Spontaneous symmetry restoration in a field theory at finite chemical potential in a toroidal topology

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We consider the massive vector N-component $(\lambda\varphi^4)_D$ theory defined on a Euclidean space with a toroidal topology. Using recently developed methods to perform a compactification of a d-dimensional subspace at finite chemical potential, we treat jointly the effects of temperature and spatial boundaries, setting forth grounds for an analysis of spontaneous symmetry restoration driven by temperature and spatial boundaries, as a function of the chemical potential. We restrict ourselves to $d = 2$, which corresponds to the heated system confined between two parallel planes (separation L) in dimensions $D = 3$ and $D = 4$. We present results, in the large-N limit, which exhibit how finite size and chemical potential affect spontaneous symmetry restoration.

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I. INTRODUCTION

We investigate spontaneous symmetry restoration induced by both temperature and spatial boundaries. In particular, we are interested in studying the influence of a finite chemical potential in the context of finite-size effects. We shall focus on the N-component vector φ^4 model within the large-N approximation for scalar fields. The symmetry restoration is carried out by starting from the two-particle irreducible (2PI) formalism [1,2] in the Hartree–Fock approximation, which sums up all contributions coming from daisy and superdaisy diagrams. We need then to implement the 2PI formalism in a space with compactified dimensions at large N, which allows us to obtain nonperturbative corrections to the coupling constant. For the sake of simplicity, we study the particular case of finite temperature and one compactified spatial coordinate. We can then, starting from the broken symmetry region, show the behavior of the renormalized mass for different values of the temperature, compactified spatial dimension, and chemical potential.

Field theories defined on spaces with some, or all, of its dimensions compactified is of interest in several branches of theoretical physics. For a Euclidean D-dimensional space, this means that its topology is of the type $\Gamma_D^d = (S^1)^d \times \mathbb{R}^{D-d}$, or its counterpart with the Minkowski signature, with $1 \leq d \leq D$, d being the number of compactified dimensions. Each of these compactified dimensions has the topology of a circle S^1. We refer to Γ_D^d as a *toroidal* topology. These theories are often associated to extra spatial dimensions, as in particle physics, where theories defined on toroidal spaces with extra spatial dimensions are employed, for instance, as a way to investigate the electroweak transition and baryogenesis [3–8]. Also, recent works involving the idea of extra spatial dimensions have been performed in low-energy physics [9,10].

On the other side, in many cases, one is concerned with theories defined on a space with a compactified subspace. A very important development of this kind, which has its roots in the late 1950s, is the systematic approach to quantum field theory at finite temperature, as proposed in Refs. [11–14]. An analogous formalism can be constructed for compactified spatial coordinates, in a D-dimensional Euclidean space. This is an idea advanced for instance in Ref. [15]. In this case, as it is stressed in Ref. [16], the compactification of spatial coordinates can describe systems confined to limited regions of space, which are interpreted as representing samples of material in the forms of films, wires, or grains. In the general case of quantum field theories in toroidal topologies, that is, with compactification of the time coordinate and of spatial dimensions, mathematical bases to deal with this situation on general grounds are consolidated in recent developments [17,18]. This provides a general framework for the results from previous works where systems at finite temperature and/or compactified spatial dimensions were considered, both for bosonic systems, as, for instance, in Refs. [19–22] and for fermionic ones [23–27].

The paper is organized as follows. In Sec. II, symmetry restoration is investigated along the lines of Ref. [2], which establishes a formula for the renormalized mass. This formula is then rewritten to take into account the compactification of the imaginary time and one of the spatial dimensions, considering the dependence with the chemical potential. We can then study the behavior of the renormalized mass starting from the broken symmetry region. In Sec. III, corrections to the coupling constant, due to

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compactification as well as the presence of a chemical potential, are discussed. In Sec. IV, we collect the results from the previous sections, and, specializing to spacetime dimensions $D = 3$ and 4, we fully determine the behavior of the critical temperature as a function of the size of the system and the chemical potential. We also present, in the same section, our conclusions and final remarks.

II. SYMMETRY RESTORATION IN A TOROIDAL SPACE

We consider the model described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial_\nu\varphi_a\partial^\nu\varphi_a + \frac{1}{2}m^2\varphi_a\varphi_a + \frac{u}{4!}(\varphi_a\varphi_a)^2, \quad (1)$$

in Euclidean D-dimensional spacetime, where m and u are, respectively, the zero-temperature mass and the coupling constant in the absence of boundaries and at zero chemical potential. Also, in Eq. (1), φ_a are scalar fields, the sum over repeated indices is carried out from $a = 1$ to N and $\nu = 1$ to D. We consider the large-N regime, such that $\lim_{N\to\infty, u\to 0}(Nu) = \lambda$, with λ fixed. To simplify the notation, we drop out the a indices, summation over them being understood in field products. In order to approach symmetry restoration for this model, we first follow the 2PI formalism developed in Refs. [1,2]. In this case, one finds a stationary condition for the effective action in the Hartree–Fock approximation which translates itself into a gap equation,

$$G^{-1}(x, y) = D^{-1}(x, y) + \frac{u}{2}G(x, x)\delta^4(x - y), \quad (2)$$

where the Fourier-transformed propagators, $D(k)$ and $G(k)$, are given by

$$D(k) = \frac{1}{k^2 + m^2 + \frac{u}{2}\phi^2}; \qquad G(k) = \frac{1}{k^2 + M^2}. \quad (3)$$

Here, $\phi = \langle 0|\varphi|0\rangle$ is the expectation value of the quantum field φ, and M is a momentum-independent effective mass.

In the 2PI formalism, the gap equation corresponds to the stationary condition, and, as such, the effective mass depends on ϕ and conveys all daisy and superdaisy graphs contributing to $G(k)$ [1,2]. Nevertheless, in order to investigate symmetry restoration, we can take instead a particular constant value M in the spontaneously broken phase. Renormalization of the mass m and of the coupling constant u can be performed along lines similar to those in Refs. [2,28], leading to the equation

$$M^2 = -m_R^2 + \frac{u_R}{2}\phi^2 + \frac{u_R}{2}G(M), \quad (4)$$

where m_R^2 and u_R are, respectively, the squared renormalized mass and the renormalized coupling constant, both at zero temperature and zero chemical potential, in the absence of boundaries, and $G(M)$ is the finite part of the integral $G(x, x) = (1/(2\pi)^D)\int d^D k G(k)$, which will be calculated using dimensional and zeta-function regularization techniques. Notice also the minus sign of the m_R^2 term, which is the same choice made in Refs. [2,28] to ensure spontaneous symmetry breaking. Therefore, Eq. (4) will be our starting point. It gives the value of the renormalized mass at the broken-symmetry phase and can be rewritten as

$$\bar{m}^2(\phi) = M^2 - \frac{u_R}{2}\int\frac{d^D k}{(2\pi)^D}\frac{1}{k^2 + M^2}, \quad (5)$$

where the *effective renormalized mass* $\bar{m}^2(\phi) = -m_R^2 + (u_R/2)\phi^2$ has been introduced. In the sequel, we will obtain the generalization of the above equation in such a way as to include the toroidal topology as well as the chemical potential, but first, we notice that restoration of the symmetry will occur at the set of points in the toroidal space where $\bar{m}^2$ is null.

We now proceed to generalize Eq. (5) to a theory defined on a space with a toroidal topology. In the general case, the system is in thermal equilibrium with a reservoir at temperature β^{-1} and confined to a $(d-1)$-dimensional spatial rectangular box of sides L_j, $j = 2, 3, \ldots, d$. We use Cartesian coordinates $\mathbf{r} = (x_1, \ldots, x_d, \mathbf{z})$, where $\mathbf{z}$ is a $(D-d)$-dimensional vector, with corresponding momentum $\mathbf{k} = (k_1, \ldots, k_d, \mathbf{q})$, $\mathbf{q}$ being a $(D-d)$-dimensional vector in momentum space. Then, the Feynman rules should be modified according to Refs. [16–18]

$$\int\frac{dk_\tau}{2\pi} \to \frac{1}{\beta}\sum_{n_\tau=-\infty}^{\infty}, \qquad k_\tau \to \frac{2n_\tau\pi}{\beta} - i\mu,$$
$$\int\frac{dk_i}{2\pi} \to \frac{1}{L_i}\sum_{n_i=-\infty}^{+\infty}, \qquad k_i \to \frac{2n_i\pi}{L_i}, \qquad i = 2, 3, \ldots, d, \quad (6)$$

where τ corresponds to imaginary time and μ is the chemical potential. We consider the simpler situation of the system at temperature β^{-1} and one compactified spatial coordinate (x_2) with a compactification length $L_2 \equiv L$. In this case, using Eq. (6), we can perform a suitable generalization of the procedure in Ref. [2], to take into account finite-size, thermal, and boundary effects in Eq. (5). The integral over the D-dimensional momentum in Eq. (5) becomes a double sum over n_τ and $n_2 \equiv n_x$ together with a $(D-2)$-dimensional momentum integral. Then, following steps similar as in Ref. [19] and using dimensional regularization to perform the integral [29], the renormalized (β, L, μ)-dependent mass in the large-N limit can be written in the form

$$\bar{m}^2(T, L, \mu) = M^2 - \frac{u_R M^{D-2}}{2} \frac{\pi^{(D-2)/2}}{4\pi^2} \frac{\Gamma(s - \frac{D-2}{2})}{\Gamma(s)} \sqrt{a_\tau a_x} \sum_{n_\tau, n_x = -\infty}^{\infty} \left[a_\tau \left(n_\tau - \frac{i\beta}{2\pi}\mu \right)^2 + a_x n_x^2 + c^2 \right]^{(D-2)/2-s} \Bigg|_{s=1}, \quad (7)$$

where we have changed variables in the integral, $k_i/2\pi M \to q_i$, and introduced the dimensionless quantities $a_\tau = (M\beta)^{-2}$, $a_x = (ML)^{-2}$, and $c = (2\pi)^{-1}$. The double sum in Eq. (7) is recognized as one of the inhomogeneous Epstein–Hurwitz zeta functions, $Z_2^{c^2}(s - \frac{D-2}{2}; a_\tau.a_x; b_\tau, b_x)$, which has an analytical extension to the whole complex s plane [30,31]; in general, for $j = 1, 2$,

$$Z_2^{c^2}(\nu; \{a_j\}; \{b_j\}) = \frac{\pi |c|^{2-2\nu} \Gamma(\nu - 1)}{\Gamma(\nu)\sqrt{a_1 a_2}} + \frac{4\pi^\nu |c|^{1-\nu}}{\Gamma(\nu)\sqrt{a_1 a_2}} \left[\sum_{j=1}^{2} \sum_{n_j=1}^{\infty} \cos(2\pi n_j b_j) \left(\frac{n_j}{\sqrt{a_j}} \right)^{\nu-1} K_{\nu-1}\left(\frac{2\pi c n_j}{\sqrt{a_j}} \right) \right.$$
$$\left. + 2 \sum_{n_1, n_2 = 1}^{\infty} \cos(2\pi n_1 b_1) \cos(2\pi n_2 b_2) \left(\sqrt{\frac{n_1^2}{a_1} + \frac{n_2^2}{a_2}} \right)^{\nu-1} K_{\nu-1}\left(2\pi c \sqrt{\frac{n_1^2}{a_1} + \frac{n_2^2}{a_2}} \right) \right]. \quad (8)$$

For us, $a_1 = a_\tau$, $a_2 = a_x$, $b_1 = b_\tau = i\beta\mu/2\pi$, $b_2 = b_x = 0$, $c = 1/2\pi$, and $\nu = s - (D-2)/2$. Replacing Eq. (8) into Eq. (7), the thermal and boundary corrected mass is obtained in terms of the original variables, β, L, μ, and of the fixed renormalized zero-temperature coupling constant in the absence of boundaries,

$$\lambda_R = \lim_{N \to \infty, u_R \to 0} (N u_R).$$

However, the first term in Eq. (8) implies that the first term in the corrected mass is proportional to $\Gamma(1 - D/2)$, which is divergent for even dimensions $D \geq 2$ [19]. This term is suppressed by a minimal subtraction, leading to a finite effective renormalized mass; for the sake of uniformity, this polar term is also subtracted for odd dimensions, where no singularity exists, corresponding to a finite renormalization.

In dimension D, the renormalized zero-temperature coupling constant in the absence of boundaries λ_R has dimension of mass^{4-D}; accordingly, we introduce a dimensionless coupling constant $\lambda'_R = \lambda_R M^{D-4}$. Also, we define the dimensionless reduced temperature t, reduced chemical potential ω, and the reduced inverse length of the system χ, in such a way that we have for any dimension D the set of dimensionless parameters defined by

$$\lambda'_R = \lambda_R M^{D-4}, \qquad t = T/M,$$
$$\chi = L^{-1}/M, \qquad \omega = \mu/M. \quad (9)$$

We then can obtain, after subtraction of the polar term, which does not depend on β, L, and μ, the corrected mass, $\bar{m}^2(D, \beta, L, \mu)$. This implies that the condition for symmetry restoration, $\bar{m}^2(D, \beta, L, \mu) = 0$, can be written in terms of the above dimensionless parameters, replacing λ'_R by the corrected coupling constant $\lambda'_R(D, \beta, L, \mu)$ [this is precisely defined in the next section, Eq. (15)], in such a way that the critical equation reads

$$1 - \frac{\lambda'_R(D, \beta, L, \mu)}{(2\pi)^{D/2}} \left[\sum_{n=1}^{\infty} \cosh\left(\frac{\omega n}{t} \right) \left(\frac{t}{n} \right)^{\frac{D}{2}-1} K_{\frac{D}{2}-1}\left(\frac{n}{t} \right) + \sum_{l=1}^{\infty} \left(\frac{\chi}{l} \right)^{\frac{D}{2}-1} K_{\frac{D}{2}-1}\left(\frac{l}{\chi} \right) \right.$$
$$\left. + 2 \sum_{n,l=1}^{\infty} \cosh\left(\frac{\omega n}{t} \right) \left(\frac{1}{\sqrt{\frac{n^2}{t^2} + \frac{l^2}{\chi^2}}} \right)^{\frac{D}{2}-1} K_{\frac{D}{2}-1}\left(\sqrt{\frac{n^2}{t^2} + \frac{l^2}{\chi^2}} \right) \right] = 0. \quad (10)$$

III. CORRECTIONS TO THE COUPLING CONSTANT

In this section, we follow the reasoning made in Ref. [21], appropriately modified to incorporate the effects from the chemical potential. We consider the zero-external-momenta four-point function, which is the basic object for our definition of the renormalized coupling constant. At leading order in $1/N$, it is given by the sum of all chains of one-loop diagrams with four external legs, which leads to the expression (we consistently define $u'_R = u_R M^{D-4}$) [32]

$$\Gamma_D^{(4)}(0, \beta, L, \mu) = \frac{u'_R}{1 + N u'_R \Pi(D, \beta, L, \mu)}, \quad (11)$$

where the dimensionless one-loop diagram is given by

$$\Pi(D, \beta, L, \mu) = \frac{\sqrt{a_\tau a_x}}{16\pi^4} \sum_{n_\tau, n_x = -\infty}^{\infty} \mathcal{I}_{n_\tau n_x}(s) \Bigg|_{s=2}, \quad (12)$$

where

$$\mathcal{I}_{n_\tau n_x}(s) = \int \frac{d^{D-2}q}{[\mathbf{q}^2 + a_\tau (n_\tau - \frac{i\beta}{2\pi}\mu)^2 + a_x n_x^2 + c^2]^s}. \quad (13)$$

Then, proceeding along lines similar to those leading to Eq. (10), we write $\Pi(D, \beta, L, \mu)$ in the form

$$\Pi(D, \beta, L, \mu) = H(D) + \frac{1}{(2\pi)^{D/2}} R(D, \beta, L, \mu),$$

where $R(D, \beta, L, \mu)$ is given by

$$\begin{aligned} R(D, \beta, L, \mu) &= \sum_{n=1}^{\infty} \cosh\left(\frac{\omega n}{t}\right)\left(\frac{t}{n}\right)^{\frac{D}{2}-2} K_{\frac{D}{2}-2}\left(\frac{n}{t}\right) \\ &\quad + \sum_{l=1}^{\infty}\left(\frac{\chi}{l}\right)^{\frac{D}{2}-2} K_{\frac{D}{2}-2}\left(\frac{l}{\chi}\right) + 2\sum_{n,l=1}^{\infty} \cosh\left(\frac{\omega n}{t}\right) \\ &\quad \times \left(\frac{1}{\sqrt{\frac{n^2}{t^2}+\frac{l^2}{\chi^2}}}\right)^{\frac{D}{2}-2} K_{\frac{D}{2}-2}\left(\sqrt{\frac{n^2}{t^2}+\frac{l^2}{\chi^2}}\right) \end{aligned} \tag{14}$$

and $H(D) \propto \Gamma(2 - \frac{D}{2})$ is a polar piece coming from the first term in the analytic extension of the zeta function in Eq. (8). Notice that from the general properties of Bessel functions, the above equations are meaningful for a reduced chemical potential satisfying the condition $0 \leq \omega < 1$. This is the allowed range of ω in Eqs. (10) and (14). We see that for even dimensions $D \geq 4$, $H(D)$ is divergent, due to the pole of the gamma function. Accordingly, this term must be subtracted to give the renormalized single bubble function $\Pi_R(D, \beta, L, \mu)$. We get, simply, $\Pi_R(D, \beta, L, \mu) = [1/(2\pi)^{D/2}]R(D, \beta, L, \mu)$. As mentioned before, the term $H(D)$ is also subtracted in the case of odd dimensions D, where no poles are present, corresponding to a finite renormalization. Using properties of the Bessel functions, we see that, for any dimension D and finite values of the chemical potential μ, $R(D, \beta, L, \mu)$ satisfies the conditions $\lim_{\beta,L\to\infty} R(D, \beta, L, \mu) = 0$, $\lim_{\beta,L\to 0} R(D, \beta, L, \mu) \to \infty$, and $R(D, \beta, L, \mu) > 0$ for finite μ and for any values of D, β, and L. Under these conditions, we define the dimensionless β-, L-, and μ-dependent renormalized coupling constant $\lambda'_R(D, \beta, L, \mu)$ at the leading order in $1/N$ as $\lambda'_R(D, \beta, L, \mu) \equiv N\Gamma_D^{(4)}(0, \beta, L, \mu)$, which, from Eq. (11), after subtraction of the polar term $H(D)$ in $\Pi(D, \beta, L, \mu)$, leads to

$$\lambda'_R(D, \beta, L, \mu) = \frac{\lambda'_R}{1 + \lambda'_R[1/(2\pi)^{D/2}]R(D, \beta, L, \mu)}. \tag{15}$$

In the next section, we will investigate the restoration of symmetry, taking into account thermal, boundary, and finite chemical potential corrections to the coupling constant as presented above.

IV. BOUNDARY AND CHEMICAL POTENTIAL EFFECTS ON THE SYMMETRY RESTORATION: COMMENTS AND CONCLUDING REMARKS

In the general situation, Eq. (10) does not allow an algebraic solution. For numerical evaluations, we fix the value $\lambda'_R = 0.50$ and take several values of the dimensionless parameters t, χ, and ω. We present and comment on the results concurrently for dimensions $D = 3$ and $D = 4$, since the behavior of the system is qualitatively the same, apart from numerical details, in both cases.

In Figs. 1 and 2, we exhibit the critical temperature as a function of the reduced inverse size of the system for different values of the chemical potential, for $D = 3$ and $D = 4$, respectively. We see from them that the behavior of the critical temperature is different, by changing the values of the chemical potential, for small and large values of χ (large and small sizes of the system, respectively). An interesting aspect, explicitly shown in Figs. 1 and 2, is the existence of a particular size of the system, L_0, corresponding to the reduced inverse size χ_0, where the critical temperature vanishes. This particular value χ_0 is *independent* of the chemical potential for both $D = 3$ and $D = 4$. This is emphasized in the lower plots of Figs. 1 and 2 which

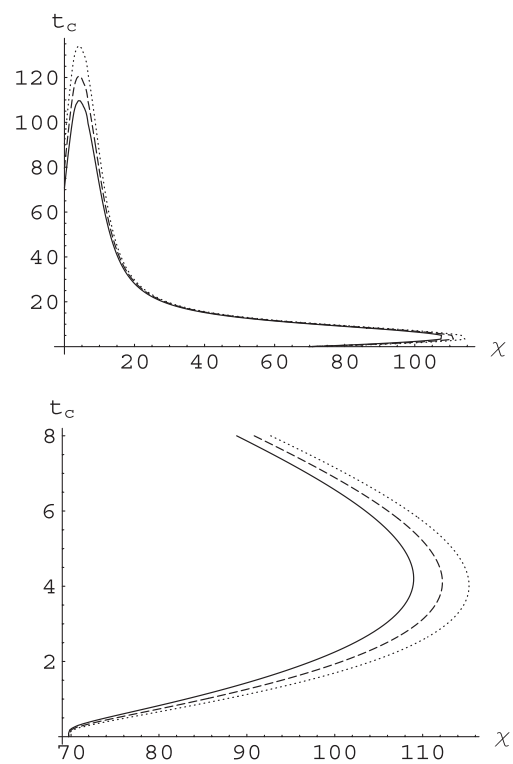

FIG. 1. Reduced critical temperature as a function of the reduced inverse size of the system for dimension $D = 3$ (upper plot). We fix $\lambda'_R = 0.5$ and take the chemical potential values $\omega = 0.1$ (full line), 0.3 (dashed line), and 0.4 (dotted line). The symmetry-breaking regions are in the "inner" side of each curve. The lower plot is a "zoom" enhancing the region of the characteristic size of the system, corresponding to $\chi \approx 69.96$.

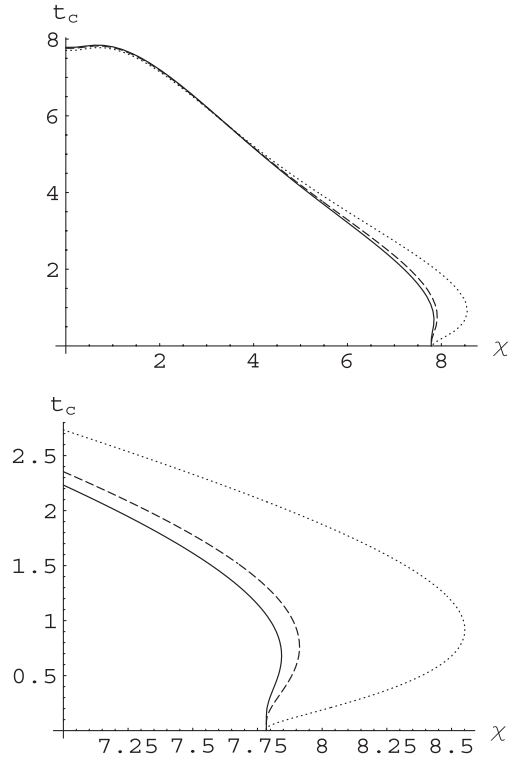

FIG. 2. Reduced critical temperature as a function of the reduced inverse size of the system for dimension $D = 4$ (upper plot). We fix the (dimensionless) coupling constant $\lambda_R = 0.5$ and take the chemical potential values $\omega = 0.0$ (full line), $\omega = 0.5$ (dashed line), and $\omega = 0.9$ (dotted line). The symmetry-breaking regions are on the inner side of each curve. The lower plot shows in detail the critical temperature in the region around the characteristic size of the system.

show in detail the domain around the characteristic value $\chi = \chi_0$. For the value of the reduced coupling constant, we take ($\lambda'_R = 0.50$), and we find $\chi_0 \approx 69.96$ for $D = 3$ and $\chi_0 \approx 7.78$ for $D = 4$.

Although these results were *a priori* unexpected, it is possible to notice that the above value of the characteristic reduced inverse size, χ_0, for both $D = 3$ and $D = 4$, can be obtained directly from Eq. (10) by solving it for $t = 0$, such that all dependency coming from the chemical potential drops out. Therefore, at $t_c = 0$, the symmetry restoration is induced exclusively as a size effect, implemented by imposing spatial periodic boundary conditions, and the chemical potential does not intervene in the value of the critical size of the system to sustain the condensed phase. Essentially what happens is that for zero critical temperature, the finite-size behavior of the physical system collapses to the one corresponding to a zero chemical potential, as is the case for a Bose-Einstein distribution.

We also see from Figs. 1 and 2 that, for each value of ω, there is a limiting smallest size of the system, $L_{\min}(\omega)$, corresponding to a largest reduced inverse size $\chi_{\max}(\omega)$, such that $\chi_{\max}(\omega) > \chi_0$, over which the transition ceases to exist.

Moreover, we can see clearly from Figs. 1 and 2 that, by effect of the spatial boundaries, χ_0 is the border between two regions: $\chi < \chi_0$ and $\chi_0 < \chi < \chi_{\max}$, with different behaviors. In the first region, the critical temperature is uniquely defined in terms of the size of the system and of the chemical potential: For each pair (χ, ω), there is only one critical temperature, while in the second region, two values of t_c may exist for the same values of ω and χ. In the region $\chi_0 < \chi < \chi_{\max}$, there are for each value of ω two possible critical temperatures, say, $t_c^{(1)}$ and $t_c^{(2)}$, with $t_c^{(2)} > t_c^{(1)}$, associated, respectively, to the lower and the upper branches of the critical curve. This means that, in this region, we have two possible transitions. For $D = 3$, we take, from inspection of Fig. 1, $t = t_c^{(1)} = 1.93$, $t = t_c^{(2)} = 6.89$, and an intermediate temperature, $t_c^{(1)} < t < t_c^{(2)}$, $t = 4.80$. For these temperatures, we plot in Fig. 3 curves of the effective potential of the system, given by $U(\beta, L, \mu) = \frac{1}{2}m^2(\beta, L, \mu)\varphi^2 + \frac{1}{24}\lambda'_R(\beta, L, \mu)\varphi^4$ (we define the quantity φ by taking $\varphi^2 = \varphi_a\varphi_a$), for fixed values of the reduced chemical potential, $\omega = 0.30$ and of the reduced inverse size of the system, $\chi = 100$. These plots confirm that two of the temperatures, $t = t_c^{(1)} = 1.93$ (full line) and $t = t_c^{(2)} = 6.89$ (dashed line), are critical temperatures corresponding, respectively, to the lower and upper branches of the critical curve; the intermediate temperature, $t_c^{(1)} < t < t_c^{(2)}$, $t = 4.80$ (dotted line), corresponds to the system in the symmetry-broken region. A similar kind of "doubling" occurs for large sizes (but *not for* $L \to \infty$) of the system. In this case, two different sizes of the system with the same chemical potential may correspond to the same critical temperature. In the region $\chi < \chi_0$, the critical temperature

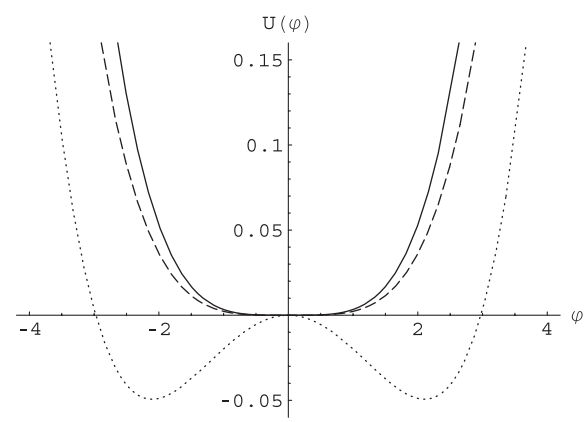

FIG. 3. Curves for the effective potential $U = \frac{1}{2}m^2\varphi^2 + \frac{1}{24}\lambda'_R\varphi^4$, for fixed values of the reduced chemical potential, $\omega = 0.30$ and of the reduced inverse size of the system, $\chi = 100$ for $D = 3$; φ and U are measured in units of $M^{\frac{1}{2}}$ and M^3, respectively. We take three values of the reduced temperature, two of them being critical temperatures, $t_c^{(1)} = 1.93$ (full line) and $t_c^{(2)} = 6.89$ (dashed line), and an intermediate one, $t = 4.80$ (dotted line), in the symmetry-broken region.

grows for increasing chemical potential for fixed χ and also for the system in unlimited space ($L \to \infty$). In this case, we may find the bulk reduced critical temperature $t_c^{\rm bulk}(\mu)$ taking the limit $\chi \to 0$ in Eq. (10).

As an overall conclusion, the results suggest that finite-size effects with finite chemical potential are relevant and greatly change the critical curves with respect to the ones for the system in bulk form. In particular, these actors lead to the appearance of a doubling of critical parameters, which, up to our knowledge, is not a trivially expected behavior. This behavior is to be contrasted with what happens with the system in bulk form, where there is always a unique critical temperature, which grows with increasing chemical potential. We also show the existence of a characteristic size of the system, which determines the existence, for each value of the chemical potential, of a minimal size sustaining the broken phase. This characteristic size, obtained from Eq. (10), is the same for all values of the chemical potential.

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