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A note on the generation of displaced number states

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Abstract

We show that a recent proposal for the generation of displaced number states [G.C. de Oliveira et al., Physica A 351 (2005) 251] is misleading and cannot be implemented experimentally. We then discuss the possibility of creating highly excited displaced number states using standard procedures.

Keywords: Displaced number states; Quantum state engineering

Engineering quantum states of the electromagnetic field, both in cavities and as traveling modes, is an important step required for the implementation of several applications of quantum optics as, for example, teleportation of states, quantum communication and quantum computation. Among many interesting quantum states of an electromagnetic field mode, some attention has been given to the displaced number state (DNS), which is defined by

$$|\alpha, n\rangle \equiv \widehat{D}(\alpha)|n\rangle, \tag{1}$$

where $|n\rangle$ denotes a Fock state, α is an arbitrary complex number and $\widehat{D}(\alpha) = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a})$ is the unitary displacement operator, $\hat{a}^\dagger$ and $\hat{a}$ being the creation and the annihilation operators, respectively.

In a recent paper [1], a suggestion for generating the DNS $|\alpha, n\rangle$ was made, which would not need prior generation of the number state $|n\rangle$. In this "alternative" proposal, the generation of the state $|\alpha, n\rangle$, as a traveling field mode, would be obtained by passing the coherent state (CS) $|\alpha\rangle$ through a non-linear medium. The appeal of such a proposal is that, starting from a coherent state (easier to be produced experimentally), one would generate the state $|\alpha, n\rangle$ with one single action. Unfortunately, such a suggestion is not experimentally feasible; it comes from a misleading physical interpretation of well-known algebraic relations satisfied by the displacement operator and its implementation would contradict fundamental principles of quantum mechanics. In this note, we present a detailed criticism to the points raised in Ref. [1]. Then, besides the low excited DNS $|\alpha, 1\rangle$, which has already been produced [2], we discuss the possibility of generating highly excited DNS using standard procedures.

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Relying on the algebraic identities satisfied by the displacement operator,

$$\widehat{D}(\alpha)(\hat{a}^{\dagger})^{n} = (\hat{a}^{\dagger} - \alpha^{*})^{n}\widehat{D}(\alpha), \quad n = 0, 1, 2, 3, \ldots, \tag{2}$$

the authors of Ref. [1] conclude correctly that

$$|\alpha, n\rangle = \frac{1}{\sqrt{n!}}(\hat{a}^{\dagger} - \alpha^{*})^{n}|\alpha\rangle\,, \tag{3}$$

by simply applying (2) to the vacuum state $|0\rangle$. This well-known result, which is the basic expression analyzed in Ref. [1] (corresponding to their Eq. (5)), has been reported earlier in Ref. [3]. The argument in Ref. [1] goes as follows.

From Eq. (3), the authors of Ref. [1] infer that: "The temporary evolution of the CS that leads to a DNS can be described in terms of the passage of the CS through a nonlinear medium; the nonlinearity of this medium being associated to the number state one wants to displace, as shown in Eq. (5)." Explicitly, they write "...one has to use a non-linear medium with level of non-linearity, g, such that g follows the relation $0 \leqslant g \leqslant n$, with g and $n \in \{0, 1, 2, \ldots\}$ ..." and, as an example, they say that "... for $n = 3$, the medium must contain terms like $(\hat{a}^{\dagger} - \alpha^{*})^{3} = (\hat{a}^{\dagger})^{3} - 3\alpha^{*}(\hat{a}^{\dagger})^{2} + 3(\alpha^{*})^{2}\hat{a}^{\dagger} - (\alpha^{*})^{3}, \ldots$", which corresponds to their Eq. (7). Continuing that reasoning, the statement "...in order to obtain the DNS for $n = 1$, the medium takes the form of a linear term $(\hat{a}^{\dagger} - \alpha^{*})^{n} = (\hat{a}^{\dagger} - \alpha^{*})^{1} = \hat{a}^{\dagger} - \alpha^{*} \ldots$" [1, Eq. (8)] is made. Since $|n\rangle = (1/\sqrt{n!})(a^{\dagger})^{n}|0\rangle$, this suggests that the Fock state $|1\rangle$ would be obtained by passing a field in the vacuum throughout any linear medium, independently of its susceptibility and length, which is an unsustainable idea. Ref. [1] then proceeds, in Section 3, showing explicitly (up to $m = 3$) that both sides of Eq. (3) possess the same representation in the Fock basis $\{|m\rangle\}$ (their results then coinciding with those of Ref. [4]), which is an unnecessary digression, since relations (2) are operator identities; the section closes with the writing of the Wigner function of the DNS, an expression (and plots) presented earlier in the literature [4].

The fundamental problem concerned with the interpretation of Eq. (3) as representing the effect of the medium on the passage of the CS (as made in Ref. [1]) is that the operator in the right-hand of Eq. (3) is not unitary. The time evolution of a field state passing through a medium must be described by a unitary operator $\widehat{U}(t) = \exp(-i\widehat{H}_{ef}t/\hbar)$, where $\widehat{H}_{ef}$ is the Hamiltonian describing the effective self-interaction of the field, which is originated from the field-matter interaction in the medium. Usually, $\widehat{H}_{ef}$ is written in terms of the frequency-dependent linear and non-linear susceptibilities of the medium and the corresponding monomials of the total electric field; thus, it contains powers of $\hat{a}_{i}^{\dagger}$ and $\hat{a}_{i}$ (for all modes involved) but, no matter the approximation one makes, it is always Hermitian in order to guarantee the unitarian nature of $\widehat{U}(t)$. Therefore, if a field in the state $|\psi(0)\rangle$ enters a non-linear medium, the output state is given by $|\psi(\tau)\rangle = \widehat{U}(\tau)|\psi(0)\rangle$ where $\tau = L/v$, L and v being the length traveled and the field velocity in the medium, respectively; and this is certainly not what Eq. (3) represents. In fact, Eq. (3) shows precisely the opposite of what is claimed in Ref. [1]: that one cannot get a DNS by just passing a coherent state through a linear or non-linear medium, since the former does not correspond to the time evolution of the latter. In fact, the authors of Ref. [1] attempt to infer dynamical ingredients of quantum theory from purely kinematical aspects of the Hilbert–Fock space.

What actually Eq. (3) tells us is that a DNS corresponds to a "number" state of the overall displaced harmonic oscillator [3]. In fact, suppressing the zero-point energy, the displaced Hamiltonian [4] is given by

$$\widehat{H}_{\alpha} = \widehat{D}(\alpha)\widehat{H}\widehat{D}^{\dagger}(\alpha) = \hbar\omega\widehat{D}(\alpha)\hat{a}^{\dagger}\hat{a}\widehat{D}^{\dagger}(\alpha) = \hbar\omega\hat{a}_{\alpha}^{\dagger}\hat{a}_{\alpha}\,, \tag{4}$$

where $\hat{a}_{\alpha}^{\dagger} = \hat{a}^{\dagger} - \alpha^{*}$ and $\hat{a}_{\alpha} = \hat{a} - \alpha$ are the displaced creation and annihilation operators, respectively. The ground (vacuum) state of $\widehat{H}_{\alpha}$ is the coherent state $|\alpha\rangle$, which corresponds to the displaced vacuum state (relative to the non-displaced Hamiltonian) $|\alpha, 0\rangle = |\alpha\rangle$; that is, $\widehat{H}_{\alpha}|\alpha\rangle = 0$. Defining the displaced number operator as $\hat{n}_{\alpha} = \widehat{D}(\alpha)\hat{a}^{\dagger}\hat{a}\widehat{D}^{\dagger}(\alpha) \equiv \hat{a}_{\alpha}^{\dagger}\hat{a}_{\alpha}$, one finds $\hat{n}_{\alpha}|\alpha, n\rangle = n|\alpha, n\rangle$ and easily obtains the relations

$$\hat{a}_{\alpha}|\alpha, n\rangle = \sqrt{n}|\alpha, n-1\rangle\,, \quad \hat{a}_{\alpha}^{\dagger}|\alpha, n\rangle = \sqrt{n+1}|\alpha, n+1\rangle. \tag{5}$$

Also, $[\hat{a}_{\alpha}, \hat{a}_{\alpha}^{\dagger}] = [\hat{a}, \hat{a}^{\dagger}] = 1$, so that $\hat{a}_{\alpha}$ and $\hat{a}_{\alpha}^{\dagger}$ satisfy the same algebra as $\hat{a}$ and $\hat{a}^{\dagger}$, as it should; the displaced Fock space is spanned by $\{|\alpha, n\rangle\}$.

Based on these observations, one can infer that the generation of the DNS $|\alpha, n\rangle$, starting from the CS $|\alpha\rangle$, would be achieved if one has a scheme for generating the number state $|n\rangle$ from the vacuum state $|0\rangle$, in a way which could also be applied to the coherent state $|\alpha\rangle$, the displaced vacuum $|\alpha, 0\rangle$. But this, certainly, does not occur by simply passing the CS through a non-linear medium. Therefore, any eventual difficulties to generate the number state $|n\rangle$, pointed out in Ref. [1] as the basic motivation for searching for alternative methods, would remain (and even in a higher degree) if one attempts to produce $|\alpha, n\rangle$ directly from $|\alpha\rangle$ using such a strategy.

Nonetheless, the generation of a DNS starting from a CS can be engendered using standard techniques, if one combines unitary evolution of entangled states with projection synthesis, involving selective conditional measurements made over part of the system; in such schemes, however, one generates a number state, starting from a CS, and then displaces it. The DNS $|\alpha, 1\rangle$ was produced experimentally as a traveling wave [2] in this way: a pulsed laser is initially separated into two beams; one beam crosses a doubler and then is downconverted to generate the single-photon state when the trigger photon is detected; the Fock state $|1\rangle$ is then displaced using a highly reflective beam splitter (HRBS) [5,6], fed from the rear by the other (relatively strong) coherent beam. Actually, any quantum state can be displaced by using a HRBS whose second port is fed by a highly excited CS [7]. Thus, DNS as traveling-mode states may be available whenever a number state is produced as a running wave. In a cavity, the action of $\widehat{D}(\alpha)$ is implemented by feeding the cavity with a classical current [8], and so low excited DNS can be produced by taking advantage of the existing schemes to generate small number states in a cavity [9].

Recently, some proposals to produce highly excited Fock states, both for trapped modes in a cavity and as traveling fields, have been presented. Such suggestions rely on the generation of specific superpositions of coherent [10] and squeezed [11] states on a circle in the phase space which, with appropriated choices of the parameters involved, reduce to excited number states. In cavities, a sequence of prepared Rydberg atoms cross the apparatus, interacting dispersively with a coherent (or a squeezed) state present initially in the high quality cavity; upon the detection of the atoms, the entangled atom-field state reduces to the circular superpositions mentioned above. For traveling modes, the apparatus consists in a sequence of Mach–Zehnder interferometers fed with the vacuum and the one-photon states, containing a Kerr medium in one of the arms which couples the injected coherent (or squeezed) state with one of the internal modes of the interferometer; detection of the photon in the outputs of the interferometers, leads to circular superpositions as traveling fields. These two schemes are complementary and Fock states of the type $|2^N\rangle$ can be generated as trapped modes or running waves. Other proposals for the generation of number states have also been presented in the literature.

The generation of highly excited DNS can be conceived as extensions of these developments, displacing Fock states using standard techniques. In any case, some important ingredients characterizing a state generation, as the *success probability* to get the desired state and the *fidelity* of the state being prepared, have to be carefully considered. A detailed analysis of the possibility of generating highly excited DNS following these lines is being developed and will be discussed elsewhere.

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