

D -meson diffusion in hadronic matter

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Abstract. We present effective-field-theory results with unitarized interactions on the D -meson transport coefficients in a gas populated by light mesons and baryons at finite temperature and baryochemical potential. The Fokker-Planck equation is used to compute the drag force, the relaxation time and the diffusion coefficients of D mesons for collisions at FAIR. At finite baryochemical potential, the combined effect of net baryonic density and sizable meson-baryon interaction makes the D mesons to relax more efficiently than in the $\mu_B = 0$ case. We also describe the connection with the quark-gluon plasma phase in adiabatic trajectories on the phase diagram at both zero and finite baryochemical potential.

1. Introduction

Some of the most useful tools to study the quark-gluon plasma (QGP) phase –formed in the early stages of a relativistic heavy-ion collision– are heavy quarks. Due to their large mass ($m_c, m_b \gg \Lambda_{QCD}, T$) they can be treated as Brownian particles interacting with lighter partons. Not only the average momentum of the heavy quarks is modified by the collisions, but also the momentum distribution is broadened. The efficiency of these two processes is quantified by the *drag force* and the *momentum-space diffusion coefficient*, respectively.

The previous picture also holds after hadronization. The role of the Brownian particle is played by a D or B meson, whereas the thermal bath is now composed of pions, kaons, nucleons and other light hadrons with whom the heavy meson may interact. Thus, the transport coefficients of D mesons [1, 2] and B mesons [3] become of key importance in the equilibration of these hadrons. The presence of these coefficients will amount on the modification of the final heavy meson spectra and affect the flow harmonics v_n and the nuclear suppression factor R_{AA} . To study these transport coefficients and quantify the energy loss of heavy quarks in the hot and dense medium we use quantum kinetic theory.

We focus on the heavy-ion physics to be produced at the Facility for Antiproton and Ion Research (FAIR) [4], where the collision energy will be lower than that for collisions at the Large Hadron Collider, for instance. Lower energies typically produce a net baryon density in the central rapidity region. This fact will imply physics at non-zero baryochemical potential in the QCD phase diagram, maybe close to the conjectured critical point. For these collisions,

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the available energy density makes difficult the production of B mesons. Therefore, in this manuscript we focus our attention to the calculation of D -meson transport coefficients [2].

In Sec. 2 we introduce the Fokker-Planck kinetic equation and detail the physical interpretation of the D -meson transport coefficients. In Sec. 3 we briefly describe the interaction of D mesons with lighter hadrons, both mesons and baryons. Finally, in Sec. 4 we present our main results and conclusions.

2. Fokker-Planck equation and transport coefficients

The collective description of D mesons is encoded in their distribution function $f(t, p)$. The temporal evolution of $f(t, p)$ follows a kinetic equation defined in the phase space, which in the case of a Brownian particle in a thermal bath composed of lighter particles is the *Fokker-Planck equation*:

$$\frac{\partial f(t, \mathbf{p})}{\partial t} = \frac{\partial}{\partial p_i} \left\{ F_i(\mathbf{p}) f(t, \mathbf{p}) + \frac{\partial}{\partial p_j} [\Gamma_{ij}(\mathbf{p}) f(t, \mathbf{p})] \right\}, \quad (1)$$

where $i = 1, 2, 3$ is the spatial index, F_i is the drag force or friction term accounting for the average momentum change, and Γ_{ij} is the momentum-space diffusion matrix which accounts for the broadening of the momentum distribution. The computation of these coefficients is performed through their microscopic expressions in terms of momentum averages,

$$F_i(\mathbf{p}) = \int d\mathbf{k} w(\mathbf{p}, \mathbf{k}) k_i, \quad \Gamma_{ij}(\mathbf{p}) = \frac{1}{2} \int d\mathbf{k} w(\mathbf{p}, \mathbf{k}) k_i k_j, \quad (2)$$

where the interaction measure $w(\mathbf{p}, \mathbf{k})$ represents the probability of a D meson with momentum \mathbf{p} to suffer a collision with a light particle, losing a momentum \mathbf{k} :

$$w(\mathbf{p}, \mathbf{k}) = g_l \int \frac{d^3q}{(2\pi)^9} n_{F,B}(E_l(q), T) [1 \pm n_{F,B}(E_l(q+k), T)] \frac{1}{2E_D(p)} \frac{1}{2E_l(q)} \frac{1}{2E_D(p-k)} \\ \times \frac{1}{2E_l(q+k)} (2\pi)^4 \delta(E_D(p) + E_l(q) - E_D(p-k) - E_l(q+k)) |\overline{\mathcal{M}}|^2, \quad (3)$$

with g_l being the spin-isospin degeneration of the light particle, $n_{F,B}$ its equilibrium distribution function following Fermi-Dirac or Bose-Einstein statistics, respectively; $E_D(p)$ is the energy of the D meson with momentum \mathbf{p} , $E_l(q)$ the energy of the light particle with momentum \mathbf{q} and $|\overline{\mathcal{M}}|^2$ is the average scattering matrix element squared of the binary collision.

If we consider an isotropic gas and focus on the *static limit* (where $p \rightarrow 0$) there is only one independent transport coefficient. For definiteness, we take it to be the drag force $F(p) = F_i p^i / p^2$, while the momentum-space diffusion coefficient Γ is related to F through the Einstein relation $\Gamma = F m_D T$ (where m_D is the D -meson mass and T the temperature of the thermal medium).

In several talks at this workshop some emphasis was put on the concept of *relaxation time*. This coefficient describes how efficient the relaxation of the non-equilibrium distribution function is, while it relaxes towards the equilibrium state (Boltzmann's H theorem). Consider for simplicity the one dimensional case of Eq. (1) and assume constant transport coefficients F and Γ . The analytical solution of the kinetic equation is

$$f(t, p) = \sqrt{\frac{F}{2\pi\Gamma(1 - e^{-2Ft})}} \exp \left[-\frac{F}{2\Gamma} \frac{(p - p_0 e^{-Ft})^2}{1 - e^{-2Ft}} \right], \quad (4)$$

where p_0 is the initial D -meson momentum. It is immediate to see that F^{-1} should play the role of an intrinsic time for the relaxation process. In particular, the average momentum reads

$$\langle p \rangle \equiv \int_{-\infty}^{\infty} dp p f(t, p) / \int_{-\infty}^{\infty} dp f(t, p) = p_0 e^{-\frac{t}{\tau_R}}, \quad (5)$$

where we have defined the relaxation time $\tau_R \equiv 1/F$, which provides a characteristic time accounting for the exponential decay of the average momentum.

We can also introduce the *diffusion coefficient in position space* D_x . The standard result for the average square displacement

$$\langle (x(t) - x_0)^2 \rangle = 2D_x t , \quad (6)$$

shows that D_x is a measure of the “speed” of the diffusion in space. A large D_x implies that the D meson can diffusively travel larger distances for a given time. In the static limit this coefficient is related to the drag force by the relation $D_x = T/(m_D F)$.

All these coefficients depend on the temperature and baryochemical potential. In a heavy-ion collision both variables change with time along the fireball’s trajectory in the phase diagram. For simplicity, we assume adiabatic trajectories –with a constant entropy per baryon consistent with the energies expected at FAIR collisions– which give us the implicit relation $\mu_B = \mu_B(T)$.

3. Interaction and effective field theories

The interaction between the D mesons and the light hadrons –as it lies in the nonperturbative and confined regime of QCD– can be described by effective models. The global symmetries of the QCD Lagrangian should be implemented into the effective models allowing a reliable description of the hadronic interaction. We take the *chiral and heavy-quark-spin symmetries* to be the fundamental guides in order to construct our effective Lagrangian (therefore, it results in a double expansion). In addition to these symmetries, our third principle is the exact satisfaction of the *unitarity condition* of the scattering matrix.

To account for the D meson–light meson scattering we introduce pions, kaons, antikaons and η mesons. Details on the effective theory based on chiral and heavy-quark-spin expansions can be found in Refs. [1, 2, 5]. Using Feynman rules we construct the perturbative amplitude (or potential) V for each scattering channel. In the meson-baryon sector we also construct the effective Lagrangian based on heavy-quark-spin symmetry which accounts for the interaction of D mesons with nucleons and Δ baryons. Details on the effective theory and the computation of the potential V can be found in [2, 6, 7, 8, 9].

In both sectors we apply a unitarization method [10] (based on a Bethe-Salpeter (BS) equation) to extend the interaction description up to temperatures $T \simeq 140$ MeV without losing physical sense. In addition, we are able to dynamically generate resonant states like the $D_0(2400)$ in the meson–meson sector and the $\Sigma_c(2800)$ and $\Lambda_c(2595)$ in the meson–baryon sector, which are automatically taken into account in the interaction. The scattering matrix element in Eq. (3) is related to the perturbative amplitude as:

$$\mathcal{M}(s) \propto \frac{V(s)}{1 - G(s)V(s)} , \quad (7)$$

where G represent the diagonal matrix of the two intermediate propagators in the BS equation.

4. Results and Conclusions

We present our main results taken from Ref. [2]. In Fig. 1 we show the drag coefficient $F(p = 100$ MeV) for a D meson as a function of the temperature when the baryochemical potential is set to zero. The largest contribution comes from the pions, as these mesons populate the gas at temperatures below the hadronization one. The kaons, antikaons and η mesons contribute appreciably only at high temperatures $T > 100$ MeV, being the baryon contribution ($N + \Delta$) negligible at $\mu_B = 0$.

The position-space diffusion coefficient D_x is shown at $\mu_B = 0$ in Fig. 2, when all hadrons are included in the calculation. We normalize this coefficient by the thermal wavelength $2\pi T$

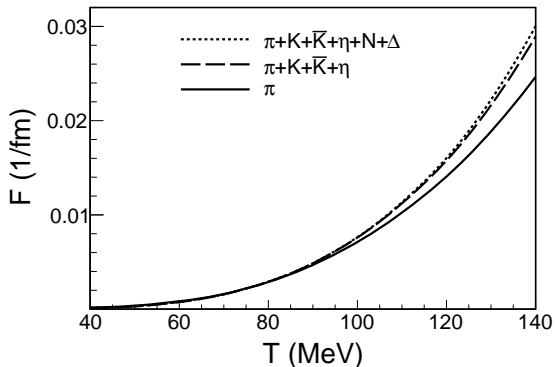


Figure 1. Drag force $F(p)$ for a D meson with momentum $p = 100$ MeV as a function of temperature for several species in the thermal bath at $\mu_B = 0$ (LHC collisions). Clearly the dominant contribution comes from pions as they are the most abundant in the bath. The contribution of baryons is practically negligible. The diffusion coefficient in momentum space is related to this coefficient via the Einstein relation $\Gamma = Fm_D T$.

to construct an adimensional number. The calculation for the QGP (Rapp and Hees) is a T -matrix calculation from Ref. [11] and the lattice-QCD computation (Banerjee, Datta, Gavai and Majumdar) is taken from Ref. [12].

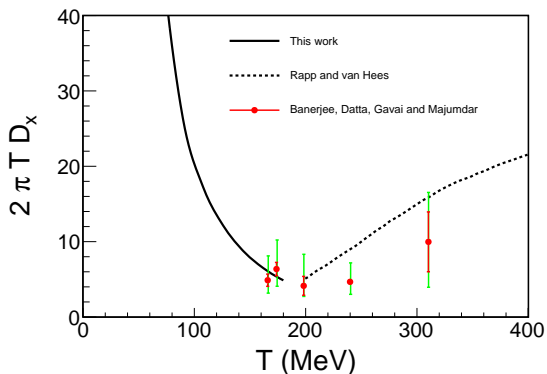


Figure 2. Diffusion coefficient multiplied by the thermal wavelength $2\pi T$ as a function of temperature, around the crossover temperature at $\mu_B = 0$. The solid line represents our results for hadronic matter with all species included. The dashed line shows the result of Ref. [11] for a quark-gluon plasma. Dots represent the result from the lattice-QCD calculation extracted from Ref. [12].

Turning to the $\mu_B \neq 0$ case –relevant for the physics at FAIR– we consider three characteristic adiabatic trajectories in the phase diagram for different initial collision energies ($\sqrt{s} \approx 5 - 40A$ GeV [4]). In the left panel of Fig. 3 we sketch our trajectories on the phase diagram. They correspond to fixed entropy per baryon of $10 - 30$ [13]. In the right panel we plot our results in the hadron gas (at low temperature) and the results of a perturbative QGP at finite quark chemical potential based on the computation of Ref. [14].

Finally, we show the relaxation time for each adiabatic trajectory plus the result at $\mu_B = 0$ as a function of temperature in Fig. 4. Notice that the presence of baryons at finite μ_B enhances the D -meson stopping and the relaxation time is accordingly reduced. Therefore collisions at FAIR put the D mesons closer to the equilibrium state than at the LHC. These relaxation times should be compared to the fireball’s lifetime which is of the order of $t_f \sim 10$ fm. In any case, the D mesons cannot totally relax through collision as $\tau_R > t_f$, confirming that D mesons should be sensible to some properties of the initial QGP.

In summary, we have presented our latest results on the transport coefficients of D mesons that describe the energy loss and diffusion of these states in a hadronic medium at finite temperature and baryochemical potential. Focusing on FAIR physics, we have obtained that the collisions with baryons make the D mesons more thermally relaxed than the case at vanishing baryochemical potential. We have obtained a nice connection with the results of QGP at high temperature, providing an indication for a possible minimum of $2\pi T D_x$ at both the crossover temperature (at $\mu_B = 0$) and at the first-order transition temperature (at FAIR collisions).

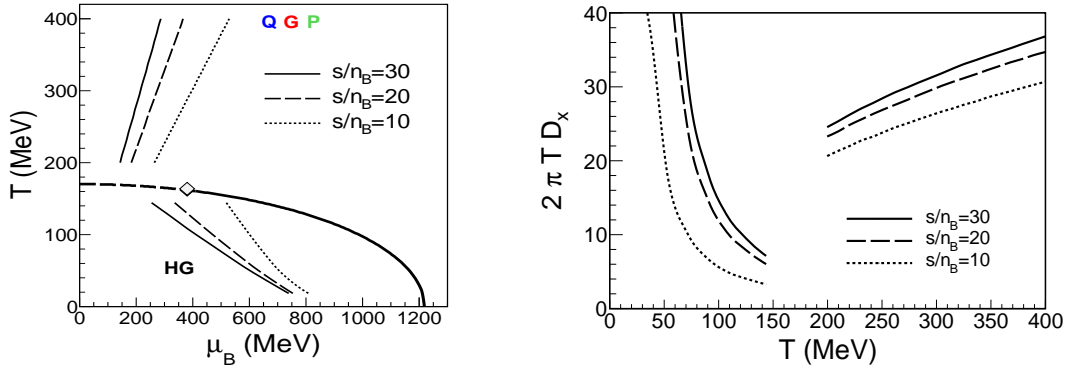


Figure 3. Left panel: Adiabatic trajectories for different initial conditions at FAIR collisions. Right panel: D_x as a function of temperature for the three adiabatic trajectories.

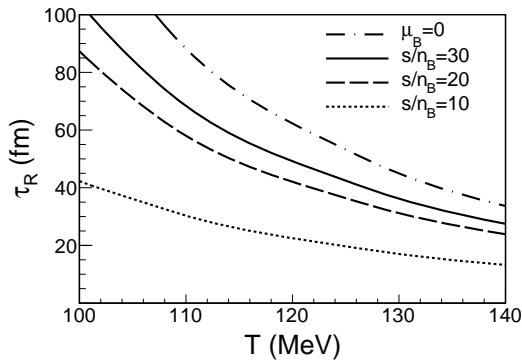


Figure 4. Relaxation time for the adiabatic trajectories as a function of temperature. The trajectories with larger baryonic density (those with smaller entropy per baryon) make the D meson to have a lower relaxation time and hence, to follow a more efficient relaxation via baryonic collisions.

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