau model in presence of an external magnetic field at leading order in $1/N$, the system being submitted to the constraint of being confined between two parallel planes a distance L apart from one another. Studies on confined field theory have been done in the literature since a long time ago. In particular, an analysis of the renormalization group in finite size geometries can be found in Ref. 7. This study is performed using a modified Matsubara formalism to take into account boundary effects on scaling laws. From a physical point of view, for $D=3$ and introducing temperature by means of the mass term in the Hamiltonian, the model studied here should correspond to a film-like material in presence of a magnetic field. We investigate the behavior of the system as a function of the separation L between the planes (the film thickness), using an extended compactification formalism in the framework of the effective potential, introduced in recent publications.^{8,9} In Ref. 10 this formalism has been employed to perform a study of the large- N β -function for superconducting films in a magnetic field. For the Ginzburg parameter $\kappa \gg 1$ (which is the case for high temperature superconductors), the Hamiltonian density of the GL model in an external magnetic field can be written in the form

$$\mathcal{H} = |(\nabla - ie\mathbf{A})\phi|^2 + m_0^2|\phi|^2 + \frac{u}{2}|\phi|^4, \quad (1)$$

where $\nabla \times \mathbf{A} = \mathbf{H}$ and $m_0^2 = \alpha(T - T_0)$, with $\alpha > 0$ and T_0 corresponding to the bulk transition temperature in absence

of external field. This Hamiltonian density describes superconductors in the extreme type II limit. In the following we assume that the external magnetic field is parallel to the z axis and that the gauge $\mathbf{A} = (0, xH, \mathbf{0})$ has been chosen. We will consider model (1) with N complex components and take the large- N limit at Nu fixed. If we consider the system in unlimited space, the field ϕ should be written in terms of the well-known Landau level basis,

$$\phi(\mathbf{r}) = \sum_{l=0}^{\infty} \int \frac{dp_y}{2\pi} \int \frac{d^{D-2}p}{(2\pi)^{D-2}} \hat{\phi}_{l,p_y,\mathbf{p}} \chi_{l,p_y,\mathbf{p}}(\mathbf{r}), \quad (2)$$

where $\chi_{l,p_y,\mathbf{p}}(\mathbf{r})$ are the Landau level eigenfunctions given with energy eigenvalues $E_l(|\mathbf{p}|) = |\mathbf{p}|^2 + (2l+1)\omega + m_0^2$ and $\omega = eH$ is the so-called cyclotron frequency. In the above equation \mathbf{p} is a $(D-2)$ -dimensional vector.

Now, let us consider the system confined between two parallel planes, normal to the z axis, a distance L apart from one another and use Cartesian coordinates $\mathbf{r} = (z, \mathbf{z})$, where \mathbf{z} is a $(D-3)$ -dimensional vector, with corresponding momenta $\mathbf{k} = (k_z, \mathbf{q})$, \mathbf{q} being a $(D-3)$ -dimensional vector in momenta space. In this case, the model is supposed to describe a superconducting material in the form of a film. Under these conditions the generating functional of the correlation functions is written as

$$\mathcal{Z} = \int \mathcal{D}\phi^* \mathcal{D}\phi \exp\left(-\int_0^L dz \int d^{D-3}\mathbf{z} \mathcal{H}(|\phi|, |\nabla\phi|)\right), \quad (3)$$

with the field $\phi(z, \mathbf{z})$ satisfying the condition of confinement along the z axis, $\phi(z=0, \mathbf{z}) = \phi(z=L, \mathbf{z})$. Then the field representation (2) should be modified and have a mixed series-integral Fourier expansion of the form

$$\begin{aligned} \phi(z, \mathbf{z}) = & \sum_{l=0}^{\infty} \sum_{n=-\infty}^{\infty} c_n \int \frac{dp_y}{2\pi} \int d^{D-3}\mathbf{q} b(\mathbf{q}) \\ & \times e^{-i\omega_n x - i\mathbf{q} \cdot \mathbf{z}} \tilde{\phi}_l(\omega_n, \mathbf{q}), \end{aligned} \quad (4)$$

where $\omega_n = 2\pi n/L$, the label l refers to the Landau levels, and the coefficients c_n and $b(\mathbf{q})$ correspond, respectively, to the Fourier series representation over z and to the Fourier integral representation over the $(D-3)$ -dimensional \mathbf{z} space. The above conditions of confinement of the z dependence of the field to a segment of length L allow us to proceed with respect to the z coordinate, in a manner analogous as it is done in the imaginary-time Matsubara formalism in field theory. The Feynman rules should be modified following the prescription,

$$\int \frac{dk_z}{2\pi} \rightarrow \frac{1}{L} \sum_{n=-\infty}^{+\infty}, \quad k_z \rightarrow \frac{2n\pi}{L} \equiv \omega_n. \quad (5)$$

We emphasize that here we are considering a Euclidean field theory in D purely spatial dimensions, we are *not* working in the framework of finite-temperature field theory. Temperature is introduced in the mass term of the Hamiltonian by means of the usual Ginzburg-Landau prescription.

We consider in the following the zero external-momenta four-point function, which is the basic object for our definition of the renormalized coupling constant. The four-point function at leading order in $1/N$ is given by the sum of all chains of single one-loop diagrams. This sum gives for the L and ω dependent four-point function at zero external momenta at the lowest Landau level approximation the formal expression

$$\Gamma_D^{(4)}(0; L, \omega) = \frac{-u}{1 + Nu\Sigma(D, L, \omega)}, \quad (6)$$

where $\Sigma(D, L, \omega)$ is the Feynman integral corresponding to the single one-loop subdiagram,

$$\Sigma(D, L, \omega) = \frac{1}{L} \sum_{n=-\infty}^{+\infty} \frac{\omega}{2\pi} \int \frac{d^{D-3}k}{(2\pi)^{D-3}} \frac{1}{[k^2 + \omega_n^2 + m^2 + \omega]^2}. \quad (7)$$

The sum over n and the integral over k can be treated using the formalism developed in Refs. 8, 9, and 11, which we resume below, adapted to the situation under study. The starting point is an expression of the form

$$U = \frac{\mu^{D-2-2s}}{(2\pi)^{2s+1}} \omega \sqrt{a} \sum_{n=-\infty}^{+\infty} \int \frac{d^{D-3}k}{(an^2 + c^2 + \mathbf{k}^2)^s}, \quad (8)$$

where we have used dimensionless quantities, $c^2 = (m^2 + \omega)/(4\pi^2\mu^2)$, $(L\mu)^2 = a^{-1}$, and μ is a mass scale. Note that our formalism makes sense only for dimensions $D \geq 3$. Using a well-known dimensional regularization formula,⁷ Eq. (8) can be written in the form

$$U = \mu^{D-2-2s} \omega \sqrt{a} f(D, s) Z_1^{c^2} \left(s - \frac{D-3}{2}; a \right), \quad (9)$$

where $f(D, s)$ is

$$f(D, s) = \frac{\pi^{D-4-2s}}{2^{2s+1}} \frac{\Gamma(s - (D-3)/2)}{\Gamma(s)}, \quad (10)$$

and $Z_1^{c^2}(s - (D-3)/2; a)$ is one of the Epstein-Hurwitz ζ -functions defined by

$$Z_K^{c^2}(u; \{a_i\}) = \sum_{n_1, \dots, n_K = -\infty}^{+\infty} (a_1 n_1^2 + \dots + a_K n_K^2 + c^2)^{-u}, \quad (11)$$

valid for $\text{Re}(u) > K/2$ [in our case $\text{Re}(s) > (D-2)/2$]. The Epstein-Hurwitz ζ -function can be extended to the whole complex s plane and we obtain, after some rather long but straightforward manipulations,

$$U = h(D, s) \omega \left[\frac{1}{4} \Gamma \left(\frac{2s-D+2}{2} \right) \left(\frac{m^2 + \omega}{2\mu^2} \right)^{(D-2-2s)/2} + \sum_{n=1}^{\infty} \left(\frac{\sqrt{m^2 + \omega}}{\mu^2 n L} \right)^{(D-2)/2-s} K_{(D-2)/2-s}(nL\sqrt{m^2 + \omega}) \right], \quad (12)$$

where

$$h(D, s) = \frac{\mu^{D-2-2s}}{2^{s+(D-2)/2} \pi^{3/2} \Gamma(s)}, \quad (13)$$

and K_ν are the Bessel functions of the third kind.

Applying formula (12) with $s=2$ to the integral $\Sigma(D, L, \omega)$, the result is that, we can write $\Sigma(D, L, \omega)$ in the form

$$\Sigma(D, L, \omega) = \omega [H(D, \omega) + G(D, L, \omega)], \quad (14)$$

where the L and ω dependent contribution $G(D, L, \omega)$ comes from the second term between brackets in Eq. (12), that is,

$$G(D, L, \omega) = \frac{1}{2^{(D+2)/2} \pi^{3/2}} \sum_{n=1}^{\infty} \left[\frac{\sqrt{m^2 + \omega}}{nL} \right]^{(D-6)/2} \times K_{(D-6)/2}(nL\sqrt{m^2 + \omega}), \quad (15)$$

and $H(D, \omega)$ is a polar parcel coming from the first term between brackets in Eq. (12),

$$H(D, \omega) \propto \Gamma \left(2 - \frac{D-2}{2} \right) \left[\frac{m^2 + \omega}{2\mu^2} \right]^{(D-6)/2}. \quad (16)$$

We see from Eq. (16) that for even dimension, $D=6$, $H(D, \omega)$ is divergent, due to the pole of the Γ function. Accordingly, this term must be subtracted to give the renormalized single bubble function $\Sigma_R(D, L, \omega)$. We get simply

$$\Sigma_R(D, L, \omega) = \omega G(D, L, \omega). \quad (17)$$

For the sake of uniformity, the term $H(D, \omega)$ is also subtracted in the case of lower dimensions D , where no poles of Γ functions are present. In these cases we perform a finite renormalization. From the properties of Bessel functions, it can be seen from Eq. (15) that for any dimension D , $G(D, L, \omega)$ satisfies the conditions,

$$\lim_{L \rightarrow \infty} G(D, L, \omega) = 0, \quad \lim_{L \rightarrow 0} G(D, L, \omega) \rightarrow \infty, \quad (18)$$

and $G(D, L) > 0$ for any values of D, L .

Taking inspiration from Eq. (6), let us define the L and ω dependent *renormalized* coupling constant $u_R(D, L, \omega)$ at the leading order in $1/N$ as

$$\Gamma_{D,R}^{(4)}(0, L, \omega) \equiv -u_R(D, L, \omega) = \frac{-u}{1 + Nu \sum_R(D, L, \omega)}. \quad (19)$$

From Eqs. (19) and (17) we can write the new L and ω dependent renormalized coupling constant

$$\begin{aligned} \omega Nu_R(D, L, \omega) &\equiv \omega \beta_R(D, L, \omega) \\ &\equiv g(D, L, \omega) = \frac{\omega \beta}{1 + \omega \beta G(D, L, \omega)}. \end{aligned} \quad (20)$$

We see from Eq. (18) that $\beta = Nu$ corresponds to the renormalized coupling constant in absence of boundaries and of external field. Note that since the coupling constant β has mass dimension of $4 - D$, then the coupling constant $g(D, L, \omega)$ has mass dimension of $6 - D$.

In order to study the critical behavior of our system, we start from the gap equation in absence of external field and boundaries, in the disordered phase,

$$\xi^{-2} = m_0^2 + \frac{V(N+2)}{N} \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 + \xi^{-2}}, \quad (21)$$

where ξ is the correlation length. In our case, in particular, in the neighborhood of the critical curve, the gap equation reduces to a (L, ω) -dependent Dyson-Schwinger equation. So, the generalization to our case of Eq. (21) in the neighborhood of criticality can be written in the form

$$\begin{aligned} \xi^{-2} &= m_0^2 + \omega + \frac{g(D, L, \omega)}{2^{(D-2)/2} \sqrt{\pi}} \frac{(N+2)}{N} \frac{1}{L} \\ &\times \sum_{n=-\infty}^{\infty} \int \frac{d^{D-3} k}{(2\pi)^{D-2}} \frac{1}{\mathbf{k}^2 + \omega_n^2 + \xi^{-2}}. \end{aligned} \quad (22)$$

In Eq. (22), we remember that $\xi^{-2} = m^2(L, \omega) + \omega$ [the pole of the propagator of ϕ in presence of a magnetic field is at $m^2(L, \omega) = -\omega$]^{1,2} and $g(D, L, \omega)$ is the renormalized (L, ω) -dependent coupling constant, which is itself a function of ξ^{-2} via the mass $m(L, \omega)$. Performing steps analogous to those leading from Eq. (8) to Eq. (12), Eq. (22) becomes

$$\begin{aligned} \xi^{-2} &= m_0^2 + \omega + \frac{g(D, L, \omega)}{2^{D/2} \pi^{3/2}} \frac{(N+2)}{N} \sum_{n=1}^{\infty} \left[\frac{\xi^{-1}}{nL} \right]^{(D-4)/2} \\ &\times K_{(D-4)/2}(nL\xi^{-1}). \end{aligned} \quad (23)$$

The coupling constant $g(D, L, \omega)$ is given by Eq. (20) with $G(D, L, \omega)$ in Eq. (15) replaced by

$$G(D, L, \omega) = \frac{1}{2^{D/2+1} \pi^{3/2}} \sum_{n=1}^{\infty} \left[\frac{\xi^{-1}}{nL} \right]^{(D-6)/2} K_{(D-6)/2}(nL\xi^{-1}). \quad (24)$$

Equations (20), (23), and (24) are, in fact, a complicated set of coupled equations involving ξ^{-2} , since $g(D, L, \omega)$ is also dependent on the L and ω inverse correlation length.

If we limit ourselves to the neighborhood of criticality, $\xi^{-2} \approx 0$, we may investigate the behavior of the system by using in both Eqs. (20) and (24) and in Eq. (23) an asymptotic formula for small values of the argument of the Bessel functions

$$K_\nu(z) \approx \frac{1}{2} \Gamma(\nu) \left(\frac{z}{2} \right)^{-\nu} \quad (z \sim 0), \quad (25)$$

which allows after some straightforward manipulations, in the large N -limit, to write Eq. (23) in the form

$$m_0^2 + \omega_c + \frac{g(D, L, \omega_c)}{8\pi^{3/2}} \Gamma\left(\frac{D}{2} - 2\right) L^{4-D} \zeta(D-4) \approx 0, \quad (26)$$

where $\zeta(D-4)$ is the Riemann ζ -function, $\zeta(D-4) = \sum_{n=1}^{\infty} (1/n^{D-4})$, defined for $D > 5$ and we have used the label c to indicate that we are in the region of criticality. Similarly, inserting Eq. (25) in Eqs. (20) and (24) $g(D, L, \omega)$ can be written for $\xi^{-2} \approx 0$ as

$$g(D, L, \omega_c) \approx \frac{\omega_c \beta}{1 + \beta \omega_c A(D, \mu) L^{6-D} \zeta(D-6)}, \quad (27)$$

where $A(D, \mu) = (1/32\pi^{3/2})\Gamma((D-6)/2)$.

We cannot obtain critical lines in dimension $D \leq 5$ by a limiting procedure from Eq. (26). For $D=3$, which corresponds to the physically interesting situation of the system confined between two parallel planes embedded in a three-dimensional Euclidean space, we can obtain critical lines, by performing an analytic continuation of $\zeta(z)$ to the values of the argument $z \leq 1$, by means of the reflection property of ζ -functions

$$\zeta(z) = \frac{1}{\Gamma(z/2)} \Gamma\left(\frac{1-z}{2}\right) \pi^{z-1/2} \zeta(1-z), \quad (28)$$

which defines a meromorphic function having only one simple pole at $z=1$. We obtain taking $m_0^2 = \alpha(T - T_0)$, for $D=3$ (the physical dimension), the critical surface

$$\alpha(T_c - T_0) + \omega_c + \frac{1}{8\pi^3} g(D=3, L, \omega_c) L \zeta(2) = 0. \quad (29)$$

Using Eqs. (27) and (28) to evaluate $g(D=3, L, \omega_c)$, the tabulated values to the several Γ and ζ functions appearing in the above formulas, we obtain

$$\alpha(T_c - T_0) + \omega_c + \frac{60\beta\omega_c L}{2880\pi + \beta\omega_c L^3} = 0, \quad (30)$$

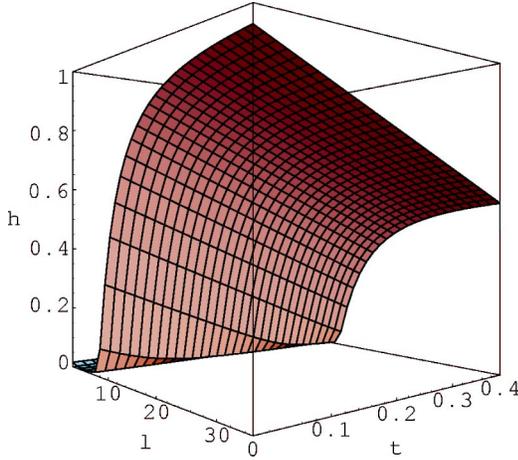


FIG. 1. Plot of the surface $h=h(l,t)$ as defined by Eq. (31), taking $B=10^4$.

α and β being the phenomenological Ginzburg-Landau parameters. Notice that as $L \rightarrow \infty$ we obtain from Eq. (30) the critical line $\alpha(T_c - T_0) + \omega_c = 0$ for the bulk.

In terms of the dimensionless quantities, respectively, reduced critical field, temperature, and film thickness, $h = \omega_c \xi_0^2$, $t = T_c/T_0$, and $l = L/\xi_0$, where $\xi_0 = (\alpha T_0)^{-1/2}$ is the zero-temperature Ginzburg-Landau coherence length, the above equation can be rewritten as

$$h(l,t) = \frac{1}{2Bl^3} \left\{ -60Bl - Btl^3 + Bl^3 - 2880\pi \left[(60Bl + Btl^3 - Bl^3 + 2880\pi)^2 + 11520\pi Bl^3(1-t) \right]^{1/2} \right\}, \quad (31)$$

where $B = \beta \xi_0$. The surface $h=h(l,t)$ is illustrated in Fig. 1. We recall that, since we have used the lowest Landau level approximation in our calculations, this surface is only meaningful for high values of the external field, that is, for low temperatures and thick films.

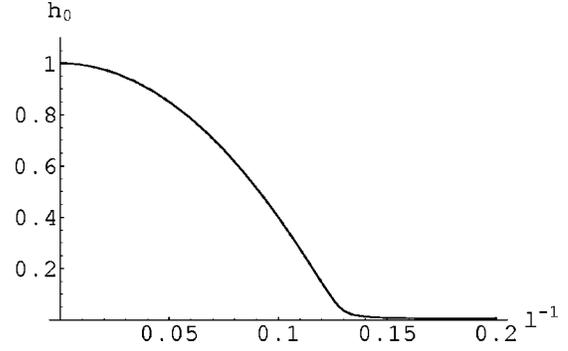


FIG. 2. Zero-temperature reduced critical field as a function of the inverse of the reduced film thickness, obtained from Eq. (31) taking $t=0$ and $B=10^4$.

We can see from Fig. 1 that each value of l defines a critical line on the $h \times t$ plane, corresponding to a film of thickness L . This set of critical lines suggests the existence of a minimal value for the thickness L below which superconductivity is suppressed. Indeed, this can be seen from the plot of the reduced critical field at zero temperature, h_0 , as a function of the inverse of the reduced film thickness, shown in Fig. 2. This behavior may be contrasted with the linear decreasing of T_c with the inverse of the film thickness in absence of external field that has been found experimentally in materials containing transition metals, for example, in Nb (Ref. 12) and in W-Re alloys;¹³ for these cases, it has been explained in terms of proximity, localization, and Coulomb-interaction effects. Notice, however, that our results do not depend on microscopic details of the material involved nor account for the influence of manufacturing aspects, like the kind of substrate on which the film is deposited. In other words, our results emerge solely as a topological effect of the compactification of the Ginzburg-Landau model in one direction.

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