Mathematics Family Tree

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In this article we present a simple argument that explains brotherhood or fraternity in terms of an exponential growth model called 'the genealogy paradox'. Looking at a family tree today, the mathematics about the calculations of how many descendants there were since the beginning of our Era shows that we should expect as many as 604 sextillion people in the past. This occurs because, in theory, a person's ancestor tree should be a binary tree, formed by the person, the first generation back with two parents, the second generation back with four grandparents, the third generation back with eight great-grandparents, and so on. Thus, there appears to be more ancestors in these early generations than available people. This astronomical number is absurd, and one explanation is related to marriages between relatives – another reason could be related to migration. In conclusion, it is easy to observe that we are all members of the same family tree, the tree of life.

We have more in common than it seems. In addition to most religions, mathematics can say with great certainty that *yes*, we are all brothers and sisters. If we were not all related, each inhabitant of the earth today would have an astronomical number of ancestors at the *Anno Domini* (AD), or at the time when Christ was born.

If we follow simple reasoning there is a curious property about the genealogy of each individual: for anyone today, there were two people to generate it, father and mother. To generate this father and this mother were needed four people: father and mother to the father, mother and father to the mother, which means four grandparents. Considering an average duration of 25 years for each generation, we find that from AD 1 there has been 80 generations. This leads us to the conclusion, completely absurd, that the existence of a single person in the early twenty-first century required the incredible number of 2^{79} (or 604 sextillion) people in AD 1. According to some astronomers, there are around 300 sextillion stars in the universe (van Dokkum and Conroy, 2010).

Although the mathematical reasoning is correct, it is clear that such a result is nonsense. Note: each individual should have two parents, four grandparents, eight great-grandparents and sixteen great-great-grandparents in a century. The evidence of exponential growth is clear: for each generation that we look back the number of people just doubled. Certainly the difference in age between generations varies widely, gradually increasing as we get into modern life – but to help with the calculations, the premise is quite acceptable. So, over the past two thousand years, for each generation that lived at least 25 years to create children, we should expect at least 80 generations until the birth of Christ, as we go back in time. This typical generation length was based on the average age of mothers at childbirth, and is approximately 26 ± 2 years (Pattison, 2001). Just for comparison, going backwards in time until the first group of anatomically modern *Homo sapiens*, Derrida *et al*. (1999) believe that there existed around 4000 generations, considering 105 years of human history.

If the calculations are consistent, what is wrong? Where is the error? An acceptable answer to this amazing situation is to consider marriages between *relatives*. Another suitable answer is related to *migration*. Although hard to admit today, it is quite plausible that this would have happened in the past (Pattison, 2001). A hundred years ago ordinary people lived in much smaller communities, long distance travel was not common and communication was much harder. The common historical inclination of small and isolated populations was to marry those within walking distance (Pattison, 2001). In practical terms, close relatives are likely to live next to somebody, strengthening family ties, to be in the same social class, to share the same religion, culture, tradition, ethnic group, language as well as to be part of the same social circle (Pattison, 2007).

This apparent 'genealogy paradox' is also named 'pedigree collapse' (Pattison, 2001). As far as the authors know, the term was coined by the American military by Robert C. Gunderson (1931–2003). It is clear from the problem that the number of ancestors in any generation can never exceed the population at any time. Basically, we have a bunch of slots on our family trees, but some of them are filled by the same people. For example, in some tribes in Amazonia it is common for natives to marry their first cousins – a term coined as 'alliance theory'. This behaviour was observed by researchers such as Claude Lévi-Strauss (1908−2009) (Lévi-Strauss, 1969), who was a visiting professor of sociology at the University of São Paulo, Brazil, between 1935 and 1939. Other recent studies in Iceland (Helgason *et al*., 2008) found that third cousins produced more children and grandchildren than more distant marriages, observing data from one and a half centuries from an isolated part of the world. Thus, in general the offspring of two first cousins has at most only six great-grandparents instead of the normal eight. This decrease in the number of ancestors is the 'pedigree collapse'.

Royal intermarriage was often practised among European noble families, habitually for interests of state. In particular, among royalty, at the *House of Habsburg* (also known as the House of Austria), the frequent requirement to only marry other royals resulted in a reduced gene pool in which most individuals were the result of extensive pedigree collapse. Alvarez *et al*. (2009) studied the Spanish Habsburg kings from an extended pedigree up to 16 generations in depth, from 1516 up to 1700, and involving more than 3000 individuals, until his last king, Charles II. In total, 9 (81.8%) of 11 marriages were consanguineous unions in a degree of third cousins or closer.

In brief, our ancestors along the way married their cousins, frequently without knowing it but in many situations intentionally. The closer the cousin, the bigger is the collapse percentage. Well known examples are related to the English naturalist and geologist Charles Darwin (18091882), that he was the grandchild of first cousins, and married a first cousin, Emma. So did the German physicist Albert Einstein (1879–1955) in his second marriage with Elsa or the American poet Edgar Allan Poe (1809–1849), with Virginia (Conniff, 2003).

In fact, a closer look reveals that moderate inbreeding has constantly been the rule, not the exception in human history – some experts assume that up to 80% of all marriages in history have been between second cousins or closer (Pattison, 2007; Conniff, 2003). Some authors, such as Derrida *et al*. (1999), affirmed, according to their model, that when we look very far in the past, about 80% of the (adult) population appears in the genealogical tree of every individual. There are good manuscripts on the subject, relating human biological origins, evolutionary development, inbreeding and genetic diversity, even considering isolated or semi-isolated populations (Pattison, 2001; 2007).

At the present time, reflections such that we are a little more than 7.3 billion people and we have much more in common than we generally realize is a great encouragement. Messages such as peace and equality between men could also be presented in mathematical terms, following this reasoning on the genealogy of each person. Due to simplicity, this argument could be learned from an early age at schools, and possibly linking mathematics and fraternity.

In conclusion, we are more related to each other than we think, despite the visible differences on belief, ideology, race, class, religion, cultural traits, language or economic *status*. As it seems naive a man looking to convince himself that, not being a branch of the same tree, is not at least a tree of the same forest. We really are fruits of the same family tree, the tree of life.

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