Fast Seismic Inversion Methods Using Ant Colony Optimization Algorithm

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***Abstract*—This letter presents ACO$_{\mathbb{R}}$-V, a new computationally efficient ant-colony-optimization-based algorithm, tailored for continuous-domain problems. The ACO$_{\mathbb{R}}$-V algorithm is well suited for application in seismic inversion problems, owing to its intrinsic features, such as heuristics in generating the initial solution population and its facility to deal with multiobjective optimization problems. Here, we show how the ACO$_{\mathbb{R}}$-V algorithm can be applied in two methodologies to obtain 3-D impedance maps from poststack seismic amplitude data. The first methodology pertains to the traditional method of forward convolution of a reflectivity model with the estimated wavelet, where ACO$_{\mathbb{R}}$-V is used to guess the appropriate wavelet as the reflectivity model. In the second methodology, we propose an even faster inversion algorithm based on inverse filter optimization, where ACO$_{\mathbb{R}}$-V optimizes the inverse filter that is deconvolved with the seismic traces and results in a reflectivity model similar to that found in well logs. This modeled inverse filter is then deconvolved with the entire 3-D seismic volume. In experiments, both the methodologies are applied to a synthetic 3-D seismic volume. The results validate their feasibility and the suitability of ACO$_{\mathbb{R}}$-V as an optimization algorithm. The results also show that the second methodology has the advantages of a much higher convergence speed and effectiveness as a seismic inversion tool.**

***Index Terms*—Acoustic impedance, ant colony optimization (ACO), fast deconvolution, reservoir characterization, seismic inversion.**

I. Introduction

INVERSION of seismic data plays a vital processing step in reservoir characterization [1]. A seismic impedance model obtained from inverting poststack amplitude data has been widely used in the process of obtaining quantitative information on acoustic properties far from well controls. Seismic inversion techniques developed to estimate impedance from recorded compressional waves have been available to geophysicists since the last three decades [2]–[4].

Many optimization techniques based on stochastic or heuristic stochastic global search have been successfully applied in these inversion methods: simulated annealing (SA) [5]–[7], genetic algorithm (GA) [8]–[10], ant colony optimization (ACO) [11], and so on. All these algorithms obtain satisfying solutions by iteration. Their main drawback is related to the complexity of the algorithm parameters' setting, whereas a significant advantage is their ease in incorporating heuristic information in the search process.

According to [11], for seismic inversion problems, SA- and GA-based algorithms generally have problems such as slower convergence speed and forming local-extreme values. On the other hand, ACO that was originally developed to discrete combinatorial optimization problems [12], [13] seems to be simpler and easier to implement, and there is no need to adjust many parameters. Furthermore, it has the advantage of enabling easy incorporation of heuristic information in the search process, and it is easily parallelizable, leading to high convergence speed with low computational resource consumption.

In this letter, we present a chosen state-of-the-art ACO algorithm, namely, ACO$_{\mathbb{R}}$ [14], [15], and propose extensions that improve its convergence speed in near-optimal solutions for seismic inversion problems. Our proposed extended algorithm, namely, ACO$_{\mathbb{R}}$-V, makes use of heuristics for good initial candidate solutions and other heuristics to exploit both local and global search steps within the original ACO$_{\mathbb{R}}$ features. ACO$_{\mathbb{R}}$-V also does not require many configuration parameters that contribute in improving the preprocessing time. The use of these heuristics results in an efficient optimization algorithm that can be used in a variety of seismic inversion methods.

Here, we show how our optimization algorithm uses seismic and well log data to estimate the wavelet's shape. Then, the candidate traces of reflectivity for the model solution are convolved with the estimated wavelet, and the result is compared to the desired seismic traces. Thus, the reflectivity model is updated by ACO$_{\mathbb{R}}$-V to minimize the difference between the modeled and observed traces [16].

We also introduce a very promising method based on backward modeling convolution. In this method, ACO$_{\mathbb{R}}$-V uses seismic amplitudes and well data to optimize the inverse filter that, when deconvolved with the seismic traces, is capable of generating a reflectivity model similar to that found in well logs. This modeled inverse filter is then deconvolved with the entire 3-D seismic volume. In our experiments, this seismic inversion workflow proved to be extremely fast and effective.

II. Revision

Proposed by Dorigo in his Ph.D. thesis [12], ACO is a probabilistic technique that has been widely used in a large class of practical and benchmark combinatorial optimization problems. This metaheuristic attempts to exploit the same behavior of real

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ants during foraging. The ants walk between the nest and the food source, following different paths, each one with different lengths. In the course of time, longer paths become less desirable, while shorter paths are highly chosen. This behavior emerges because ants deposit a substance called pheromone on the way where they walk through. The quantity of this substance can be noticed by other ants which probabilistically choose a path to follow by using this information. The ants on the shortest paths come and go in less time than those on the largest paths, which implies more pheromone concentration in these short paths. Dorigo *et al.* in [13] explain how the shorter path prevails at ants' behavior by making an analogy with artificial ants that are used in ACO metaheuristic.

Due to its good performance, several ACO variations for general continuous-domain problems were proposed in the last years, and some of them showed state-of-the-art results for benchmark problems [14], [15], [17], [18]. In recent years, swarm intelligence optimization algorithms have been gradually introduced into the field of geophysics [19], [11]. Yuan *et al.* [11] state that swarm intelligence techniques have advantages, such as the absence of centralized control and an overall model. Swarm intelligence techniques are also capable of providing new interpretations for solving complex nonlinear inversion problems; however, the potential of ACO algorithms in seismic inversion problems was not largely explored.

The capacity to incorporate heuristic information is a key issue in a search optimization algorithm, and it is essential to allow finding good solutions with a minimum number of function evaluations [20].

Socha in [14] proposed an extended ACO algorithm for continuous domains called $\mathrm{ACO}_{\mathbb{R}}$. In this algorithm, the main data structure is the *population archive*. The population archive is a list that contains a specified number of k (algorithm parameter) solutions for the problem. The idea of the population archive is to have a probability distribution for each problem's variable, so that we have an individual probability density function for each variable of the solution and the combination of them represents the knowledge of the colony. This probability distribution guides the search through the solution space. For each problem's variable, we can obtain a mean value and a standard deviation from the population archive. Using a normal distribution sampling method with the mean value defined by the chosen solution s_l [(2)], each ant generates a new candidate solution. The standard deviation is calculated as the average distance for each variable of the chosen solution to all the other solutions in the archive.

At the beginning of the algorithm, the population archive can be initialized with random or heuristically guided values for the n variables s_l^i $(i = 1, 2, \ldots, n)(l = 1, 2, \ldots, k)$. During the run, the solutions in the archive are used by the ants to generate new ones. Those newly constructed solutions can replace the worst ones present in the archive. By doing so, the probability distribution of each variable is changed. This vector of solutions is kept orderly at all times

$$f(s_1) \geq f(s_2) \geq \cdots \geq f(s_l) \geq \cdots \geq f(s_k) \tag{1}$$

where s_1 and s_k are the first and the last solutions in the archive, respectively, and $f(s_l)$ is the value of the objective function to the lth solution. The best solutions are preferably chosen in the construction step, given as

$$p_l = \frac{\omega_l}{\sum_{i=1}^{k} \omega_i} \tag{2}$$

where p_l is the probability of solution s_l to be chosen and ω_i $(i = 1, 2, \ldots, k)$ is defined as

$$\omega_i = \frac{1}{qk\sqrt{2\pi}} e^{-\frac{(i-1)^2}{2q^2k^2}} \tag{3}$$

and represents the amount of pheromone on the solution s_i. The variable q is a parameter of the algorithm, where low values of q imply that the best solutions are preferably chosen. On the other hand, high values for q imply that the solutions will be chosen with more equal probability.

In the construction of a new candidate solution, for each variable of this new solution, the sampling process uses the value s_l^i, which is the dimension i of the solution s_l [chosen according to (2)], as the mean of a normal probability distribution to sample the value of dimension i for the new candidate solution. The standard deviation in the sampling process is defined as

$$\sigma_l^i = \xi \sum_{e=1}^{k} \frac{\left|s_e^i - s_l^i\right|}{k-1} \tag{4}$$

where ξ is an algorithm parameter so that a high value of ξ implies a low convergence speed.

When the values for all dimensions of this new solution have been sampled, the candidate solution is evaluated and kept in an auxiliary population archive. Only at the end of the iteration, when all the ants have constructed their solutions, these new solutions are moved from the auxiliary archive to the population archive, and the same number of worst solutions is removed from the population archive. This construction step is repeated at each iteration, and at each iteration, each ant generates a new candidate solution.

In the next section, we present an extension of the $\mathrm{ACO}_{\mathbb{R}}$ algorithm that we called $\mathrm{ACO}_{\mathbb{R}}$-$V$. By incorporating a variety of heuristics, our extension can achieve a high performance when applied to seismic inversion problems. More implementation details can be seen in [21].

III. Methodology

Most of the seismic inversion methods are based on forward modeling convolution. If we assume a convolutional model for the seismic trace

$$s(t) = r(t) \otimes w(t) \tag{5}$$

where $s(t)$ is the observed seismic trace, $r(t)$ is the true reflectivity, and $w(t)$ is the wavelet, then the goal of seismic inversion is to find the inverse operator $v(t)$ which, when convolved with the seismic traces $s(t)$, gives the reflection coefficient [22].

The first part of our seismic inversion method is *wavelet estimation*. $\mathrm{ACO}_{\mathbb{R}}$-$V$ uses the acoustic impedance logs from the drilled wells and seismic traces in the neighborhood from each well position. With these data, the goal is to find the best vector of amplitude values that, when convolved with the reflectivity of the well logs, results in synthetic seismic traces similar to those observed around the well logs. In the

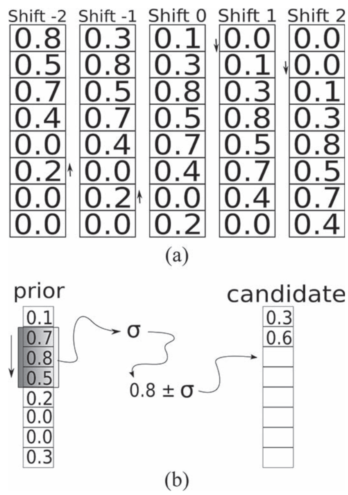

Fig. 1. Shift and sliding window heuristics. (a) Shift heuristic with parameter value of two. (b) Sliding window heuristic.

presented methodology, we do not use any prior model to begin the optimization of the estimated wavelet; thus, the candidate solutions initially generated by the algorithm are random values inside a defined domain restriction (amplitude value limitation). Moreover, heuristic methods for generating a prior model of the wavelet could be easily used to generate the initial population of candidate solutions, which could accelerate the convergence of the algorithm.

The second step in this methodology involves the utilization of the estimated wavelet from the previous step and the ACO$_{\mathbb{R}}$-V to do a *trace-by-trace* inversion. The process starts from the seismic traces in the neighborhood of the well log. Once the reflectivity pattern is known at well position, the algorithm uses the presence of lateral continuity of rock layer characteristic (except by occasional presence of faults) and initializes the population archive with solutions similar to the well. The appropriate choice of the initial population of candidate solutions can accelerate the convergence speed of the algorithm when compared with a random initialization of the population. Therefore, we developed two heuristics to initialize the population archive: the *shift* and the *sliding window*, as can be seen in Fig. 1.

Fig. 1(a) represents the process where ACO$_{\mathbb{R}}$-V creates several solutions similar to the well but has slipped some positions in both the vertical directions. We do this to consider an eventual dip of a rock layer, a situation where the horizontal change of the acoustic impedance model just differs by a small displacement when compared with the previous position and the shift heuristic can generate solutions closer to the optimal solutions, accelerating the convergence. The number of shifted solutions that ACO$_{\mathbb{R}}$-V creates is an algorithm parameter, and a large number implies several slips with all the intermediate values of shift heuristic from the negative value of the parameter up to the number of the parameter. In other words

$$\text{shift} = n \begin{cases} \text{shift} = -n \\ \text{shift} = -n+1 \\ \cdots \\ \text{shift} = n. \end{cases} \tag{6}$$

Fig. 1(b) represents the sampling of new solutions whose values are near the well. This heuristic uses a sliding window where all the values inside it are used to calculate a standard deviation, and then, the values for a new solution are sampled, setting the value in the center of the window as the mean. As the window slides down in the trace, new values are sampled, and the result is a candidate solution close to but not equal to the well. The sliding window heuristic allows sampling far values in parts of the impedance log with high variation, while in areas with low variation, the sampled values of the candidate solution are similar to those of the same time position in the well log.

To take advantage of the proposed heuristics, the ACO$_{\mathbb{R}}$-V algorithm starts the inversion process from seismic traces beside the drilled well. This start position is required owing to the use of the well log information in both heuristics described in the previous paragraph. When the optimization is finished for these traces in the neighborhood of the well, these optimized impedance trace solutions are considered as pseudowells, and the inversion process continues for the other seismic traces' neighbors from these new ones, until all the seismic traces have their respective optimized impedance trace, i.e., the optimization process starts near the well and propagates to the borders, using the already optimized traces as wells for the heuristics previously mentioned.

The other approach that we propose for reservoir characterization is *deconvolution filter estimation*. It works similar to ACO$_{\mathbb{R}}$-V wavelet estimation, the only difference being the goal. ACO$_{\mathbb{R}}$-V wavelet estimation looks for the best signal that convolved with the reflectivity vector of the well and results in a synthetic seismic trace close to the observed seismic trace from the same well. By contrast, ACO$_{\mathbb{R}}$-V deconvolution filter estimation looks for the best signal that deconvolved with the observed seismic trace from the well and results in a synthetic reflectivity vector close to the reflectivity vector of the well.

IV. Tests and Results

For the tests presented in this section, we used an inline from a synthetic cube, which was generated based on realistic rock properties. The seismic and the impedance volumes were provided to us by an oil company as part of a research cooperation project. The used inline has values measured for 250 different depth positions in time for each one of the 199 traces in horizontal displacement, i.e., two matrices of 250 × 199, one with seismic amplitude values and the other with acoustic impedance values. The impedance model of the described inline is presented in Fig. 2, and its respective poststack seismic traces are presented in Fig. 3. These seismic traces have 30% of noise relative to the amplitude of the signal simulating a real seismic acquisition.

In the ACO$_{\mathbb{R}}$-V algorithm parameters, we used the population size $k = 50$, $q = 0.0001$, $\xi = 0.85$, and number of ants $= 2$, which are the values recommended in the original paper of ACO$_{\mathbb{R}}$ [15]. This indicates that the tuning of parameters is not critical in the algorithm. For the number of variables n, in the wavelet estimation and deconvolution filter estimation methods, we used $n = 41$ and $n = 31$, respectively. Each variable or "point" is an amplitude value in the candidate solution vector that shapes the signal. These values for n were defined after some previous tuning runs. For the trace-by-trace inversion, $n = 250$ was used in a way that each candidate solution represents a reflectivity trace of the inline.

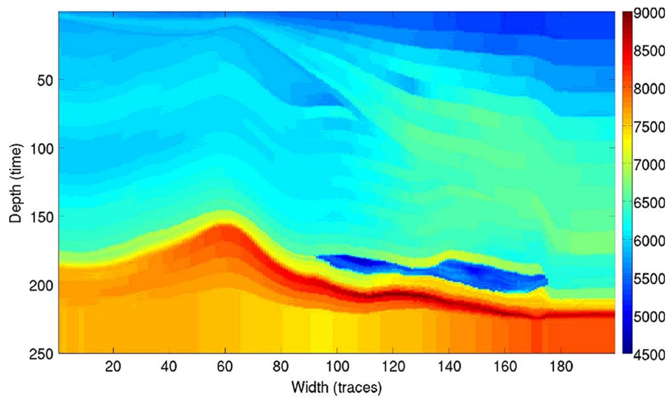

Fig. 2. Acoustic impedance model with the desired values.

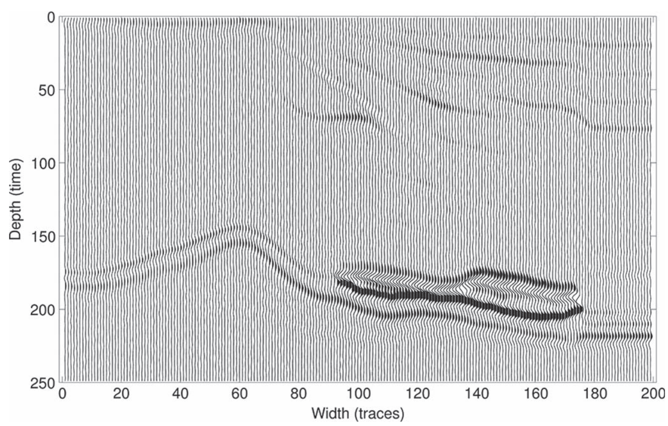

Fig. 3. Used seismic traces with 30% of noise.

The stopping criterion for all the experiments was a measure of how much improvement was obtained in the fitness function in the last 500 iterations. When this measure is lower than 10^{-6}, the algorithm assumes that a convergence situation was reached. We defined the value 10^{-6} as the threshold after watching the fitness evolution in previous tuning runs. Other algorithm parameters are the shift $= 5$ and the size of the sliding window $= 5$, both defined after some previous tuning runs.

We considered a well at a position of 100 in the horizontal axis (width), and it is the only well log information that we use in the experiments.

All the simulations were performed in a Sun Ultra 27 Workstation with eight processors of 3.2 GHz of frequency, 8 MB of cache, and 12 GB of RAM DDR3, running an Ubuntu Linux 10.04 64-b operating system, and the implementation language was C++.

The better (lower error) wavelet estimated by the ACO$_{\mathbb{R}}$-V algorithm is presented in Fig. 4. The run took 6 s to perform 30 replications to estimate the wavelet.

To optimize the acoustic impedance model, the ACO$_{\mathbb{R}}$-V algorithm run took 3 min and 2 s to generate four replications, and the result is an average of them. The process used the wavelet estimated in the previous step to perform the convolution with candidate solutions during optimization. Fig. 5 shows the acoustic impedance model obtained with its low-frequency band until 3 Hz replaced by reflection-moveout velocity information and a velocity–density relationship to convert velocity to acoustic impedance. This is done because this low-frequency band is absent in the seismic traces [23].

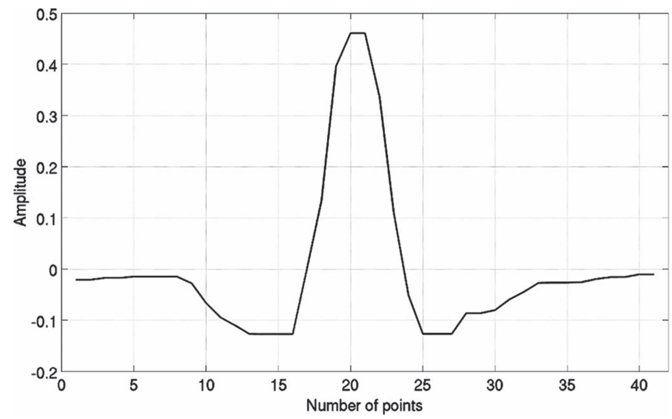

Fig. 4. Wavelet estimated by ACO$_{\mathbb{R}}$-V algorithm.

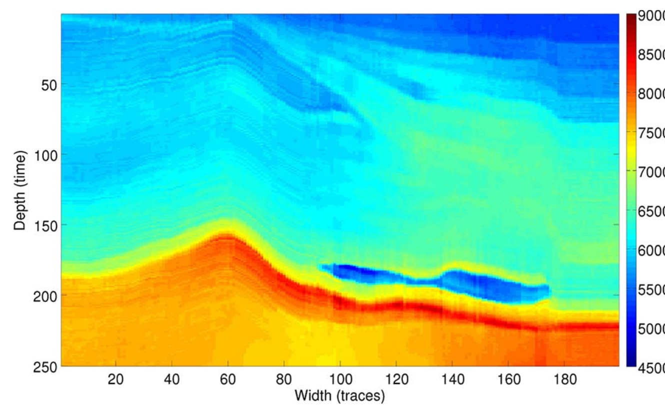

Fig. 5. Acoustic impedance model obtained from the inverted volume.

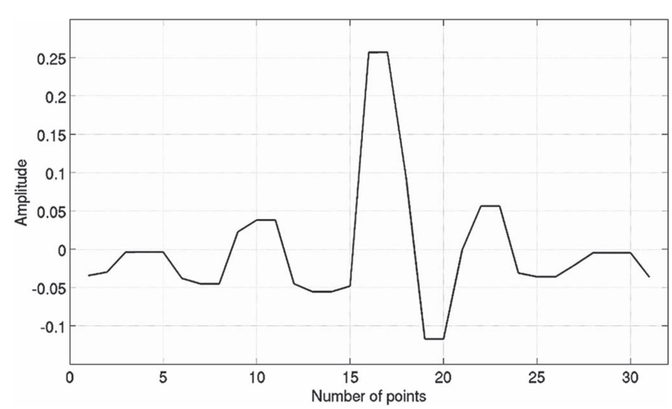

Fig. 6. Deconvolution filter estimated by the ACO$_{\mathbb{R}}$-V algorithm.

The correlation between the desired acoustic impedance model and that achieved by the ACO$_{\mathbb{R}}$-V algorithm has a coefficient $R = 0.97809$. The correlation was performed with both models in the frequency band limited between 6 and 40 Hz because we are evaluating only our inversion method and not the velocity analysis cited in the previous paragraph. Furthermore, if the correlation is done in the obtained model with the low frequency, the correlation coefficient is $R \approx 0.99$.

The filter estimated by the ACO$_{\mathbb{R}}$-V algorithm is presented in Fig. 6, and it took 18 s to optimize 30 filters. As for the wavelet, we use the filter with the lower error inside all the

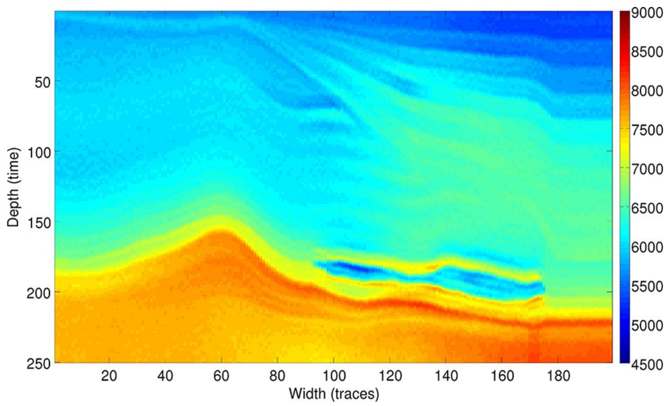

Fig. 7. Acoustic impedance model obtained from the deconvolved volume.

30 parallel replications. This filter was deconvolved with all the seismic traces of the volume to generate the achieved acoustic impedance model. Fig. 7 shows the acoustic impedance model obtained with its low-frequency band until 3 Hz replaced by the information from velocity analysis, as we did for the trace-by-trace inversion.

The correlation between the real acoustic impedance model and that achieved by the ACO$_{\mathbb{R}}$-V deconvolution filter has a coefficient $R = 0.95604$. The correlation was performed with frequency-band-limited models as we did in the previous experiment.

These models optimized trace by trace by the ACO$_{\mathbb{R}}$-V and deconvolution filter can support a fast decision in improving estimation of reserves and making decisions regarding the development of the field [2], [24].

Despite that the presented experiments were conducted over synthetic data, we also applied our techniques to real data, obtaining a similar performance. We did not present the real-data experiments owing to the confidentiality agreement with the oil company. The results of our methods were compared with the results of state-of-the-art techniques used by the oil company.

V. Conclusion

In this letter, we have proposed the use of the swarm intelligence technique, i.e., ACO, to seismic inversion problems. ACO is a metaheuristic method that presents state-of-the-art results for benchmark problems; however, it was so far not extensively explored in application to industry problems.

ACO$_{\mathbb{R}}$ was chosen for being an extension of the ACO for a continuous domain with close similarity to the original ant-based idea. The modifications that we did in the technique, which we have named ACO$_{\mathbb{R}}$-V, create several initial solutions by heuristics described in Section III, and we believe that this behavior implies an improvement to the convergence speed of the algorithm based on several previous tuning runs. The proposed heuristics provide good initial candidate solutions instead of random ones.

All the techniques, i.e., wavelet estimation, trace-by-trace optimization, and deconvolution filter estimation, produce relevant results because they show solution values that are close to the desired values in a small amount of time. The ACO$_{\mathbb{R}}$-V algorithm can be an alternative to traditional methods with a fast response and without the need of setting many parameters.

Research about the applications of ACO$_{\mathbb{R}}$-V in seismic inversion problems with more constraints, such as the derivation of shear impedance, porosity, and other rock properties, is ongoing, and it will be the subject of future publications.

References

[1] R. P. Srivastava and M. K. Sen, "Fractal-based stochastic inversion of poststack seismic data using very fast simulated annealing," *J. Geophys. Eng.*, vol. 6, no. 4, pp. 412–425, Dec. 2009.
[2] G. Robinson, "Stochastic seismic inversion applied to reservoir characterization," *CSEG Recorder*, vol. 4, pp. 37–40, Jan. 2001.
[3] E. P. Leite and A. C. Vidal, "3D porosity prediction from seismic inversion and neural networks," *Comput. Geosci.*, vol. 37, no. 8, pp. 1174–1180, Aug. 2010.
[4] "Understanding stochastic seismic inversion," Earthworks Environ. Resource. Ltd., Salisbury, U.K., 2006, Tech. Rep.
[5] S. Kirkpatrick, C. D. Gelatt, and M. P. Vecchi, "Optimization by simulated annealing," *Science*, vol. 220, no. 4598, pp. 671–680, May 1983.
[6] M. K. Sen and P. L. Stoffa, "Nonlinear one-dimensional seismic waveform inversion using simulated annealing," *Geophysics*, vol. 56, no. 10, pp. 1624–1638, Oct. 1991.
[7] X.-Q. Ma, "Simultaneous inversion of prestack seismic data for rock properties using simulated annealing," *Geophysics*, vol. 67, no. 6, pp. 1877–1885, Nov. 2002.
[8] J. Holland, *Adaptation in Natural and Artificial Systems*. Ann Arbor, MI: Univ. Michigan Press, 1975.
[9] M. J. Porsani, P. L. Stoffa, M. K. Sen, and R. Chunduru, "Fitness functions, genetic algorithms and hybrid optimization in seismic waveform inversion," *J. Seismic Explorat.*, vol. 9, no. 2, pp. 143–164, 2000.
[10] P. L. Stoffa and M. K. Sen, "Nonlinear multi-parameter optimization inversion of plane-wave seismograms using genetic algorithms," *Geophysics*, vol. 56, no. 11, pp. 1794–1810, Nov. 1991.
[11] S. Yuan, S. Wang, and N. Tian, "Swarm intelligence optimization and its application in geophysical data inversion," *Appl. Geophys.*, vol. 6, no. 2, pp. 166–174, Jun. 2009.
[12] M. Dorigo, "Optimization, learning and natural algorithms," Ph.D. dissertation, Dip. Elettronica e Informazione, Politecnico di Milano, Milano, Italy, 1992.
[13] M. Dorigo, V. Maniezzo, and A. Colorni, "Ant system: Optimization by a colony of cooperating agents," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 26, no. 1, pp. 29–41, Feb. 1996.
[14] K. Socha, "ACO for continuous and mixed-variable optimization," *Ant Colony Optim. Swarm Intell.*, vol. 3192, pp. 25–36, 2004.
[15] K. Socha and M. Dorigo, "Ant colony optimization for continuous domains," *Eur. J. Oper. Res.*, vol. 185, no. 3, pp. 1155–1173, Mar. 2006.
[16] P. C. H. Veeken and M. Da Silva, "Seismic inversion methods and some of their constraints," *First Break*, vol. 22, no. 6, pp. 47–70, Jun. 2004.
[17] H. Huang and Z. Hao, "ACO for continuous optimization based on discrete encoding," *Ant Colony Optim. Swarm Intell.*, vol. 4150, pp. 504–505, 2006.
[18] M. Kong and P. Tian, "A direct application of ant colony optimization to function optimization problem in continuous domain," *Ant Colony Optim. Swarm Intell.*, vol. 4150, pp. 324–331, 2006.
[19] S. Yuan, N. Tian, Y. Chen, H. Liu, and Z. Liu, "Nonlinear geophysical inversion based on ACO with hybrid techniques," in *Proc. 4th Int. Conf. Natural Comput.*, Oct. 2008, pp. 530–534.
[20] W. Bangerth, H. Klie, M. F. Wheeler, P. L. Stoffa, and M. K. Sen, "On optimization algorithms for the reservoir oil well placement problem," *Comput. Geosci.*, vol. 10, no. 3, pp. 303–319, Sep. 2006.
[21] C. R. Conti, M. Roisenberg, and G. S. Neto, "ACOR-V—An algorithm that incorporates the visibility heuristic to the ACO in continuous domain," in *Proc. IEEE CEC*, 2012, pp. 2910–2917.
[22] D. W. Oldenburg, T. Scheuer, and S. Levy, "Recovery of the acoustic impedance from reflection seismograms," *Geophysics*, vol. 48, no. 10, pp. 1318–1337, Oct. 1983.
[23] M. Becquey, M. Lavergne, and C. Willm, "Acoustic impedance logs computed from seismic traces," *Geophysics*, vol. 44, no. 9, pp. 1318–1337, Sep. 1979.
[24] H. Beucher, D. Renard, B. Doligez, M. Pontiggia, and G. Bellentani, "The effect of methodology on volumetric uncertainty estimation in static reservoir models," *AAPG Bull.*, vol. 92, no. 10, pp. 1359–1371, Oct. 2008.