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TEXTO PARA DISCUSSÃO Nº 21

SRAFFA AND THE INVERSE TRANSFORMATION PROBLEM: AN EMPIRICAL EXPERIMENT TO BRAZIL - 1969

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1983

"The exchange of commodities at their values, or approximately at their values requires a _much lower stage_ than their exchange at their prices of production, which requires a definite level of capitalist development (...) It is appropriate to regard the values of commodities as not only _theoretically_ but also _historically_ antecedent (_prior_) to the prices of production"

(F. Engels)

"What kind of correspondence between logic and history would it be necessary to demonstrate in order to prove that Marx's transformation of values into prices of production, although logical in form, also possessed a significant historical dimension?"

(R. Meek)

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PART 1 - The Model

1.1 - Introduction : The so-called Sraffa model has made its appearance in 1960, when the italian economist Piero Sraffa published his book "Production of Commodities by Means of Commodities". (1) Sraffa had spent some 43 years of hard work and research, during which had collected and edited (1951 - 10 volumes) "The Works and Correspondence of David Ricardo", bringing light to the economic perspective of the classics and inducing a "come-back" to the study of Ricardo and Marx.

In Joan Robinson's words :(2) "It is no wonder that this book took a long time to write. It will not be read quickly. Addicts of pure economic logic who find their craving ill-satisfied by the wishy-washy products peddled in contemporary journals have here a doubled-distilled elixir that they can enjoy drop by drop, for many a day."

In fact, since its publication, this book has provoked considerable turbulence on the academic - and to some extent to the non-academic - economic world and was a departing point for a long discussion throughout the 60', known as "Cambridge Controversy", where the central subject was the "reswitching of tecniques" (3), what would make the whole marginalist neoclassical theoretical framework be shaken, in addition to attack the concept of "capital" as a factor of production - that could be measured unambiguosly - and from which one could calculate the "remuneration of the capital

(1) P. Sraffa, "Production of Commodities by Means of Commodities" - The Syndics of the Cambridge University Press - Cambridge 1960.

(2) J. Robinson - Prelude to a Critique of Economic Theory - Oxford Economic Papers - Vol 13 - 1961 - pp 7-14 - in "A Critique of Economic Theory" p. 197 E.K. Hunt & J. Schwartz - Penguin Education

(3) P. Sraffa - op cit . Chap XII

factor" along the well-known relation of marginal productivities.

Apparently it seems that the bulk of the attention payed to Sraffa's work has fallen on these subjects, giving rise to an increasing volume of criticism to the marginalist or/and neoclassical theory, almost always departing from his brilliant conclusions on his Chapter XII. On the one hand, there's no doubt that this was one of the intentions of the author, who declares explicitly in his Preface: "It is, however, a peculiar feature of the set of propositions now published that, although they do not enter into any discussion of the marginal theory of value and distribution, they have nevertheless been designed to serve as the basis for a critique of that theory". (4)

Nevertheless, on the other hand we think that a series of aspects of his work were, on general, surprisingly not well grasped and if we do not take into consideration what Sraffa has called his "theoretical cements", we could be led into incompatible and erroneous interpretations of his model that would even be contradictory to the authors' positions and theoretical formation.

As we believe that it is necessary to apprehend the "sraffian climate" we will try to present a brief discussion of the assumptions of the model, aiming to relate them to the "natural historical sources" from which they seem to derive

1.2 - Assumptions

In an apparent tentative to avoid preconceptions - no matter where they come from - Sraffa has adopted a somewhat original form of exposition and created his own terminology. This must be brought to our attention, because we must be aware of this for not making confusions between the usual meaning of the concepts and Sraffa's usage of them.

For instance, we are facing an equilibrium model, where the

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(4) P. Sraffa - op cit p. VI

term "price" must not be taken as "market prices". In fact, we'll show that "price" in Sraffa's model is much nearer the Marxian "production prices" than the observed - or neoclassically adopted - "market prices". Another important fact is that whenever we have a surplus production on our system, Sraffa doesn't make any conjecture on whether this surplus is incorporated in the next period or not, not being an object of his analysis the destination that this surplus is going to have - productive or improductive consumption, increase on inventories or means of production's stock, or purely losses or littering. In Sraffa's words : "No changes in output and (...) no changes in the proportions in which different means of production are used by an industry are considered, so that no question arises as to the variation or constancy or returns. The investigation is concerned exclusively with such properties of an economic system as do not depend on changes in the scale of production or in the proportions of "factors" ". (5)

Logically the reason for this is to avoid bringing about the old problem of decreasing returns to scale - initially formulated by Ricardo for agricultural economies and after that adopted by the marginalist school, whose application to the industrial production was and is a "sine qua non" condition for the paretian optimality solution. In fact Sraffa had already worked on this subject, questioning the theoretical legitimacy of this extension in an article published in 1926. (6) Therefore it is clear that the author was willing not to enter again in this kind of discussion since for him it was unnecessary to make ANY assumption related to the returns to scale to proceed in his analysis. (7)

"The marginal approach requires attention to be focused on change, for without change either in the scale of an industry or

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(5) P. Sraffa - Op cit pp V - Preface - Sraffa's italics

(6) P. Sraffa - "The Laws of Returns Under Competitive Condition"- The Economic Journal - 1926

(7) P. Sraffa - Op Cit - p.V Preface - Sraffa's italics

in the "proportions of the factors of production" there can be neither marginal product nor marginal cost. In a system in which, day after day, production continued unchanged in those respects, the marginal product of a factor (or alternatively the marginal cost of a product) would not merely be hard to find - it just would not be there to be found."

Sraffa assumes an isolated system : there's no exchange with sectors that are not explicitly in the system. This brings about the necessity of self-regeneration for (at least) the reproduction of the initial conditions of production by the end of each period, say, an year.

In our description, (and later, in our simulation) we will adopt just the assumptions set forth by Sraffa in the Part I of his book, that is : a) each industry produces one and just one product, in addition to be the only one which produces this product; b) all the means of production utilized during the period of production are scrapped - or totally consumed - by the end of the period, i.e., the capital is circulating.

It is implicitly supposed competition among producers, although this doesn't mean perfect, imperfect, oligopolistic or something like that. We have simply to imagine that the producers compete among them in a way that we can observe a tendency for their rate of profits to be levelled throughout the system. (8)

Finally, the fundamental assumption - from which the author takes the title of his work - all the commodities are produced using labor and other commodities, what immediately confer to labor his former ricardian status of only factor of production, only source of value.

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(8) A tentativelly extension to non-competitive markets could be found in Sylos-Labini, P. - " Introduzione di forme di mercato non concorrenziàl nello schema di Sraffa e passaggio alla riproduzione allargata " - 1969 - mimeo. original in italian.

1.3 - The Production of Subsistence - 2 Products

To begin with, Sraffa asks us to imagine a very simple society that produces exactly enough to survive, and where we could find just 2 sectors or "industries" : wheat and iron. Let us suppose that these products enter in their productions after exchange, no matter if as seeds and tools or as foodstuff or housing for the workers. Therefore we have :

Wheat industry - 280 qr. wheat @ 12 t. iron → 400 qr. wheat
Iron industry - 120 qr. wheat @ 8 t. iron → 20 t. iron
400 qr. wheat 20 t. iron

where @ means composition

As we see that everything that is produced in a period is totally consumed in the next, without altering the scale of production, we are facing a production of subsistence or simple reproduction.

Being our first objective to settle some concepts, we would like to pose the question : what is the wheat's price? And iron's? But, how to find these prices if we do not have the demand and supply curves for these products? Furthermore in what effective conditions are these markets? Without any doubt the frequent use of the marginalist approach make us to formulate these questions in these ways.

In fact, though the use-values that the industries exchange * are irrefutably a subjective category, there is a unique exchange-value (for these "surpluses" not consumed in the industries where they were produced), that will allow to the system the repetition of the productive process. Therefore we are pushed to face a necessary equivalence for the continuity of the system, traduced on

10 qr. wheat ⊜ 1 t. iron see figure 1.1

1.4 - Relative Prices and Equilibrium Prices

We have obtained therefore "relative prices" for the products wheat and iron that must independ on the monetary standard used.

120 qr.. wheat=12 t.. iron
or
10 qr.. wheat=1 t.. iron
that is

$$Pt=\frac{Pf}{10}$$

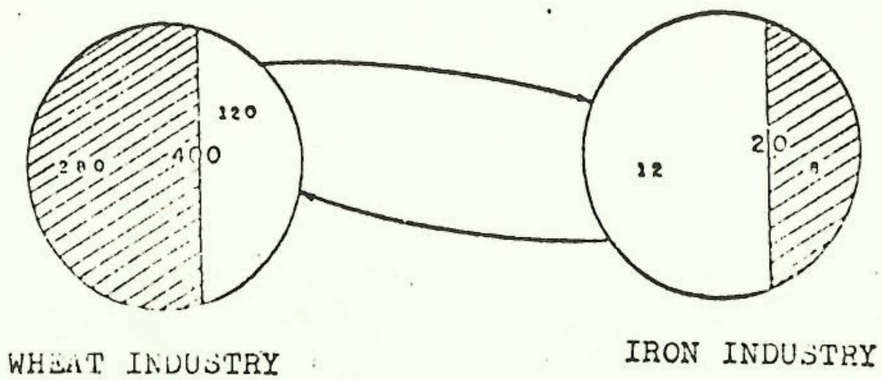

FIGURE 11.

Furthermore, these prices must be interpreted as "equilibrium prices" and it is easy to show (as does R. Meek (9)) that, being relative prices, they would be in the same proportion as the quantities of labor, direct or indirectly used up in their productions. This means that the exchange value of each final product will equal the value of the social labor expended, and the ratio of prices will have the same magnitude of the ratio of labor quantities "embodied" in each product.

It would be interesting to note that if the iron industry decides to rise its prices from pi = 10 pw to pi = 12 pw (receiving now the same 120 qr. of wheat in exchange for just 10 t. of iron) making a "profit", in the next period the wheat industry would not be able to produce its traditional 400 qr. - because its iron input has fallen by 1/6 - and would not be able to supply the 120 qr. of wheat for exchange with the iron industry. In these conditions the persistence of iron's "market prices" artificially above its "equilibrium price" would gradually lead the system to colapse (with sequentially diminishing levels of production of wheat and iron, together with increasing stocks of iron) or alternatively to a fall of iron's "market prices" to a level below its "equilibrium price" until the proportions adjust.

In fact, this is not just a mental exercise; this is the dynamic idea of this equilibrium price that should be kept in mind. We can see that it is not necessary that the producers have any awareness of their equilibrium prices : the fact is that on the long-run, after sucessive trial-and-error processes, fluctuation of market prices upward and downward, we verify a reasonable (and necessary) accordance between long run market prices and the equilibrium prices <u>for the system to keep reproducing itself</u>. Therefore, as far as we went, the term <u>price</u> in Sraffa is used in the same sense as VALUE in Marx, "natural price" in Adam Smith, or "necessary price" in the physiocrats and must be never confounded with "market price".

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(9) R. Meek - " Mr. Sraffa's Rehabilitation of Classical Economics" Scottish Journal of Political Economy - Jun/ 1961

1.5 - The Production of Subsistence - General Case

Posing the problem in general terms, we have a system where the commodities a, b, ... , k are produced according to :

$$
\begin{array}{ccccccccc}
Aa & \oplus & Ba & \oplus & \dots & \oplus & Ka & \rightarrow & A \\
Ab & \oplus & Bb & \oplus & \dots & \oplus & Kb & \rightarrow & B \\
\multicolumn{7}{c}{\text{-------------------------}} & & \\
Ak & \oplus & Bk & \oplus & \dots & \oplus & Kk & \rightarrow & K \\
\hline
A & & B & & \dots & & K & &
\end{array}
$$

where A,B,...,K are the total quantity of commodities a,b,...,k produced annually and Aa, Ba,..., Ka the total quantities of the commodities a, b, ..., k used up annually in the total production of commodity a and so on. Of course it is not necessary to assume that every commodity enter directly in the production of one particular commodity, what means that some of those coefficients may be zero.

According to the assumptions of self-regeneration and simple reproduction the contour conditions must be satisfied:

$$
\begin{array}{l}
Aa + Ab + \dots + Ak = A \\
Ba + Bb + \dots + Bk = B \\
\text{-----------------------} \\
Ka + Kb + \dots + Kk = K
\end{array}
$$

We can now associate to this physical production system a price system (in our case still equal to their exchange values), where the unknowns to be determined pa,pb,...,pk will be respectively the prices of commodities a,b,...,k :

Aa pa + Ba pb + ... + Ka pk = Apa

Ab pa + Bp pb + ... + Kb pk = Bpb

Ak pa + Bk pb + ... + Kk pk = Kpk

It is clear that in face of the contour conditions above presented, the system has one degree of indeterminacy, that is, from the k equations we have in our price system, just (k - 1) are linearly independent. We could infer this conclusion in a somewhat more intuitive way if we observe that if we equate the value of the products exchanged and received by one industry we would be refering to an interface of the "exchange polygon" and that given K industries, we could find just (K - 1) independent interfaces, once that the k^{th} would be immediatly determined by the others.

Therefore, in Figure 1.2 we show an example for 3 industries, that can be easily extended to a general case of K industries.

1.6 - The Choosing of a Measure for Value

We have arrived to a linear system with (k - 1) equations linearly independent and k absolute prices, respectively, pa, pb,..., pk, and immediately we are forced to conclude that it is impossible to obtain K absolute prices. This should be expected, since as we have seen in section 1.4 we have to deal with "relative prices" and therefore it will be necessary to elect one of the commodities as a "numéraire" and use it as money (that is, we make its price equal to one), calculating the remaing (K - 1) relative prices in relation to it. Therefore we could find a relative-prices system compatible and determined with (K - 1) linearly independent equations and (K - 1) unknowns.

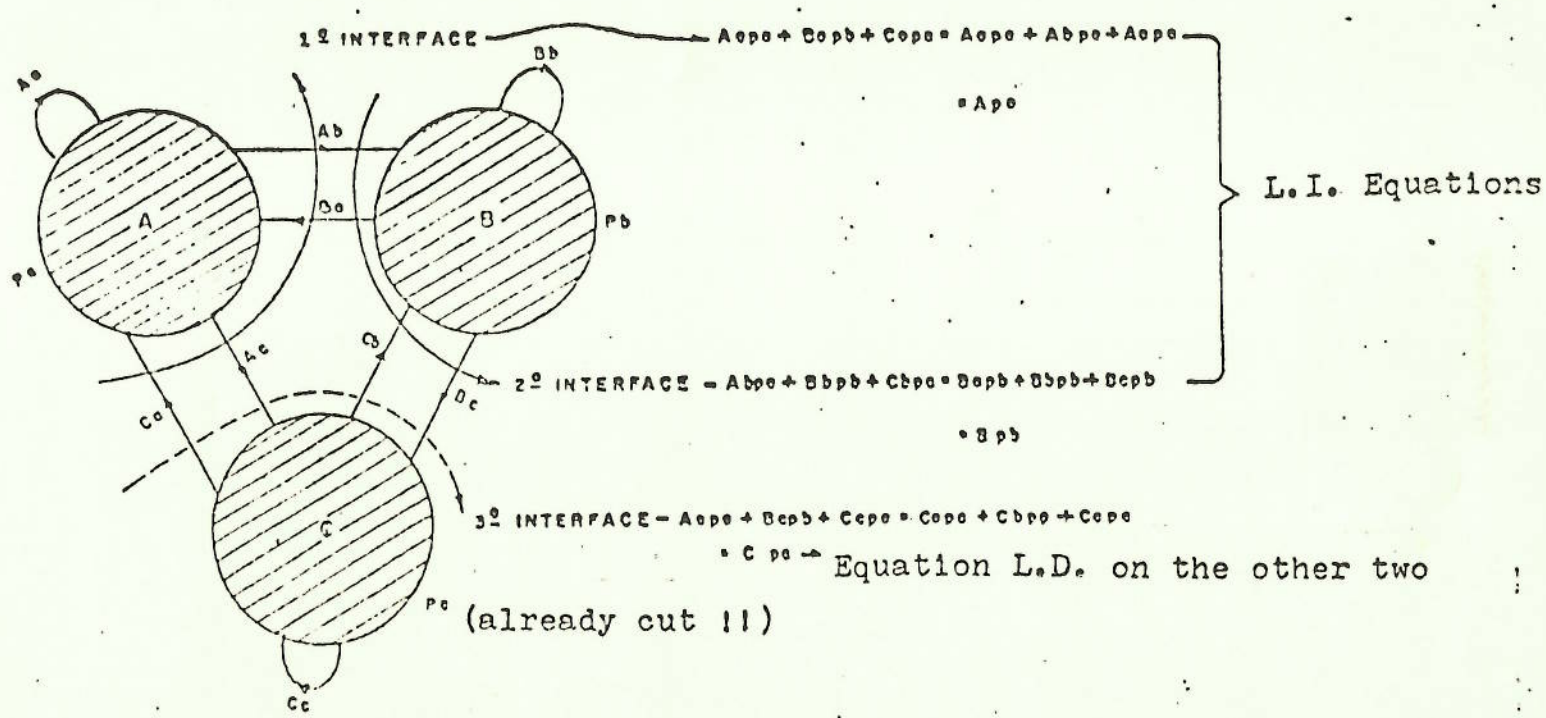
1º INTERFACE
Aapa + Bapb + Capa = Aapa + Abpa + Aapa
= Apa
2º INTERFACE = Abpa + Bbpb + Cbpa = Bapb + Bbpb + Bcpb
= Bpb
3º INTERFACE – Aapa + Bcpb + Ccpa = Capa + Cbpa + Ccpa
= C pa → Equation L.D. on the other two
(already cut !!)
L.I. Equations
A
B
C
Aa
Ab
Ba
Bb
Ac
Ca
Cb
Bc
Cc
Pa
Pb
Pc

FIGURE – 1.2

$$Aa + Ba\ p'b + \ldots + Ka\ p'k = A$$

Let $pa = 1$

$$Ab + Bb\ p'b + \ldots + Kb\ p'k = Bp'b$$

We have $pi' = \frac{pi}{pa}$

- -

$$Ak + Bk\ p'b + \ldots + Kk\ p'k = Kp'k$$

Where we could choose any (K - 1) equations from the existing K and where the money is the commodity a .

It would be worthwhile to remember that on doing so, we lap the crucial problem of choosing a measure for value and this is nothing but the dilemma that Ricardo tried to solve throughout his life and works.. (10)

1.7 - Production with a Surplus

Now Sraffa ask us to imagine an economic system where what is produced is more than the minimum necessary for self-regeneration and reproduction bringing about an "excess" or surplus. We therefore have a physical system like :

$$Aa \oplus Ba \oplus \ldots \oplus Ka \rightarrow A$$

$$Ab \oplus Bb \oplus \ldots \oplus Kb \rightarrow B$$

- -

$$Ak \oplus Bk \oplus \ldots \oplus Kk \rightarrow K$$

but now subject to the following contour conditions:

---

(10) "When commodities varied in relative value it would be desirable to have the means of ascertaining which of them fell and which rose in real value, and this could be effected only by comparing them one after another with some invariable standard measure of value, which should itself be subject to none of the flutuations to which other commodities are exposed." D. Ricardo - The Principles of Political Economy and Taxation - Everyman's Library - Ch. 1 - Section VI p. 27

$$Aa + Ab + \dots + Ak \leq A$$

$$Ba + Bb + \dots + Bk \leq B$$

$$Ka + Kb + \dots + Kk \leq K$$

where at least one must be a strict inequality.

Unfortunately, if we try now to apply a linear prices system like we have done in section 1.5 we would get an algebraic system of K linearly independent equations, on just (K - 1) unknowns, that is to say, a not compatible or inconsistent system.

At this point Sraffa proposes (and we will try to justify this assumption in the next section) that the surplus should be distributed equally among the industries in the proportion of the means of production utilized, giving rise to a "general rate" of profit "r", (11) uniform for all industries. Doing so, the prices system is modified to :

$$(Aa\ pa + Ba\ pb + \dots + Ka\ pk)(1 + r) = Apa$$

$$(Ab\ pa + Bb\ pb + \dots + Kb\ pk)(1 + r) = Bpb$$

$$(Ak\ pa + Bk\ pb + \dots + Kk\ pk)(1 + r) = Kpk$$

and now we are facing a non linear algebraic system, compatible and determined, with K independent equations and K unknowns : (K - 1) relative prices and the "general rate of profit" r (12).

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(11) It should be more appropriate to call r a "general rate of surplus" because, up to that point this r is not the rate of profit that Sraffa is going to addopt later.

(12) From this point on, it should be said that we cannot consider Sraffa "prices" as equal the commodities' labor-values. By now we should be aware of this fact and later we will come back to this subject, trying to point how it is possible to conciliate these two concepts.

1.8 - The Equalization of the Rates of Profit (13)

The assumption of a general tendency for the rates of profit to equalize on a capitalist system is not much of a point of divergence inside the economic theory. From the classics, like Adam Smith and Ricardo, the mercantilists and physiocrats to the marxists and the neoclassical school, this assumption is made with slightly differents meanings.

If we assume that capital is competitive (and it is not necessary to assume "perfect competition"; on the contrary, if our system is closed and self regenerative, there happens to be a mutual interdependence among the industries and the conflicting bargaining powers of the firms is what we understand by competition) and has some mobility which implies that the remuneration of each unity of capital tends to be the same in all of it applications. This means that the equilibrium rate of profits (r), which is the oscillation axis for the real rates of profit, must be the same in all the sectors, although we observe "conjuncture" deviations. Any difference, upward or downward, gives rise to capital movements - looking for the maximal rate of profits possible - which tends to reinstate the equilibrium.

We can now summarize Sraffa's conclusions up to now (14) in the following way : Given a self-reproducing system where K commodities are produced and some surplus is present, we can find (K - 1) equilibrium relative prices and an uniform equilibrium rate of profits (r), just starting from its production's technical coefficients, i.e., from the <u>real</u> physical conditions of production, <u>independently</u> of what can happen thereafter ("a posteriori") in

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(13) " The really difficult question is this: how is this equalization of profits into a general rate of profit brought about, since it is obviously a result rather than a point of departure? " K. Marx - Capital - IV - Ch. X - p. 174

(14) P. Sraffa - op. cit pp 4-5

the sphere of circulation. (15)

1.9 - Basic and Non-Basic Products

At this point, Sraffa calls our attention for an effect of the surplus' appearance : we can now produce commodities which are not utilized neither as means of productions nor as subsistence goods (16). There appears the "luxury good".

Trying to exemplify, let us imagine a self-reproductive system with surplus production where the commodities A, B and C are produced along :

$$
\begin{array}{ccccccc}
2A & \oplus & 3B & \oplus & C & \rightarrow & 7A \\
A & \oplus & 2B & \oplus & 2C & \rightarrow & 6B \\
2A & \oplus & B & \oplus & C & \rightarrow & 5C \\
\hline
5A & & 6B & & 4C & &
\end{array}
$$

where the contour conditions are satisfied and the surplus is $(2A + C)$.

The associated price system is:

$$(2pa + 3pb + pc)(1 + r) = 7pa$$

$$(pa + 2pb + 2pc)(1 + r) = 6pb$$

$$(2pa + pb + pc)(1 + r) = 5pc$$

with determinate solutions for 2 relative prices and the surplus ("profit") rate.

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(15) This affirmative, there's no doubt, shakes our neoclassical beliefs and the marginalist concept of <u>price</u> and capital remuneration

(16) P. Sraffa - op cit. pp 7-8

Let us suppose now that all the system's surplus is used up in the production of a new commodity D, which is, as we said, a "luxury good". Our production system turns out to be :

$$2A \oplus 3B \oplus C \oplus 0D \rightarrow 7A$$
$$A \oplus 2B \oplus 2C \oplus 0D \rightarrow 6B$$
$$2A \oplus B \oplus C \oplus 0D \rightarrow 5C$$
$$2A \oplus 0B \oplus C \oplus 0D \rightarrow 8D$$

$$7A \quad 6B \quad 5C \quad 0D$$

and the total surplus of this economy is now 8 D produced by the new industry.

Similarly, we can associate a price system to this production system, that is :

$$(2pa + 3pb + pc)(1 + r) = 7pa$$
$$(pa + 2pb + 2pc)(1 + r) = 6pb$$
$$(2pa + pb + pc)(1 + r) = 5pc$$
$$(2pa + \quad + pc)(1 + r) = 8pd$$

and we can see that if we eliminate the last equation the relative prices of A, B and C and the rate of "profits" will not be affected.

The conclusion we reach is that "these products have no part in the determination of the system. Their role is purely passive "..." This can be seen if we eliminate from the system the equation representing the production of a "luxury good". Since by the same act we eliminate an unknown (the price of that good) which only appears in that equation, the remaining equations will still form a determinate system which will be satisfied by the solutions of a larger system." (17)

Therefore, if a commodity enters direct or indirectly in the production of all the others commodities it will be called a basic product. If not we will call it a non-basic product.

1.10 - Subsistence Wage and Surplus Wage

The treatment that Sraffa gives to wages is peculiar and merits some reflexion:

Ricardo, along the classical tradition, considered that "the natural price of labor is that price which is necessary to enable the laborers, one with another, to subsist and to perpetuate their race, without either increase or diminution" (18) linking therefore the wage to a concept of physiological subsistence. It was Marx who first noted the possibility for the wage to be above this minimum physiological level and that its determination would depend on historical, moral and concrete factors. Thus, after saying that the wage is the remuneration of the labor-power at its value and that "the value of labor power is determined, as in the case of every other commodity, by the labor-time necessary for (its) production, and consequently also (its) reproduction" (19); Marx goes on characterizing what he understands by subsistence: (20)

"For (the individual) maintenance he requires a given quantity of the means of subsistence. Therefore the labor-time requisite for the production of labor-power reduces itself to that necessary for the production of those means of subsistence"..." If the owner of labor power works to-day, to-morrow he must again be able to repeat the same process in the same conditions as regards health and strenght.

(17) P. Sraffa - op cit. pp. 7-8
(18) D. Ricardo - op cit. Ch.V , p. 52
(19) K. Marx - Capital, I - Ch.VI p. 170
(20) K. Marx - idem p. 171 - emphasis added

His means of subsistence must therefore be sufficient to maintain him in his normal state as a laboring individual..His natural wants, such as food,.clothing, fuel and housing, vary according to the climatic and other physical conditions of his country.. On the other hand, the number and extent of his so-called necessary wants, as also the modes of satisfying them, are themselves the product of historical development, and depend therefore to a great extent on the degree of civilization of a country, more particularly on the conditions under which, and consequently on the habits and degree of confort in which, the class of free laborers has been formed. In contradistinction therefore to the case of other commodities, there enters into the determination of the value of labor-power a historical and moral element.. Nevertheless,. in a given country, at a given period,. the average quantity of the means of subsistence necessary for the laborer is practically known."

We think it is worth to note here that although Marx calls the "basket of goods" necessary for the labor-power's reprocuction as subsistence goods, the sense encountered here is somewhat more general than the simple concept of a minimum physiological subsistence.

Therefore, when Sraffa says that "we must now take into account the other aspect of wages since, besides the ever-present element of subsistence, they may include a share of the surplus product" (21) we could be initially tempted to explicitly divide the wage in two parts: a (physiological) subsistence wage and a "surplus" wage.

Nevertheless, as we have seen from the above long quotation of Marx, it is difficult to arbiter which portion of the remuneration of the labor-power is for "indispensable necessities", what indeed would open a full discussion on what is really indispensable and what is just a share in the surplus product.

Sraffa, however,. avoiding any restrictive assumption, decides

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(21) P. Sraffa - op cit. p. 9

to opt for not expliciting the wage on its alleged two components, treating it as if it could be varied upward and downward, until a minimum of (physiological) subsistence is reached, in the same way as Marx treated the remuneration of the labor-power (22) that means that wages dispute with profits the division of the net product (in the ricardian sense) of the economy, and the quantities of labor in each industry must be explicitly shown, in the place of the respective quantities of subsistence goods. Therefore, the historical and moral characteres and the necessary objectivity that conditionate the determination of the valor of the labor-power are preserved in Sraffa's exposition

1.11 - The Auxiliary Diagram

It would be interesting to introduce now an "auxiliary diagram" that will make easier the visualization of the next sections (See figure 1.3)

As we can see from the figure, the total value of a commodity is composed of two components: one, the value added by direct (present) labor on the period of production, and the other, the value of the means of production used up in its production (which we assume as total by consumed to be coherent with the assumption of circulating capital) .

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(22) "The minimum limit of the value of labor power is determined by the value of the commodities, without the daily supply of which the laborer cannot renew his vital energy, consequently by the value of those means of subsistence that are physically indispensable. If the price of labor-power fall to this minimum, it falls below its value, since under such circunstances it can be maintained and developed only in a crippled state." K. Marx - Capital I - p. 173

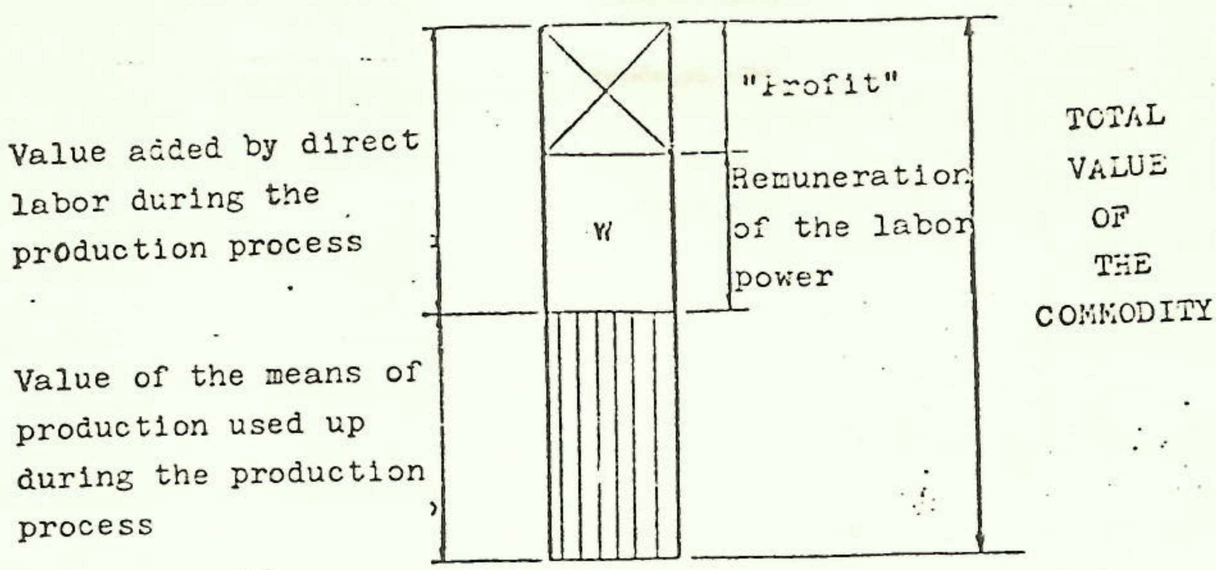

FIGURE - 1.3

The remuneration of the labor-power (the wage) is necessarily less than (or at most equal to) the value added by direct labor during the production period, what may give rise to the appearance of a "profit" (23) for the industry considered. It is apparent that, as we have said, the wages and profits begin disputing the net product of direct labor and therefore we can represent wage as the proportion of the net product that remunerates labor, or the percentage of the value added that is effectively paid to the workers. Therefore $w = .4$ means that 40% of the net product is paid as wages and 60% as profits.

It is important to note that up to now all these values are quantified in hours of labor.

1.12 - Observation on Terminology

The fact that the diagram in the preceding section and the

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(23) Later we are going to redefine this concept more properly.

explanation of the composition of the value of a commodity found in Marx are somewhat alike is not accidental.

In fact, if we try to express the same concepts in a marxian terminology, we will see that what for Sraffa is "value of the means of production", "wages" and "profits", for Marx is "constant capital", "variable capital" and "surplus value" respectively. Also the way of presenting the division of the net product is different, for Marx defines a "rate of surplus value" or "rate of exploitation", that is just the ratio between the surplus-value and the variable capital. Therefore, when Sraffa tell us that w = .4, Marx would say that "the rate of surplus-value is 150%". Taking into consideration what was said, we will make use of both terminologies in the next sections.

1.13 - Wages Paid After Production

"We shall also hereafter assume that the wage is paid post factum as a share of the annual product, thus abandoning the classical economists' idea of a wage "advanced" from the capital". (24)

On doing so, Sraffa alters the classical concept of "rate of profit" (25). The idea of wages being paid before the production can already be found in the physiocrats : Quesnay, in his well-known "Tableau Economique" addopts the wages as a part of the advanced payments the producers must make. Adam Smith and Ricardo also make the same assumption, the last arguing that "the workers could not survive if this is not done".

Marx is consistent with that classical tradition and uses this formulation in all of his works, although he explicitly says that: "In every country in which the capitalist mode of production

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(24) P. Sraffa - op cit - p. 10

(25) This is going to introduce some modifications on the schemes of transformation of values into prices (see section 1.18), as they are presented on Capital - Volume IV. Several marxists have criticized Sraffa for doing that but, as we will see, even Marx have admitted this hypothesis as probably more realistic for the capitalist system he was seeing.

reigns, it is the custom not to pay for labor-power before it has been exercised for the period fixed by the contract (...) the use-value of the labor-power is advanced to the capitalist : the laborer allows the buyer to consume it before he receives payment of the price; he everywhere gives credit to the capitalist (...). The labor-power is sold, although it is only paid for, at a later period" (26).

Summing up, the classics, as they thought that the wages came from the advanced capital, calculated the rate of profits also over the wages. In marxian terms, the rate of profits of the classics would be the surplus-value divided by the sum of the constant capital and the variable capital. When Sraffa, assumes that wages are paid "post-factum", he excludes the payment of the rate of profits over wages and calculates it just as surplus-value divided by constant capital. (See figure 1.4) We consider a constant capital equal to 20, a variable capital also equal to 20 and a surplus-value equal to 10, in our example. Note that the rate of profits calculated " a la Sraffa" is always bigger than the calculated by the classics, once that, being wages paid after the production, a smaller amount of capital is necessary to start the production.

It should be clear that nowadays Sraffa's assumption is completely corroborated by everyday practice.

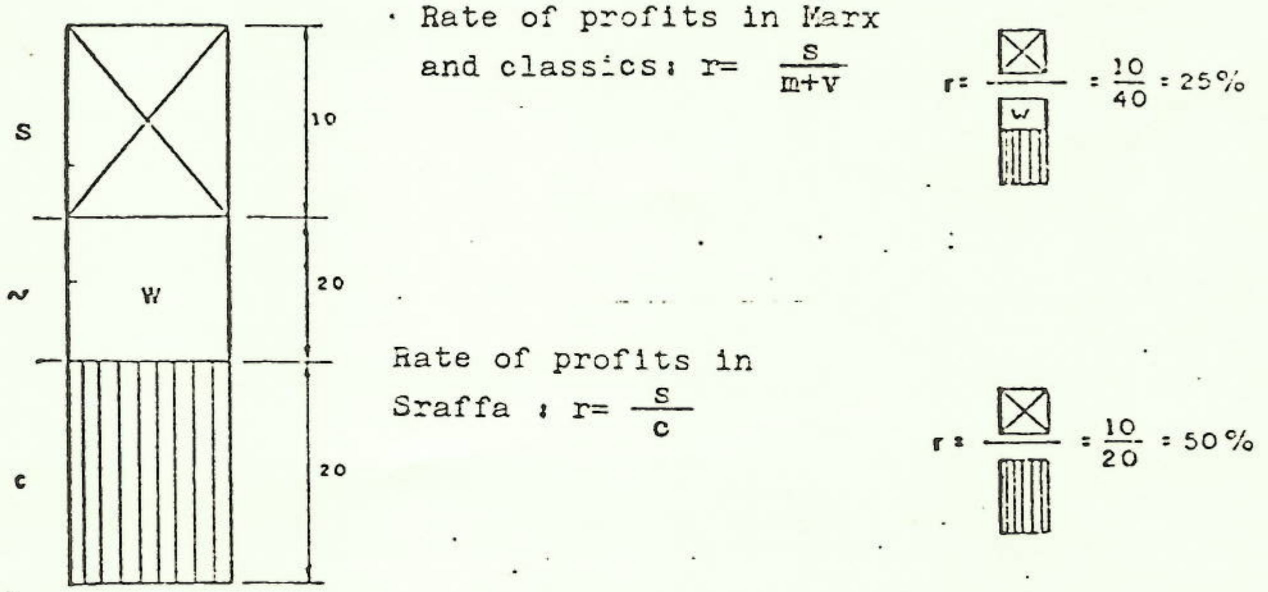

FIGURE - 1.4

(26) K. Marx - Capital I - pp 174-5

1.14 - Quantity and Quality of Labor

We are going now to make explicit the quantities of labor used up in each industry, putting them on the places of the respective quantities of commodities paid as wage. For thus we assume that labor is "uniform in quality or, what amounts to the same thing, we assume any differences in quality to have been previously reduced to equivalent differences in quantity so that each unit of labor receives the same wage." (27)

On doing so, Sraffa admits the possibility of converting skilled work on non-skilled work, in a similar way as Marx makes the convertion from "complex labor" to "simple labor". (28) Furthermore it is also implicitly assumed that labor has mobility, in such a way that the remuneration of non-skilled labor tends to be uniform.

Let us call La, Lb, ..., Lk the annual quantities of labor employed in the industries that produce a, b, ..., k, respectively. If we use Sraffa's normalization, expressing them as proportions of total annual labor employed in the society, we have

$$La + Lb + ... + Lk = 1$$

If we define w as the wage per unit of labor and if we express it on the same measure of value as prices (see section 1.17 below) we can say that (La.w) has, dimensionally, the same nature as (A.pa), for example, and these quantities can therefore be added and treated algebraically.

1.15 - The Equations of Production

Finally we can complete the model. The system of physical

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(27) P. Sraffa - op cit. p. 10

(28) Marx, like Adam Smith and Ricardo, had used a long-run scale of stablished wages to reduce "ex-post" complex labor to simple labor.

coefficients, with labor appearing explicitly is now:

$$Aa \oplus Ba \oplus \ldots \oplus Ka \oplus La \rightarrow A$$

$$Ab \oplus Bb \oplus \ldots \oplus Kb \oplus Lb \rightarrow B$$

$$Ak \oplus Bk \oplus \ldots \oplus Kk \oplus Lk \rightarrow K$$

subject to the contour conditions for self-reproduction :

$$Aa + Ab + \ldots + Ak \leqslant A$$

$$Ba + Bb + \ldots + Bk \leqslant B$$

$$Ka + Kb + \ldots + Kk \leqslant K$$

$$La + Lb + \ldots + Lk = 1$$

leading us to the "system of production prices" (27.a)

$$(Aa\ pa + Ba\ pb + \ldots + Ka\ pk)(1 + r) + Law = Apa$$

$$(Ab\ pa + Bb\ pb + \ldots + Kb\ pk)(1 + r) + Lbw = Bpb$$

$$(Ak\ pa + Bk\ pb + \ldots + Kk\ pk)(1 + r) + Lkw = Kpk$$

1.16 - The National Income

In a similar way as we do when we work with keynesian theory, it is now possible to define a "physical national income", in commodity terms. For this we just have to discount from the value of

(27a) On a sense very near of marxian's as we will see.

each commodity produced their means of production's "transferred values". Thus, the physical national income could be written as:

$$RN\ (physical) = \left[A - (Aa + Ab + \ldots + Ak)\right] \cup$$

$$\cup \left[B - (Ba + Bb + \ldots + Bk)\right] \cup \ldots \cup$$

$$\cup \left[K - (Ka + Kb + \ldots + Kk)\right]$$

and its magnitude expressed in "production prices" is :

$$RN = \left[A - (Aa + Ab + \ldots + Ak)\right] pa +$$

$$+ \left[B - (Ba + Bb + \ldots + Bk)\right] pb +$$

$$+ \ldots + \left[K - (Ka + Kb + \ldots + Kk)\right] pk$$

1.17 - The National Income as a Measure of Prices

When we put the labor on its explicit form and defined the wage as a proportion of the net product (or national income), we have made two important restrictions: the labor was normalized and its total sum was made equal to unity; the wage was allowed to vary from $w = 0$ to $w = 1$. After that we can no more choose any commodity for "numéraire" of our system.

In fact, when the wages are equal to one, in the model, all the net product is being paid as wages, and the profits fall to zero. If we want our production prices system of section (1.16) to be compatible with this situation we have to look for a measure for prices that could guarantee this desired result.

Let us write again the system from section (1.16) and sum the equations member by member :

$$(Aa\ pa + Ba\ pb + \dots + Ka\ pk)(1 + r) + Law = Apa +$$

$$(Ab\ pa + Bb\ pb + \dots + Kb\ pk)(1 + r) + Lbw = Bpb +$$

$$\dots$$

$$(Ak\ pa + Bk\ pb + \dots + Kk\ pk)(1 + r) + Lkw = Kpk +$$

$$\Big[(Aa + Ab + \dots + Ak)\ pa + (Ba + Bb + \dots + Bk)\ pb + \dots + (Ka + Kb + \dots + Kk)\ pk\Big]\ (1 + r) + (La + Lb + \dots + Lk)w = Apa + Bpb + \dots + Kpk.$$

Rearranging the terms, putting the rate of profits (r) on an explicit form, and remembering that La + Lb + ...+ Lk = 1 we have the equation in page 26.

For $w = 1 \Rightarrow r = 0$ and it is <u>necessary</u> (28a) that:

$$A - \Big[(Aa + Ab + \dots + Ak)\Big]\ pa + B - \Big[(Ba + Bb + \dots + Bk)\Big]\ pb + \dots + K - \Big[(Ka + Kb + \dots + Kk)\Big]\ pk = 1$$

(28a) Sraffa does not emphasizes this necessity although he explicitly says that he is using the national income evaluated at productions prices as a measure for prices. op. cit. p. 11

$$r = \frac{\left[A-(Aa + Ab + \ldots + Ak)\right]pa + \left[B-(Ba + Bb + \ldots + Bk)\right]pb + \ldots + \left[K-(Ka + Kb + \ldots + Kk)\right]pk}{(Aa + Ab + \ldots + Ak)\ pa + (Ba + Bb + \ldots + Bk)\ pb + \ldots + (Ka + Kb + \ldots + Kk)\ pk} =$$

$$= \frac{W}{(Aa + Ab + \ldots + Ak)\ pa + (Ba + Bb + \ldots + Bk)\ pb + \ldots + (Ka + Kb + \ldots + Kk)\ pk}$$

that is, it is indispensable that we take the national income at production prices as our measure for prices and wage. Therefore, aggregating this equation to our system of production, we have:

$$(Aa\ pa + Ba\ pb + \ldots + Ka\ pk)(1 + r) + Law = Apa$$

$$(Ab\ pa + Bb\ pb + \ldots + Kb\ pk)(1 + r) + Lbw = Bpb$$

$$(Ak\ pa + Bk\ pb + \ldots + Kk\ pk)(1 + r) + Lkw = Kpk$$

$$A - \left[(Aa + Ab + \ldots + Ak)\right] pa + B - \left[(Ba + Bb + \ldots + Bk)\right] pb +$$

$$+ \ldots + K - \left[(Ka + Kb + \ldots + Kk)\right] pk = 1$$

and now we have (K + 1) independent equations and (K + 2) unknowns: wage, rate of profits and K relative prices (now in relation to the national income). The system still has one degree of indeterminacy that will just be given by the distribution of the net product between wages and profits. Once determined this distribution, the system admits equilibrium solutions for its unknowns.

1.18 - The Transformation of Values into Prices of Production

We have said in section 1.7 that once we have an equilibrium rate of profit present in the system, the prices do not coincide with the commodities' labor-value anymore.

Before trying to elucidate better this problem, we have to answer to the following question: how can one find the labor-values of the commodities, given an economic system specified by their physical coefficients of means of production and labor?

This question can be easily answered, begining again with the physical system of production and forming a "labor-value system" where the wages are equal to one and therefore the rate of profits is zero. Thus, we have :

$$Aa\ pa + Ba\ pb + \ldots + Ka\ pk + La = Apa$$

$$Ab\ pa + Bb\ pb + \ldots + Kb\ pk + Lb = Bpb$$

$$Ak\ pa + Bk\ pb + \ldots + Kk\ pk + Lk = Kpk$$

subject to the same contour conditions as the production prices system and using, in the same way, the National Income as measure of prices and wage. "We thus revert, in effect, to the system of linear equations from which we started, with the difference that the quantities of labor are now shown explicitly instead of being represented by quantities of necessaries for subsistence. At this level of wages the relative values of commodities are in proportion to their labor cost, that is to say to the quantity of labor which directly and indirectly has gone to produce them." (29)

In fact, this "labor-value system" is not only useful for the calculation of the commodities' labor-values, but gives rise to an interesting reflexion: it can describe the equilibrium behavior of an economy on a historical moment where we already find the broad diffusion of commodity (30) production although the selling and buying of labor-power is not yet disseminated. This is the period of simple production of commodities, and accordingly to F. Engels' Supplement to Volume III of Capital, was on which Marx had based his law of value found on Volume I.

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(29) P. Sraffa - op cit. p. 12

(30) In the sense that a considerable proportion of the production is aimed to be exchanged by other products, and loses its use-value for the producer having for him, just its exchange-value.

"The Marxian law of value holds generally (...) for the whole period of simple commodity production, that is, up to the time when the latter suffers a modification through the appearance of the capitalist form of production. Up to that time prices gravitate towards the values fixed according to the Marxian law and oscillate around those values, so that the more fully simple commodity production develops, the more the average prices over long periods uninterrupted by external violent disturbances coincide with values within a negligible margin. Thus the Marxian law of value has general economic validity for a period lasting from the begining of exchange, which transforms products into commodities, down to the XV century of the present era." (31)

Nevertheless the appearance of a general rate of profits and the metamorphosis of money into capital make the market prices to deviate sensibly from their labor-values, gravitating now around a new point of equilibrium: the production prices. In Marx's words:

"The whole difficulty arises from the fact that commodities are not exchanged simply as commodities , but as products of capitals, which claim participation in the total amount of surplus-value proportional to their magnitude (...) And this claim is to be satisfied by the total price for commodities produced by a given capital in a certain space of time." (32)

Illustrating this rationing with auxiliary diagrams of section 1.11, let us assume that a system produces 3 commodities A, B and C according to figure 1.5. We are also assuming, for simplicity, equal total labor-values for all three commodities but the commodity A uses relatively "more" direct labor than means of production, and the commodity C uses relatively "less" direct labor than means of production (33). Imagine that the wage is w = .5, that is to say,

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(31) F. Engels - "Supplement to Capital III" on Engels on Capital, New World Paperbacks, 1974- pp 109 - 10.
A. Brody also makes a similar comparison: "Such (...) a system could be thought of as a community of artisans and peasants in medieval times." Three types of price Systems - Economics of Planning - Vol 5 - nº 3 - 1965 - p. 59
(32) K. Marx - Capital III - p. 175
(33) In fact we have to think of the means of production as "indirect (past) labor" to obtain homogeneous and comparable quantities.

the rate of surplus-value is 100%.

If the commodities A, B and C should exchange at their labor-values, we would observe immediatly the appearance of different rate of profits on the industries A, B and C (see figure 1.5), there appearing a "deficit" in industry C and a "surplus" in industry A. (34)

But, as we have seen in section 1.8, there is a tendency for the rate of profits to be equalized, and that could be obtained if the commodities from the industries where we can expect a "surplus" should be sold below their labor-values and the commodities from the industries where we expect a "deficit" should be sold above their labor-values. Making these adjustments on the industries A and C, we have now the configuration of figure 1.6 where we observe a commom rate of profits (r) for all the system. (35)

It is apparent that "the key to the movement of relative prices (...) lies in the inequality of the proportions in which labor and means of production are employed in the various industries. It is clear that if the proportions were the same in all industries no price-changes could ensue, however great was the diversity of the commodity-composistion of the means of production in different industries." (36)

That fact was already observed by David Ricardo on the third edition of his "Principles of Political Economy" and he considered a necessary and primordial condition to give an answer to the problem of distribution for being able to calculate its effects on prices, given changes in wages. Therefore, when presenting for the first time the "modifications" observed on the relative prices, as

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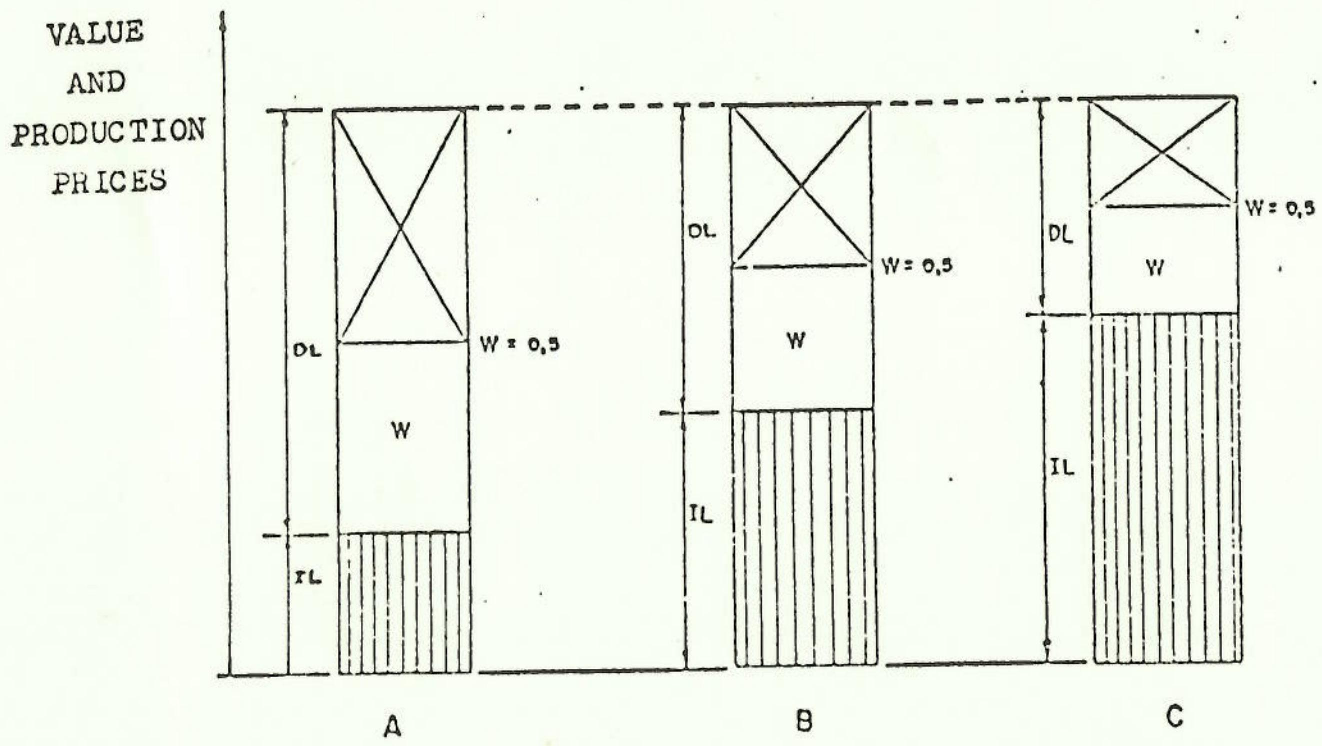
(34) P. Sraffa - op. cit.. p. 12-3

(35) It should be noted that the industry B was assumed to be using an intermediate proportion between labor and means of production such that its rate of profit rb is equal to the mean rate of profits r. Therefore, the price of commodity B does not vary and remains equal to its value.

(36) P. Sraffa - op cit. p. 12



Being r calculated as r = ⊠ / ▥, it is clear that $r_a > r_b > r_c$.

FIGURE – 1.5

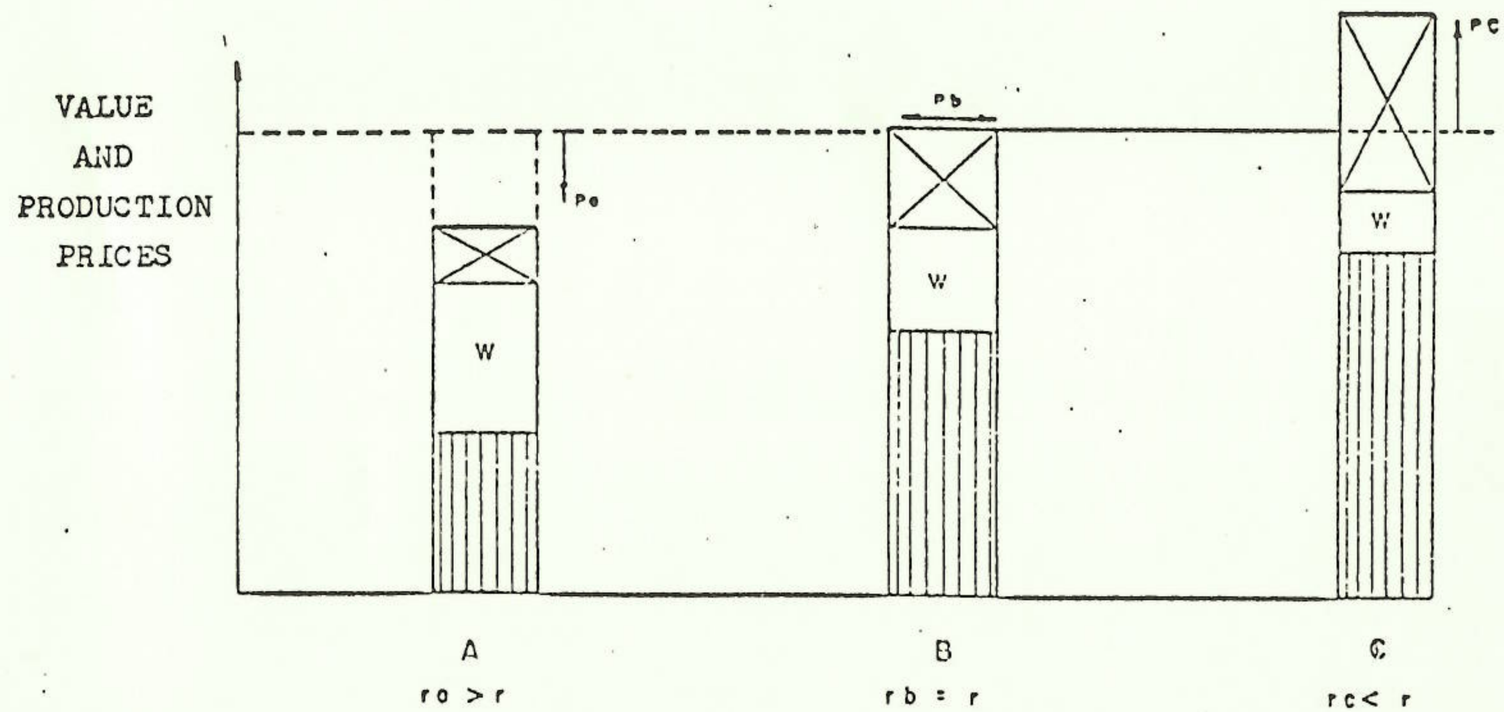


FIGURE-1.6

dependent upon the different technical conditions of production, particularly relating them to the use of fixed capital. Ricardo tried to refute Adam Smith's theory of the "sum of components", rejecting the possibility of treating the sphere of exchange relations as an isolated system, and trying to ground the explanation for these exchanges on prices to differ from their values, on the conditions and circumstances of production .(37)

In 1847, when writing "The Poverty of Philosophy" Marx states this fact for the first time, subject which would be the central focus of his 3rd Volume of Capital many years later.

1.19 - The Proportion Between Labor and Means of Production

From what we have said in the latter section it should be clear the importance of knowing the different proporitons of labor and means of production utilized on the industries of our system, for the complete understanding of the transformation of values into production prices.

For a similar reason, Marx has created the concept of "organic composition of capital" that is defined on Chapter VIII of Capital Vol III as the ratio of the constant to the variable capital (37a) We are going to show that it is possible to stablish a unequivocal relation between the sraffian concept of "proportion between labor and means of production" (on the sense of proportion between direct and indirect labor used up on the production) and the marxian concept of "organic composition of capital".

Using the diagram on figure 1.7 we can say that, by definition, the proportion between labor and means of production on the observed industry will be equal to $\frac{DL}{IL}$, i. e., $\frac{s + v}{c}$. On the other hand, also by definition, the organic composition of capital

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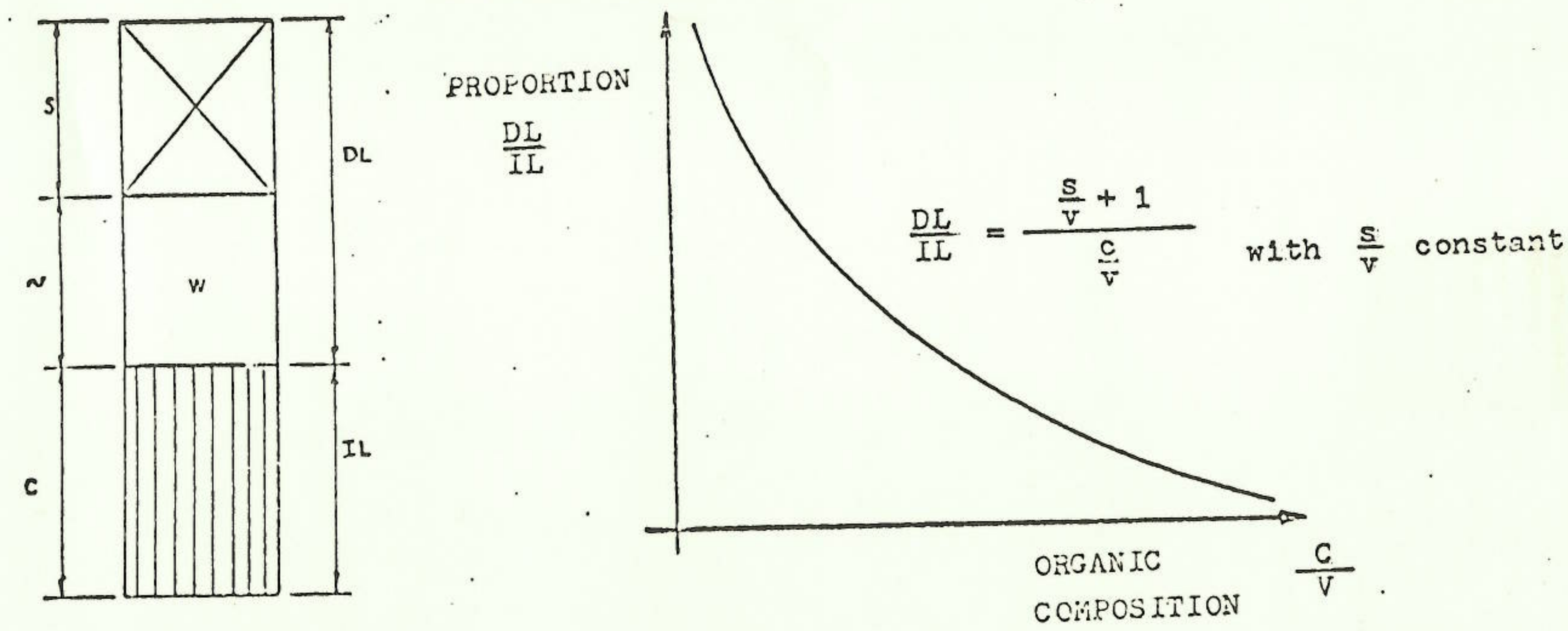
(37) D. Ricardo - op cit - Chap. 1 . Sections IV and VII.

(37a) Although some authors define organic composition of capital as $\frac{c}{c + v}$, on Chapter VIII of Volume III of Capital Marx define it as $\frac{c}{v}$. See page 144 and ss.

PROPORTION BETWEEN LABOR AND MEANS OF PRODUCTION : $\frac{DL}{IL} = \frac{s+v}{c}$

ORGANIC COMPOSITION OF CAPITAL : $\frac{c}{v}$



FIGURE—1.7

is given by the ratio $\frac{c}{v}$. Finding the relation between those two ratios, we have:

$$\frac{DL}{IL} = \frac{s + v}{c} = \frac{\frac{s}{v} + 1}{\frac{c}{v}}$$, that is

$$\frac{\text{Proportion between labor and means of production}}{} = \frac{(\text{Rate of Surplus-Value}) + 1}{\text{Organic Composition of Capital}}$$

that taken to a pair of cartesian coordinates gives us a branch of a hyperbole (38). Therefore we can say that it is possible to express the proportion between labor and means of production as a strictly decreasing function of the organic composition of capital. In general, we could say that "lower" proportions between labor and means of production correspond to "higher" organic composition of capital and vice-versa.

1.20 - The Critical Proportion

As we have seen, the prices will tend toward an equilibrium point that is above their labor-values for commodities produced on industries with "low" proportions between labor and means of production (industries with "deficits") and below their labor-values for commodities produced on industries with "high" proportions between labor and means of production (industry with "surpluses").

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(38) This hyperbole will be equilateral if w = 1, that is, if the rate of surplus value is zero and the profits have just disappeared. We can say that varying the rate of surplus-value we can obtain a family of hyperboles, that give us the unequivocal relation for a given value of the rate of surplus-value.

Well, if there exist proportions that give rise to "deficits" and proportions that give rise to "surpluses", it is likely to exist, at least theoretically, a "critical proportion", between those, that would demarcate a frontier betweenindustries with "deficits" and "surpluses", and that, once addopted by an hypotetical industry would bring about a rate of profits exactly equal to the mean rate of profits of the system. (39) This industry, that Marx would have said to possess the mean organic composition of capital, could produce a commodity with the peculiar property of having its production price always coinciding with its labor-value, being, therefore, the most indicated for a "numéraire", thanks to its invariability characteristics.

It must be remembered that up to now, the Sraffa's model as presented in section 1.17 uses the national income at production prices as a measure, and that this measure standard will have its own magnitude altered if the distribution of the net product between wages as profits should vary.

This problem, the finding of an invariant measure of value, that could be used as an unambiguous standard was the "blue rose" of Ricardo, who has died without having given an answer to this question. He express his doubts on a letter to Mill in the following way: (40)

"Commodities (even our measure of value) alter in relative value not on account only of an alteration in the quantity of labor expended on them, but also on account of the variations (in the division of the net product) between wages and profits."

The first answer to Ricardo's meditations was presented by Marx, on the Volume III of Capital, as we are going to see on next section.

1.21 - The Transformation, in Marx

The problem of transformation labor-values into production

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(39) In the example given in Figures 1.5 and 1.6, we have, in fact, assumed that this industry using that "critical proportion was industry B.
(40) D. Ricardo - "Works and Correspondence of D. Ricardo" t. IX Letter to Mill - p. 386 - P. Sraffa ed.

prices appears when Marx intend to study the process of capitalist production as a whole.. (41)

The theory of value that he had found in Ricardo, and that was the foundation for the writings of Volume I,. put labor on a first plane, as human productive activity, and makes it the basis for the explanation of exchange-value. Therefore,. throughout Volume I it is assumed that commodities are exchanged at their labor-values (that is, proportionally to the direct and indirect social labor embodied on them). When trying to explain the process of capitalist production as a whole (where the production prices depart from labor-values), Marx is concerned with exhibiting the quantitative relations that exist between the conditions of production and "the real exchange values" (the market prices),. because otherwise it would lack a link between the analysis in labor-value term of Volume I and the real, observed phenomena.

It must be said that on doing so Marx makes <u>the law of value to function as a law of formation of production prices</u>,. regulating the production process..

The first critique came from Böhm-Bawerk (42) that charged the labor-value system present on Volume I as inconsistent and incompatible with the production prices system of Volume III.. Recently, Paul Samuelson,. has approached this subject with unusual irony (43).

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(41) K. Marx - Capital III

(42) "(.The writings on Capital III) bear evident traces of having been a subtle and artificial after-thought contrived to make a preconceived opinion seem the natural outcome of a prolonged investigation" and further,. "I think everyone who reads (this work) with an impartial mind will get the impression that the writing is, so to speak, demoralized" - Böhm-Bawerk - "Karl Marx and the Close of his System" - P. Sweezy ed 1949 - p. 69 and 93-100

(43) "Contemplate two alternative and discordant systems.. Write down one. Now transform by taking an eraser and rubbing it out. Then fill in the other one- Voilá ! You have completed your transformation problem." P. Samuelson in "Understanding the Marxian Notion of Exploitation" - Collected Papers of Paul Samuelson - p. 277. MIT,. Cambridge - Mass.. 1972 - 2nd ed.

trying to characterize this problem as a "false dilemma".

What seems to lack to those eminent critics is some historical perspective. What characterizes the concept of production prices is not the fact that it is "more complex" than the labor-value concept, but the fact that the former is a transformed form of labor-value, whose intervention is imposed as an effect of the separation of the productive unities and the separation between workers and means of production. Production prices are the law of value's effect when operating on a capitalist mode of production, i.e., in a system where the commodity production is fully developed.

Marx did not offer a rigorous demonstration of some "deviation rule" between production prices and labor-values. As he did not know much algebra, he tried the arithmetic simulation through numeric examples : the absence of an algebraic treatment of the problem has prevented him from reaching the completion of the transformation. Nevertheless, as we are going to see, his assumptions on the tendencies for production prices to deviate from their labor-values have opened a way for a complete and rigorous solution of the problem.

1.22 - An Example (44)

To illustrate the transformation, in Marx, we will use the auxiliary diagrams. (see figure 1.8). Let us suppose, as we did in figures 1.5 and 1.6, that 3 commodities, A, B, and C are being produced, with the same total value (40 labor-hours), but produced with different proportions between direct and indirect labor. Let these proportions be 3, 1 and 1/3 respectively (corresponding to organic compositions of capital equal to 2/3, 2 and 6) and let the wages be $w = .5$ (corresponding to a rate of surplus-value equal to 100%).

We can see that the system could pay a $r = 50\%$ mean rate of profit and that industry A tends to get a higher, the industry C a lower, and industry B an exactly equal rate of profit as compared with this mean r. The solution, after the transformation, would make the rates of profit to equalize at the mean $r = 50\%$ with the

(44) In this example, we calculate the rate of profits as $r = \frac{s}{c}$, using the sraffian assumption of "post factum" paid wages.

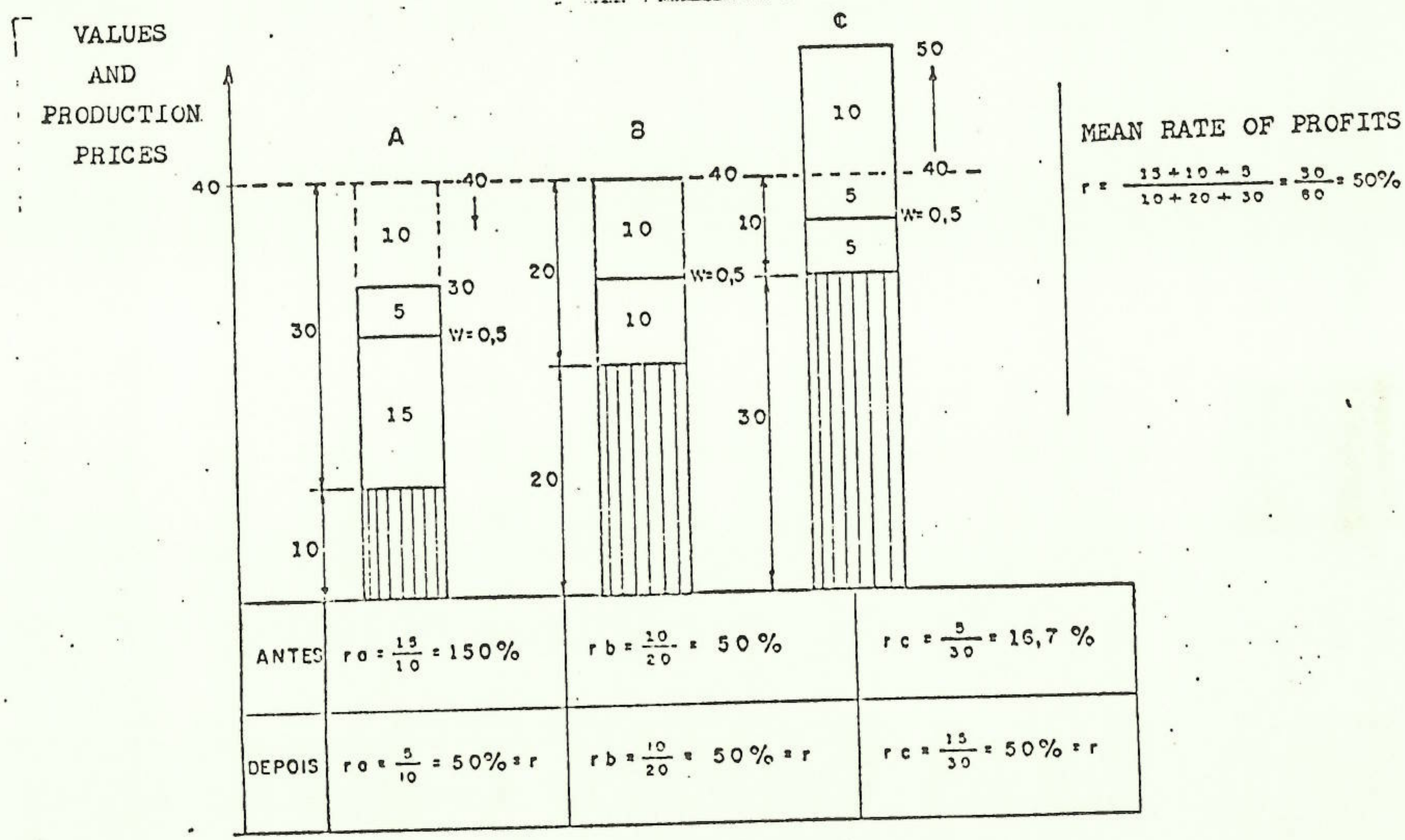
VALUES AND PRODUCTION PRICES
A
B
C
40
30
10
10
5
15
30
W=0,5
20
20
10
10
W=0,5
10
30
10
5
5
W=0,5
50
MEAN RATE OF PROFITS
$r = \frac{15+10+5}{10+20+30} = \frac{30}{60} = 50\%$
ANTES
$ra = \frac{15}{10} = 150\%$
$rb = \frac{10}{20} = 50\%$
$rc = \frac{5}{30} = 16,7\%$
DEPOIS
$ra = \frac{5}{10} = 50\% = r$
$rb = \frac{10}{20} = 50\% = r$
$rc = \frac{15}{30} = 50\% = r$

FIGURE — 1.8

price of A decreasing to 30, the price of B remaining the same (and equal to its labor-value 40) and the price of C increasing to 50.

It is important to note that the price of commodity B did not vary because it is using the "critical proportion" (or mean organic composition of capital) of the system.

1.23 - Bortkiewicz's Objection

Another aspect of Marx's solution for the transformation of labor-values into production prices was also criticized by Böhm-Bawerk, but now with reasonable argumentation: the marxian solution did not include the value of constant and variable capitals, but only the commodities' final value. In other words, it was claimed that the full transformation would also require the transformation of the imputed values - including the labor-power - into production prices terms, and that therefore those production prices should be determined altogether and interdependently, i.e., as solving a system of simultaneous equation (45). The first to show that this was possible to be done was Bortkiewicz (46), but the "transformation" at which it submits the labor-values is such that after completing it, we loose all the traces of <u>value as a chronometric quantity of labor</u>. In other words, after Bortkiewicz's works were published, it became common to argue that the labor theory of value was incompatible with a "full and simultaneous transformation" that embraced products and inputs, once that we would find no link between prices and physical quantities of labor.

In fact, the individual prices of commodities are not more than the reflexion of how commodities do exchange, being therefore,

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(45) In fact, even Marx was aware of the innadequation of his transformation schemes and just before he died, he was studying mathematics aiming to acquire the necessary tools for the general solution. See M. Godelier - "Rationality and Irrationality in Economy"

(46) L. von Bortkiewicz - "Value and Price in the Marxian System" - International Economic Papers, nº 22 - p. 33

relative. But commodities' values, on the other hand, have a kind of absolute character, once they can be physically measured,. in hours of labor. The univoque correspondence between values and prices could be determined only if we could find a counting unity that should maintain a link between a chronolcgical quantity of labor and the prices of production.

It should be self-evident by now,. the real nature of Ricardo's problem : when he was looking for his invariable measure of value he had discovered that without a commodity that possessed the peculiar characteristic of maintaining its price always equal to its value - and from which we could stablish comparisions with the others - he could not maintain the labor theory of value's coherence. (46a) Marx, when trying to solve this problem, has attributed those characteristics of invariability to the commodity whose provenience was from the industry operating with the mean organic composition of capital, but did not suceed on transforming the inputs..

Bortkiewicz,. as we have seen,. has a merit of looking for the full transformation, but while doing so, cannot avoid the disappearance of the labor-value.. From the publication of his article on,. a series of propositions had been made, trying to compatibilize the transformations of inputs and the labor-theory of value, but all the "invariance principles" advanced by those authors (Seton,. Sweezy,. Lange,. Dobb, Meek, Winternitz,. among others) relied on arbitrary restrictions as making the organic composition of capital of one of the sectors to be by assumption equal to the mean, or making the output-capital ratio, or output-labor ratio invariant. In fact, none of those who were concerned with this problem, since Bortkiewicz a half-century ago,. had succeded to reach a solution which was not just approximate, or, alternatively,. which did not broke with the labor-theory of value.

(46a) This was expressedly said on a manuscript from his death's year: "Absolute Value and Exchangeable Value" - published in "Works and Correspondence of D. Ricardo" 1951 - Vol IV pp - 361 - 412 P. Sraffa editor - where Ricardo seems to admit that the invariability of a measure of value was not only impossible to be found pratically, but was also an "a priori" impossibility.

1.24 - The Production Prices After the Complete Transformation

The question posed by Bortkiewicz,, with no doubt brings about such a complicated manipulation that, without using linear algebra and input-output techniques, it is almost impossible to solve..

In fact, if we start again with the configuration of figure 1.5 and assume now a complete transformation of values into production prices, we could find us facing totally different - and to some extent unexpected - situations as compared with figure 1.6.

Figure 1.9 shows to us what would happen with an industry with low organic composition of capital (and therefore likely to obtain a "surplus", with ra > r,. forcing its prices for below their values) if we should consider the transformation of constant and variable capital. Three situations should be expected :(a) The aggregate cost of the means of production remains equal to its aggregate value : in this situation the production price of the commodity would be smaller than its value,. as in figure 1.6. (46b) (b) the aggregate cost of the means of production became smaller than its aggregate value (47) : in this case the production price will also be smaller than its value, but will be still lower than forecasted in figure 1.6 . (c) The aggregate cost of the means of production became greater than its aggregate value (48) : in this case the production price could be greater or smaller than its value, depending on how much the aggregate cost of the means of production departs from its aggregate value.. In Figure 1.9 c,. we assume that the aggregate cost has risen enough to make the prices of commodity A be above its value, although this industry has, by assumption, a low organic composition of capital.

(46b) We could say that from now on, the wages are not 50% of the monetary value obtained at production prices, although they still represent that proportion in physical terms..

(47) Meaning that in a weighted mean these means of production come from industries with below-mean organic composition of capital.

(48) Meaning, similarly, that in a weighted mean they come from industries with above-mean organic composition of capital..

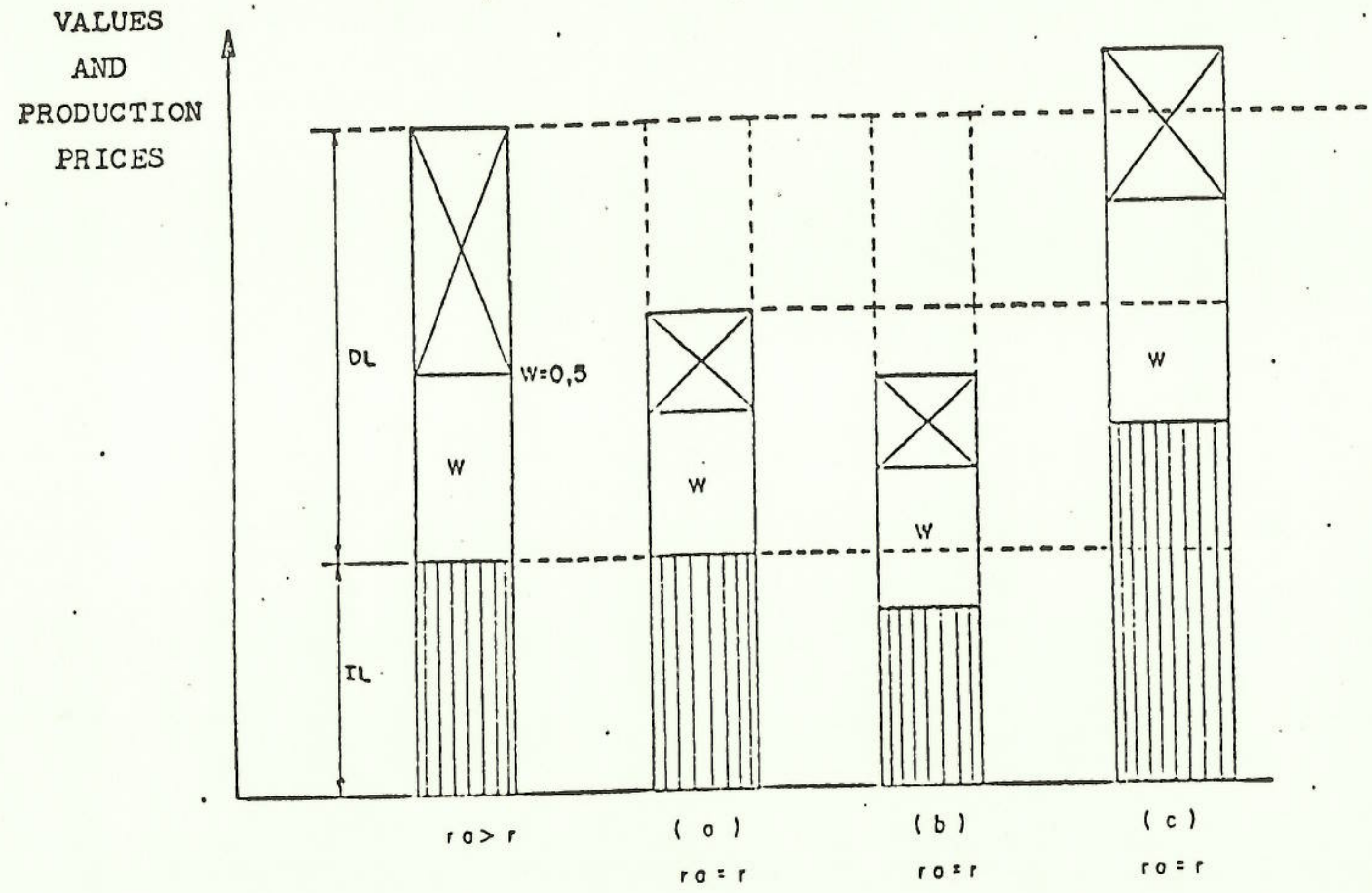
VALUES
AND
PRODUCTION
PRICES
DL
IL
W=0,5
W
W
W
W
ro > r
(a)
ro = r
(b)
ro = r
(c)
ro = r

FIGURE -1.9

Therefore, the simplistic initial conclusion could be misleading, once the equilibrium prices seem to stop following a clear rule for deviations. In Sraffa's words: "the relative price-movements of two products come to depend, not only on the "proportions" of labor to means of production by which they are respectively produced, but also on the "proportions" by which those means have themselves been produced, and also on the "proportions" by which the means of production of those means of production have been produced, and so on . The result is that the relative price of two products may move, with the fall of wages, in the opposite direction to what we might have expected on the basis of their respective "proportions"." (49)

Furthermore, we should note that the commodity whose value should coincide with its production prices whatever the level of wages, should not only be provenient from an industry operating at "critical proportions" between labor and means of production (mean organic composition), but also its means of production should have been produced at that same proportion and so on. This conclusion, reached before Sraffa's work was published, reinforced the argument that all the labor theory of value was dependent on the existence of an industry which, besides operating on a mean organic composition of capital, had all of their means of production produced on this same organic composition and so on, leading to the conclusion that it was very unlikely to find a commodity produced in such a circunstances.

1.27 - Sraffa's Solution

Sraffa's production prices system, as we have seen in section 1.18 is a labor-value system when the wage is equal to unity. As long as we assume a wage smaller than one, and the appearance of a rate of profits, the relative prices that solve the system behave as if they were submitted to the full transformation, given the characteristics which his system has. But we still have a big problem : using the national income as a measure unit, we cannot stablish

(49) P. Sraffa - op. cit. p. 15

a permanent (and fixed) relation between production prices and labor-values. Facing this problem, Sraffa - after agreeing with Ricardo on that "it is not likely that an individual commodity could be found which possessed even approximately the necessary requisites" (50) - decides to build a "composite commodity" that could be used as an invariable measure of value, in which the eventual variations of prices of the simple commodities which enter in its composition cancel themselves out mutually.

This "composite commodity" should be a set of commodities chosen in such a way among the existing commodities and in such a quantity that the various component commodities are represented in their means of production <u>in the same proportion</u> as they appear as products. In this "standard commodity", as Sraffa calls it, there is physical homogeneity between the net product and the means of production, once they are aggregates with the same commodity composition. As a consequence, we can determine the rate of net product (that obviously coincides with the maximal rate of profit; when wages are zero) <u>in physical terms</u>, <u>independently of prices</u>, as did Ricardo with wheat, which was assumed to be output and input at the same time (51).

On doing so, Sraffa has provoked some equivocated interpretations which tried to see his work as an alternative to the labor theory of value (52). On the contrary, when he tried to find a solution for the transformation problem and built a production prices system compatible with a chronometric measure in labor-time, Sraffa not only did not abandon the labor theory of value, but gave to it much more strong and solid foundations...

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(50) P. Sraffa - op cit. p. 18

(51) It can be proved that this maximal rate of profits (R) coinc[illegible] with the ratio between direct and indirect labor used in the indus[illegible] which should operate at the"critical proportion".

(52) Claudio Napoleoni in his work:"Economic Thought of the Twentiet[illegible] Century", after saying that Sraffa's book "has a decisive importance in the history of economic thought" proceeds to say that "Sraffa completely avoids the labor theory of value which, as is more eviden[illegible] in Marx, is at the root of the formal difficulties of classical econ[illegible] mics." p. 166 - John Wiley - NY . 1972.

1.28 - Sraffa's Model : Matrix Notation

In order to put Sraffa's system in matrix form, let us define some symbols:

Qij = quantity of commodity i directly used up in the production of commodity j.

Lj = quantity of direct labor (already submitted to Sraffa's normalization) used up in the production of commodity j.

qj = total quantity of commodity j produced in the system.

$$i = 1, \ldots, n$$
$$j = 1, \ldots, n$$

Therefore, assuming that n commodities are produced, we can write our physical system as:

$$\sum_{i=1}^{m} Qij + Lj = qj, \text{ for } j = 1, \ldots, n$$

subject to the conditions for self-reproduction:

$$\sum_{j=1}^{n} Qij \leq qi \,, \quad qi > 0 \quad \text{(all commodities are produced)}$$
$$Qij \geqslant 0$$

and $$\sum_{j=1}^{n} Lj = 1$$ (Sraffa's normalization), where $Lj > 0$ (all commodities use direct labor to be produced.)

Defining the matrices:

$Q = [Qij]$, $q = (q_1, q_2, \ldots, q_n)'$ and $u = (1, 1, \ldots, 1)'$

we can write:

(I) $\boxed{q = Qu + f}$, where f is the vector $f = (f_1, f_2, \ldots f_n)'$ such that $f = q - Qu$ is the vector of final product (physical nationa[l] income, as defined in section 1.16) or the net product of our syste[m].

If we define now the following matrices and vectors:

$A = [Aij]$ such that $Aij = \dfrac{Qij}{qj}$ the matrix of technological coefficients, and

$lj = \dfrac{Lj}{qj}$ such that $\begin{cases} Aij \geqslant 0 & \text{for } i \neq j \\ 0 \leq Aij < 1 & \text{for } i = j \\ lj > 0 \end{cases}$

$l = (l_1, l_2, \ldots, l_n)$

$$\hat{q} = \begin{bmatrix} q_1 & 0 & \ldots & 0 \\ 0 & q_2 & \ldots & 0 \\ 0 & 0 & \ldots & q_n \end{bmatrix} ; \quad L = (L_1, L_2, \ldots, L_n)$$

obtaining the identities : $Q = A\hat{q}$ we could write our physical system as :

(II) $$\boxed{q = Aq + f}$$

If we express the labor explicitly, we can write the production prices system associated with that physical system :

(III) $$\boxed{p'A\hat{q}\ (1 + r) + w\ l\ \hat{q} = p'\hat{q}}$$

where $p' = (p_1, p_2, \dots, p_n)$ is the vector of prices of commodities $1, \dots, n$,

r = is the equilibrium rate of profits,

w = is the wage (normalized along Sraffa's method)

and where the unit for measurement of prices and wages, still is the national income, that is :

(IV) $$\boxed{\sum_{i=1}^{n} \left[qi - \sum_{j=1}^{n} Aij\ qj \right] pi = 1} \quad \text{or} \quad \boxed{p'f = 1}$$

If we now post-multiply the matricial equation (III) by $\hat{q}^{-1}$, we get :

$$p'A\ (1 + r) + wl = p' \Rightarrow p' \left[I - A(1 + r) \right] = wl$$

If $\left[I - A(1 + r) \right]$ is not singular (what is always true in Sraffa's normalization if A is productive) we can invert it and then write :

(V) $$\boxed{p' = wl \left[I - A(1 + r) \right]^{-1}}$$

which is an expression for the n relative prices (given equation (IV) as normalization), once we have a determined distribution of the net product between wages and profits.

1.29 - The Standard Commodity

The search for invariable measure of value was, as we have seen, intented by all the economists that adopted the labor theory of value, from Ricardo to nowadays. The solution advanced by Marx, who proposed as an invariable standard the commodity produced on the industry which possessed the organic composition of capital equal to the mean of the system was just a first approximation — in the sense that he did not provide a transformation of inputs — and was not able to hold when we tried to find the complete transformation. As Sraffa had emphasized, for a commodity to have its production prices continually equal to its labor-value, it would be necessary not only that it should be produced by an industry which possessed the mean organic composition of capital, as well as all of its means of production and the worker's consumption goods should also have been produced in industries which possessed this very same organic composition, and so on "ad infinitum".

Therefore, two possibilities are posed : either all industries work at the same organic composition of capital (and then equal to the system's mean); or we assume that a given commodity, produced at an organic composition of capital just equal to the system's mean, only uses itself as mean of production and workers' consumption good. Well, the first hypothesis is clearly unlikely to happen, for it is apparent that the organic compositions vary from industry to industry. The second hypothesis, also seems completely absurd at the first sight. However, it is here that Sraffa has found the solution for his problem: "It is not likely that an individual commodity could be found which possessed even approximately the necessary requisites. A mixture of commodities, however, or a composite commodity, would do equally well; it might do even better, since it could be blended to suit our requirements, modifying its composition so as to smooth out a price-bulge at one wage-level or to fill in a depression at another level". (53)

Along this line, Sraffa proceeds especulating on how should that composite commodity be :"the perfect composite commodity (...) in which the requirements are fulfilled to the letter, is one which consists of the same commodities (combined in the same proportions) as does the aggregate of its own means of production - in other words, such that both product and means of production are quantities of the self-same composite commodity". (54)

It should be clear by now that if we want to define a composite commodity - that should behave like a simple commodity, but

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(53) P. Sraffa - op cit. p. 18

(54) P. Sraffa - op cit. p. 19

indeed used just itself as mean of production - that condition of constant proportionality between the different commodities that are its components is a "sine qua non" condition for the very maintainance of that composite commodity's identity, while product and while means of production. If not, the input-commodity would be different from the output-commodity, and we would not have satisfied the condition for it to be "the same commodity".

1.30 - The Construction of That Commodity

At this point Sraffa shows us how to obtain a composite commodity which is able to satisfy the above requirements.(55) Beginning with a system which produces iron, coal and wheat, along the following structure:

	90t.. iron ⊕	120t.. coal ⊕	60qr. wheat ⊕	3/16 labor	→ 180t.. iron
	50t.. iron ⊕	125t.. coal ⊕	150qr. wheat ⊕	5/16 labor	→ 450t. coal
	40t. iron ⊕	40t. coal ⊕	200qr. wheat ⊕	8/16 labor	→ 480qr. wheat
Totals	180t.. iron	285t.. coal	410qr. wheat	1 labor	

he suggests that "we must take, with the whole of the iron industry, 3/5 of the coal industry and 3/4 of the wheat-growing one" (56), leading us to the resulting system:

	90t.. iron ⊕	120t.. coal ⊕	60qr.. wheat ⊕	3/16 labor	→ 180t. iron
	30t.. iron ⊕	75t.. coal ⊕	90qr.. wheat ⊕	3/16 labor	→ 270t. coal
	30t.. iron ⊕	30t.. coal ⊕	150qr. wheat ⊕	6/16 labor	→ 360qr. wheat
Totals	150t.. iron	225t.. coal	300qr. wheat	12/16 labor	

It is easy to conclude that now the proportions in which iron, coal and wheat enter on their own means of production as a whole (150 : 225 : 300) are the same on which these commodities are produced (180 : 270 : 360). Everything goes like if the subsystem

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(55) P. Sraffa - op cit. pp. 18-20

(56) It should be noted that Sraffa gives those fractions, without telling yet how and where fo find them. P. Sraffa - op cit. p. 19

constituted by those three "slices" of the real industries should behave as a single industry that used as mean of production just the very commodity it produces. If, for example, we define an unit of this composite commodity as being (2t. iron ⊕ 3t. coal ⊕ 4qr. wheat), we could say that this subsystem consumes 75 units of that composite commodity, in order to produce 90 units of this same commodity in each production period. A composite commodity that behaves in that way is called "standard commodity" by Sraffa.

1.31 - The Standard System and the Standard National Income

The physical subsystem obtained in the above section, although being real and observable inside the total physical system, has a particularity that could bring about some difficulties for comparative evaluations : it does not employ all the available labor force. In the given example, only 3/4 of the national labor is present. For uniformity reasons, Sraffa defines as Standard System the set of equations (or "industries") taken on a proportion that makes the standard commodity to appear (like in the example), but using the total annual available labor in the economic system.

Therefore, if we want to obtain the Standard System for the example above, we must multiply each equation by 4/3, obtaining :

	120t. iron ⊕	160t. coal ⊕	80qr. wheat ⊕	4/16 labor	→ 240t. iron
	40t. iron ⊕	100t. coal ⊕	120qr. wheat ⊕	4/16 labor	→ 360t. coal
	40t. iron ⊕	40t. coal ⊕	200qr. wheat ⊕	8/16 labor	→ 480qr. wheat
Totals	200t. iron	300t. coal	400qr. wheat	1 labor	

We should note that the proportions remain the same (200 : 300 : 400) and (240 : 360 : 480).

Now, we are able to define the Standard National Income, that will be used as invariable measure of value, as being the national income obtained from the standard system. Naturally it is also composed by the "standard commodity", that in the example is :

SNI (physical) = (40t.. iron) U (60t..coal) U (80qr..wheat) where iron, coal and wheat, of course, still are in the same proportion of the inputs and outputs (2 : 3 : 4) ..

It is worth to remember that in the standard "industry" (now system), the ratio between net product and the means of production is equal to the system's maximal rate of profits (R), once, the standard commodity's price is always equal to its labor-value.. By the same token, given the proportions constancy, the rate of surplus of each one of the simple commodities which compound the standard commodity is also equal to R.

1.32 - The Dual System of Multipliers

"To restate it in general terms, the problem of constructing a Standard commodity amounts to finding a set of k suitable multipliers which may be called $q_a, q_b, \ldots, q_k$, to be applied respectively to the production-equations of commodities «a», «b», ..., «k»" (57).

Therefore, beginning with the physical system below :

$$Aa \oplus Ba \oplus \ldots \oplus Ka \oplus La \rightarrow A$$

$$Ab \oplus Bb \oplus \ldots \oplus Kb \oplus Lb \rightarrow B$$

$$Ak \oplus Bk \oplus \ldots \oplus Kk \oplus Lk \rightarrow K$$

the problem now is to find multipliers that, applied to the system's equations, give rise to equal rate of surplus (R) for all the standard system's commodities. Therefore we can write the dual system of multipliers in the following way :

$$(Aa\ qa + Ab\ qb + \ldots + Ak\ qk)(1 + R) = Aqa$$

$$(Ba\ qa + Bb\ qb + \ldots + Bk\ qk)(1 + R) = Bqb$$

$$(Ka\ qa + Kb\ qb + \ldots + Kk\ qk)(1 + R) = Kqk$$

remembering the requirement to use all the available labor, that is:

$$La\ qa + Lb\ qb + \ldots + Lk\ qk = 1$$

We have now a system with (K + 1) independent equantions,. from which the K multipliers $q_r, q_b, \ldots, q_k$ and the maximal rate of profits R can be determined.

Applying those multipliers, thus determined, to the original physical system, we obtain the standard physical system:

$$Aa\ qa \oplus Ba\ qa \oplus \ldots \oplus Ka\ qa \oplus La\ qa \rightarrow Aqa$$

$$Ab\ qb \oplus Bb\ qb \oplus \ldots \oplus Kb\ qb \oplus Lb\ qb \rightarrow Bqb$$

$$Ak\ qk \oplus Bk\ qk \oplus \ldots \oplus Kk\ qk \oplus Lk\ qk \rightarrow Kqk$$

from which we can stablish the standard production price's system

$$qa\left[(Aa\ pa + Ba\ pb + \ldots + Ka\ pk)(1+r) + Law\right] = qa\ Apa$$

$$qb\left[(Ab\ pa + Bb\ pb + \ldots + Kb\ pk)(1+r) + Lbw\right] = qb\ Bpb$$

$$qk\left[(Ak\ pa + Bk\ pb + \ldots + Kk\ pk)(1+r) + Lkw\right] = qk\ Kpk$$

1.33 - The Invariable Measure of Value: The Standard National Income

We can now make explicit what is the composite commodity whose value is invariable - either when taken as proportional to the direct

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(57) P. Sraffa - op cit., p. 23

and indirect amount of labor dispended on its production or evaluated at production prices - and that will serve as a standard of measure for the production prices and wage of the original system.

As we have seen on section 1.32, it will be the standard national income, whose magnitude taken on production prices, we will equate to unity, that is:

$$\left[qa\ A - (qa\ Aa + qb\ Ab + \dots + qk\ Ak) \right] pa +$$

$$+ \dots + \left[qb\ B - (qa\ Ba + qb\ Bb + \dots + qk\ Bk) \right] pb +$$

$$+ \dots + \left[qk\ K - (qa\ Ka + qb\ Kb + \dots + qk\ Kk) \right] pk = 1 .$$

1.34 - <u>The Construction of the Standard Commodity: Matrix Notation</u>

Retaking the notation presented in section 1.28, our problem now is to find n multipliers ki, that can satisfy the dual system:

(VI) $\boxed{(Qk)\ (1 + R) = \hat{q}\ k}$

subject to $k'L = 1$
where $k = (k_1, k_2, \dots, k_n)'$ is the vector of multipliers that shall be applied to equations 1,2,....,n.

Remembering that : $Q = A\hat{q}$, we may write (VI) as

(VII) $\boxed{A\hat{q}k\ (1 + R) = \hat{q}k}$

If we now define a matrix $B = [Bij]$ such that $Bij = \frac{Qij}{qi}$ (the so-called output-quotas matrix) we may write that :

(VIII) $\boxed{B = \hat{q}^{-1}A\hat{q}}$

Pre-multiplying (VII) by $\hat{q}^{-1}$, we get:

$\hat{q}^{-1}A\hat{q}k\ (1 + R) = k$, or taking(VIII) $\Rightarrow$

(IX) $\boxed{Bk\ (1 + R) = k}$

But equation (IX) is a characteristic equation, and if we re-write it in the form

$$\text{(X)} \quad \left[I \left(\frac{1}{1 + R} \right) - B \right] k = 0$$

we would see that $\frac{1}{1 + R}$ is the dominant characterist root (eigenvalu of matrix B and k is the associated right-hand characteristic vector (eigenvector).

Aplying these multipliers to the original physical system, and calling $\hat{k}$ a diagonal matrix of multipliers, we can write the standard production prices system as:

(XI) $p'A\hat{q}\hat{k}\ (1 + r) + wl\hat{q}\hat{k} = p'\hat{q}\hat{k}$, or, post-multiplying by $(\hat{q}\ \hat{k})^{-1} \Rightarrow p'A\ (1 + r) + wl = p'$ or $p' = wl \left[I - A\ (1 + r) \right]^{-1}$

that is, the production prices system is still the same (as should be expected) although now our standard measure unit is $p'\ (q\hat{k} - Aq\hat{k}) = 1 \Rightarrow p' \left[(q - Aq)\ \hat{k} \right] = 1$ or $p'f\hat{k} = 1$ (XII), way in which we can express the Standard National Income for our system..

1.35 - Conclusion

Our objective is not to have exposed in detail all the aspec of Sraffa's model, but just to emphasize some of them that seemed to be important for our empirical application on the next part of this paper..

It should be valuable to make reference here, before finishi that Sraffa finds it possible to measure commodity prices and wages in a way that assures the validity of the labor theory of value, with even less reference to the Standard Commodity : "There is available (...) a more tangible measure for prices of commodities which makes it possible to displace the standard net product (...). This measure (...) is the quantity of labor that can be purchased by the Standard net product (...) the resulting prices of commodities can be indif- ferently regarded as being expressed either in the Standard net product or in the quantity of labor which at a given level of the rate of profits is known to be equivalent to it." (58)

Finding a linear relation linking the rate of profits and wage through the maximal rate of profit as $\boxed{r = R(1 - w)}$ (XIII) Sraffa then express his brilliant conclusion:

"All the properties of an <u>invariable</u> <u>standard</u> <u>of</u> <u>value</u> (...) are found in a variable quantity of labor, which, however, varies according to a simple rule which is independent of prices: this unit of measurement increases in magnitude with the fall of the wage, that is to say with the rise of the rate of profits, so that, from being equal to the annual labor of the system when the rate of profits is zero, it increases without limit as the rate of profits approaches its maximum value R." (59)

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(58) P. Sraffa - op cit. p. 32

(59) idem

PART 2 - Empirical Application

2.1 - Introduction : Aggregation

For our empirical application of Sraffa's model we have chosen an input-output matrix built for Brazil, for the year 1969. (1) The original matrix has 25 sectors and, being the first attempt to build such a matrix made in Brazil, has many deficiencies, which will be discussed along our analysis. As the authors explicitly put it. "In some sectors we had some problems that were partially solved, but its final structure seem to be not completely reliable. Among those, we would point mainly : crude minerals sector, agriculture, building material sector, services and utilities & urban transport. The last two were aggregated into a single sector under a "non-discriminated" label for they presented serious statistical discrepancies." (2)

To make the manipulation easier and the presentation neater, we decided to aggregate the original matrix into 10 sectors. For doing so, we have used some of the recommendations found on Chenery and Clark's Interindustry Economics (3) for unbiased aggregation.

Basically our criterion was based on 2 indexes : a) the "degree of basicity" (in Sraffa's sense) of the industry analysed, and calculated as total intermediate demand divided by total final demand. b) The proportion of wages on value added. The argument for that was, on the one hand to preserve the basic-non-basic character of the commodities in the system, avoiding biased mixing; on the other hand it was necessary not to aggregate industries where the relation wage-value-added should be very different, once that the departure could be just apparent (linked to prices variation) not meaning a pure variation in the rate of surplusvalue (do not forget

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(1) This matrix is presented on the article "Matriz de insumo - produto do Brasil" by Antonio Leao and Others - Revista Brasileira de Economia - March 1973 - Rio de Janeiro - Brazil

(2) Idem p. 4 - our English version

(3) H. Chenery and P. Clark -"Interindustry Economics"- 1959 - John Wiley & Sons Inc..

that our matrix was expressed on market prices !)

Those indices for the 25 sectors were found to be:

Sector		Basicity Index	Wage-Value Added Ratio
1	Minerals	2.21	.75
2	Non-Metallic	..71	.24
3	Metallurgy	16.26	.47
4	Machinery	.84	.51
5	Eletric and Communication Equip.	.77	.34
6	Transport Equipment	.68	.39
7	Wood	6.19	.49
8	Furniture	.36	.74
9	Paper & Paper-board	70.34	.34
10	Rubber Products	1.17	.45
11	Hides, Skins and Leather	74.28	.39
12	Chemicals	51.53	.19
13	Medicinal and Pharmacy Products	.61	.59
14	Perfumery Products	.82	.29
15	Plastics	.88	.41
16	Textiles	3.19	.54
17	Apparel, Footwear	.97	.87
18	Foodstuffs	.74	.13
19	Beverages	.90	.44
20	Tobaco products	.41	.16
21	Editorial, Printing	.41	.28
22	Others	3.13	.66
23	Building Materials	~0	.50
24	Agriculture	.52	.39
25	Non-discriminated	.47	.52

Along those lines the final aggregation made was:

Aggregated Sector	Basicity Index	Wages-Value Added	Original Sectors
1- Raw Materials	9.89	.46	1- Minerals 3- Metallurgy 7- Wood
2- Intermediate Products I	54.87	.21	9- Paper and Paperboard 11- Hides, Skins and Leather 12- Chemicals
3- Intermediate Products II	.85	.29	2- Non-metallic 10- Rubber Products 15- Plastics
4- Intermediate Products III	.75	.36	4- Machinery 5- Eletric & Communication Equip. 6- Transport Equipment
5- Consumer Goods I	.68	.43	13- Medicinal & pharmacy Products 14- Perfumery Products
6- Consumer Goods II	1.44	.49	8- Furniture 16- Textiles 17- Apparel, Footwear 21- Editorial, Printing
7- Consumer Goods III	.74	.15	18- Foodstuffs 19- Beverages 20- Tobacco Products
8- Building Materials	~0	.49	23- Building Materials
9- Agriculture	.51	.37	24- Agriculture
10- Non-discriminated	1.11	.40	22- Others 25- Not-discriminated

Obviously, a lot of problems had to be overcame. Particularly problematic was the aggregation of Metallurgy, Chemical, Editorial and "Others". We finally decided, not before some hesitations, to aggregate them as specified above, on grounds of "basicity" and/or wage-value added relations, or, additionally on no ground at all, as we have done with sector "Others", that was aggregated with sector "non- discriminated". However, although at a first sight we have some clear heteregeneity among the aggregated sectors, we think, we are much more able to justify the above aggregation than all the "more alike" one that has ocurred to us.

The 10 by 10 input-output aggregated matrix obtained is shown in table 2.1, along with values for total, intermediate and final demand, and the composition of value added per aggregated sector.

2.2 - The Simulation Model

It was built a semi-conversational simulation model which was stored in the Computer Center of Boston University. This simulation model, fed with 10-by-10 dimensioned matrices Q or A (along the convention presented in section 1.28), vector q or f and direct labor vector (normalized along Sraffa's method), provides a complete output for the physical system, and simulates the production prices variation for different values of wages. The program also furnishes the normalization multipliers (see sections 1.32 and 1.34) and preceeds to "standardize" the system, presenting again the production prices variations for different values of wages, but now measured in the standard commodity "or in "quantities of labor that can be purchased by the standard net product". Others programs written for the TSP language provided us with graphical representations for that simulations and the "factor-prices frontier".

A complete listing of these programs will be given on Appendix 1.

2.3- Problems of Simulation: The Inverse Transformation Problem

Up to now we have as data an aggregated matrix (10 by 10) measured in market prices on the one hand, and, on the other hand,

	SECTOR 1	SECTOR 2	SECTOR 3	SECTOR 4	SECTOR 5	SECTOR 6	SECTOR 7	SECTOR 8	SECTOR 9	SE[illegible]
SECTOR 1	2 561.0	156.3	328.5	1 462.3	38.8	184.0	213.2	984.9	500.6	688.3
SECTOR 2	269.7	1 730.0	681.2	256.4	234.8	2 040.0	1 021.9	115.0	818.2	72.7
SECTOR 3	70.1	53.1	347.0	246.5	99.1	71.9	87.5	5.9	329.5	959.3
SECTOR 4	65.4	.7	1.4	2 435.6	.2	.2	.5	.1	89.2	2 352.2
SECTOR 5	.5	115.5	2.7	1.0	288.1	2.8	48.9	.1	24.5	537.6
SECTOR 6	35.6	47.2	81.5	49.5	13.5	2 866.0	24.1	4.1	1 905.9	1 471.8
SECTOR 7	.6	60.4	1.8	.4	50.7	1.4	1 750.6	.6	14.4	1 773.9
SECTOR 8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	.0
SECTOR 9	7.1	114.0	185.1	1.0	3.7	237.4	684.1	.1	1 668.0	8 508.6
SECTOR 10	887.0	783.4	228.0	680.9	228.2	1 062.7	476.4	114.2	3 817.7	100 860.3
Total int. consumption	3 932.8	3 061.5	1 865.4	5 133.8	957.1	6 445.4	4 307.2	1225.0	10 168.0	116 920.6
Imports	460.7	819.3	215.4	849.9	232.8	281.4	163.7	0.0	162.4	6 863.9
Wages	1 845.3	722.8	817.1	2 018.8	572.0	2 277.5	601.1	680.4	8 366.4	33 207.9
Profits	1 334.1	2 040.1	1 572.3	2 580.8	517.2	1 446.6	2 696.4	684.5	10 320.3	28 136.5
Ind. Tax.	318.3	314.9	319.4	696.5	206.7	542.9	673.4	70.7	883.0	20 061.8
Depreciation	503.8	390.9	138.7	276.0	41.4	394.7	154.7	7.8	2 779.7	2 810.3
Total Value Added	4 001.5	3 468.7	2 847.5	5 572.1	1 337.3	14 613.5	4 126.3	1443.4	22 349.4	83 357.2
Total	8 395.0	7 349.5	4 928.3	11555.8	2 527.2	11 340.3	8 597.2	2668.4	32 679.8	207 141.7

	Interm. Demand	Final Demand	Total Demand
SECTOR 1	7 624.3	768.7	8 395.0
SECTOR 2	7 218.2	131.3	7 349.5
SECTOR 3	2 269.9	2 654.4	4 928.3
SECTOR 4	4 945.8	6 610.0	11 555.8
SECTOR 5	1 021.7	1 506.5	2 527.2
SECTOR 6	6 696.3	4 644.0	11 340.3
SECTOR 7	3 654.7	4 942.5	8 597.2
SECTOR 8	0	2 668.4	2 668.4
SECTOR 9	11 109.1	21 270.7	32 679.8
SECTOR 10	109 174.8	97 966.9	207 141.7
Total int. consumption	154 016.8	143 166.4	297 183.2

TABLE 2.1

a simulation model which asks for magnitudes as the technological matrix (A), the quantity vector (q) and the normalized amount of direct labor (l).

The question that is now posed is: can we "convert" this market-price-matrix into matrix A? If yes, how and at what cost can this "convertion" be made?

Looking for an answer, let us define a market prices vector $p^* = (p_1^*, p_2^*, \ldots, p_n^*)$ and a matrix $X = [Xij]$ such that $Xij = p_i^* Qij$. We can therefore write:

$\boxed{X = \hat{p}^* Q}$ (XIV) , where $\hat{p}^*$ is a diagonal matrix of market prices. Defining also the vector of total quantities at market prices as $x = (x_1, x_2, \ldots, x_n)'$ such that $xi = p_i^* qi$, we can also write:

$$\boxed{x = \hat{p}^* q} \quad \text{(XV)}$$

Surely we cannot determine the magnitudes of Q and q because we do not have the magnitude of the market prices vector p*. However a first step toward a way to solve this problem can be taken along Morishima and Seton's line (4) : in their article on the "inverse transformation problem", no matter how misleading conclusions they have reached, a neat way to find the "output-quotas" matrix (B in section 1.34) is advanced.

As we remember, the matrix B was defined as:

$B = [Bij]$ such that $Bij = \frac{Qij}{qi}$

Therefore: $\boxed{B = \hat{q}^{-1} Q}$ (XVI)

Writing the identity matrix as $I = \hat{p}^{*-1}\hat{p}^*$ and manipulating (XVI) algebraically, we get :

$$B = q^{-1} I Q = \quad B = q^{-1} p^{*-1} p^* Q = \quad \boxed{B = (\hat{p}\hat{q})^{-1} \hat{p} Q} \quad \text{(XVII)}$$

Taking (XIV) and (XV) into consideration, (XVII) can be written as

$$\boxed{B = \hat{x}^{-1} X} \quad \text{(XVIII)}$$

(4) M. Morishima, F. Seton - "Aggregation in Leontief Matrices and the Labor - Theory of Value" - Econometrica - April 1961 - pp203-[illegible]

This means that for each element $Bij = \frac{p^{*}i\ Qij}{p^{*}i\ qi} = \frac{Qij}{qi}$, the market prices $p^{*}i$ cancel them out and we conclude that the matrix B is invariant when market prices change.

The matrix B calculated this way is seen in table 2.2. (p.66)

The usefulness of Morihima and Seton's work finish here, once that most of its assumptions advanced from this point on are either incompatible with Sraffa's or relied on too arbitrary hypothesis.

We find ourselves with a 10-by-10 matrix that could not "pretend " to be matrix A, once that

$$\boxed{A = \hat{q}\ B\ \hat{q}^{-1}} \qquad (XIX)$$

and whatever q vector we choose for transforming the matrix B into matrix A would be as arbitrary as picking randomly supposed market prices, from other sources...

2.4 - The Inverse Transformation Problem Along Sraffa's Normalization

In order to avoid introducing extraneous information and to keep the coherence of Sraffa's normalization, it was needed therefore a way to solve this "inverse transformation" so that our system could be simulated. Surprisingly this "inverse transformation" can come almost straightforward[4], provided that we give proper interpretations for the vectors total quantities and prices which are produced as solution.

To begin with, let us write again the expression (III) of section 1.28 :

$p'\ A\hat{q}\ (1 + r) + wl\hat{q} = p'\hat{q}$, or, remembering that $L = l\hat{q} \Rightarrow$

$\boxed{p'A\hat{q}\ (1 + r) + wL = p'\hat{q}}$ (XX) but $A = \hat{q}\ B\hat{q}^{-1}$ and therefore

$\boxed{p'\hat{q}B\ (1 + r) + wL = p'\hat{q}}$ (XXI)

Taking now a particular value of distribution, such that w = 1 and r = 0 (the labor-value system) we have the expression (XXI) converted into:

$$\boxed{p'\hat{q}B + L = p'\hat{q}} \qquad \text{(XXII)}$$

Our question still is, how to determine p and q, without making any arbitrary assumption? Well, it should be thought now, what meaning should have p and q in our aggregate matrix? They clearly could not refer to an individual commodity, since as we have seen (in section 2.1), the sectors are all composed of heterogeneous goods. But then, how to measure different goods with a single number? The answer is : measuring them in quantities of labor. But how?

Let us take expression (XXII) and make the assumption that $p_i = 1$, $i = 1, \ldots, n$, i. e., $p' = u'$. We have therefore:

$$u'\hat{q}B + L = u'\hat{q} \Rightarrow q'B + L = q' \Rightarrow \boxed{q' = L\,(I - B)^{-1}} \qquad \text{(XXIII)}$$

What does this mean? This mean simply that beginning with the labor-value system written as (XXII) we decide to measure not the physical number of commodities in absolute terms, but, in the proportion of each "composite commodity" produced in each sector which is necessary to exchange by a given amount of labor (made equal for all the sectors when we put $p' = u'$). In other words, when we made the prices (now just index prices) to be equal to one, in the labor-value system, we have established a given amount of labor (which we do not have even to know how is measured - in hours, days or seconds) and we are asking how do the proportions of the composite commodities produced with this amount of labor compare. In fact the absolute values of the qi's found by the process do not have any useful meaning in itself. But the proportions between any two of them are in the same proportions as the quantities of different goods produced with the same quantity of labor (Say, 3 trucks and 22 television sets for 1.000 hours of labor).

Therefore, expression (XXIII), while not giving us "real quantities" (what would be almost foolish to expect) does give us some valuable information : we can know how the "composite commodities" of each sector should exchange at their labor-values with the others "composite-commodities" of our system.

In this sense, we conclude that we cannot obtain our "true" quantity vector, but we could provide a vector whose components are in the same proportions of the components of the "true" quantity vector, should the latter be measured in labor-time.

This conclusion should be well understood for there is no tautology or assumptions in it extraneous to the labor theory of value.

2.5 - The Determination of the Normalized Quantities of Labor

Although we have found a neat expression for q in (XXIII) we still do not know the value of the vector $L = (L_1, L_2, \ldots, L_n)$, such that $\sum_{i=1}^{n} Li = 1$, along Sraffa's normalization.

If we remember that Sraffa makes the assumption that all the labor is homogeneous, or conversely, that heterogeneous qualities of labor have been converted into different quantities of homogeneous labor, so that any unit of labor receives the same remuneration, (see section 1.14) we could see that the normalized quantities of labor already reduced to homogeneous labor can be easily calculated from table 2.1. We just have to see that the figures given as wages in this table could be thought as homogeneous quantities of labor multiplied by the same wage rate. If we divide each one of these figures by the total wages-payment in the economy, wages will cancel out and the magnitudes we obtain for each sector have much of the same significance than Sraffa's normalized quantities of labor.

Therefore we have:

Sector	1	2	3	4	5	6	7	8	9	10	Total
[illegible]ents as Wages	1,845.3	722.8	817.1	2,018.8	572.0	2,277.5	601.1	680.4	8366.4	33207.9	51,109.3
Normalized Quantities of Labor	.0361	.0141	.0160	.0395	.0112	.0446	.0118	.0133	.1637	.6497	1

2.6 - The Calculation of the Technological Matrix

We are now able to calculate our technological matrix (A) from the output-quotas matrix B and the relative labor quantities q given by expression (XXIII).

Noting, that, by (XIX): $A = \hat{q}B\hat{q}^{-1}$, that is $A_{ij} = B_{ij}.\frac{q_i}{q_j}$, the matrix A calculated that way is the same as if we have the "real" quantity vector, once <u>the proportions $\frac{q_i}{q_j}$ are maintained in our solution if quantities are measured in labor-time.</u>

The matrix A therefore calculated is presented on table 2.3.

2.7 - Simulation of the Model for Varying Distribution of the Net Product

We are now able to feed the computer with the necessary data for the simulation in two steps:

a) To calculate $p' = wl\,[I - A(1 + r)]^{-1}$ with $p'f = 1$ as measure for prices and wages. (not standardized system)

b) To calculate $p' = wl\,[I - A(1 + r)]^{-1}$ with $p'f\hat{k} = 1$, as the invariable measure for prices and wages, where k is given by

$$\left[I\left(\frac{1}{1 + R}\right) - B\right]k = 0$$

(standardized system)

The Matrix B is:

Sectors	1	2	3	4	5	6	7	8	9	10
1	.3363	.0205	.0431	.1980	.0051	.0242	.0280	.1203	.1970	.0004
2	.0434	.2783	.1096	.0413	.0378	.3202	.1644	.0195	.1310	.0117
3	.0168	.0121	.0790	.0561	.0220	.0164	.0100	.0013	.0750	.2103
4	.0065	.0001	.0001	.2433	.0000	.0000	.0000	.0000	.0089	.2350
5	.0002	.0553	.0013	.0005	.1390	.0013	.0234	.0000	.0117	.2575
6	.0034	.0045	.0078	.0047	.0013	.2725	.0023	.0004	.1812	.1400
7	.0001	.0078	.0002	.0001	.0065	.0002	.2256	.0001	.0019	.2286
8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
9	.0002	.0036	.0059	.0000	.0001	.0075	.0216	.0000	.0527	.2690
10	.0049	.0043	.0013	.0038	.0013	.0059	.0025	.0006	.0212	.5597

TABLE 2.2

The matrix A is

Sectors	1	2	3	4	5	6	7	8	9	10
1	.3363	.0405	.1031	.1641	.0104	.0178	.0501	.3010	.0557	.0037
2	.0220	.2783	.1327	.0179	.0726	.1222	.1487	.0276	.0188	.0002
3	.0067	.0100	.0790	.0200	.0359	.0050	.0149	.0016	.0089	.0037
4	.0076	.0002	.0003	.2433	.0000	.0000	.0000	.0000	.0029	.0112
5	.0001	.0288	.0008	.0001	.1380	.0003	.0110	.0000	.0009	.0022
6	.0046	.0121	.0254	.0055	.0067	.2725	.0056	.0016	.0696	.0077
7	.0001	.0086	.0003	.0000	.0138	.0001	.2256	.0002	.0003	.0052
8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	.0007	.0252	.0500	.0000	.0013	.0195	.1366	.0000	.0527	.0387
10	.1205	.2097	.0765	.0799	.1214	.1967	.1143	.0435	.1473	.5597

TABLE 2.3

The results of the simulation are presented in the following pages. The symbol's convention is the same as adopted in section 1.28.

It is important not to forget the interpretation that should be given to the "physical" results. (see section 2.4 above)

The price frontier in this system (along with $r = R. (1 - w)$, as seen in equation (XIII) of part I) is seen in Figure 2.1.

From page 73 to page 82 we can see the plots (figures 2.2 to 2.11) of the production prices for the 10 sectors for variations of wages, from one (when production prices are equal to labor-value) to zero. We observe that the production prices react differently to wages variation.

If we, furthermore, try to express the production prices' variation (along with their labor-values) in a scale comparable to which the market prices are measured, we can see approximately how do the total revenue of each sector, taken at market prices, compare with the equilibrium revenue (at production prices) and with its labor-value. For those comparisions we have taken a distribution of the net product between wages and profits, such that w = .35. Although this value could be not exact - for it was estimated from the input-output matrix - we know from an immediate observation of figures 2.2 to 2.11 that in this region all the production prices lines are sufficiently flat to make this approximation reliable. Of course we should not expect the market prices to coincide with its respective production prices. (see section 1.5) In this sense, some departure from the theoretical production prices is nothing but natural. Nevertheless, a large departure could mean a stagnant sector,* which could, by its turn, be an indication of bottlenecks (present or futures) in the economic system. From page 83 to page 92 we can see those comparisions (figures 2.12 to 2.21)

With respect to their labor-value, the departure of production prices, and market prices, for the 10 sectors, at w = .35 is summarized in table 2.4 below.

(CONTINUED IN PAGE 93)

putz

THIS SEMI-CONVERSATIONAL PROGRAM
SIMULATES SRAFFA'S MODEL

EC 814-Joao Damasio------------------------------------Prof.O.Kyn

Do you want to enter data from the terminal ?
(Answer yes or no)
no
Do you want to have the results of the physical system printed?
Answer yes or no
yes
THE MATRIX Q IS

2.4297	.1475	.3101	1.3216	.0368	.1741	.2017	.0304	1.4103	0.55[illegible]
.1584	1.0133	.3992	.1507	.1376	1.1955	.5005	.0674	.4707	0.035[illegible]
.0482	.0364	.2376	.1684	.0620	.0400	.0600	.0039	.2265	0.65[illegible]
.0547	.0007	.0009	2.0483	0.0000	0.0000	0.0000	0.0000	.0730	1.[illegible]
.0007	.1049	.0024	.0008	.2615	.0029	.0443	0.0000	.0220	0.4[illegible]
.0351	.0441	.0764	.0463	.0127	2.6659	.0225	.0039	1.7723	1.3[illegible]
.0007	-.0313	.0009	0.0000	.0262	.0019	.0020	.0005	.0075	0.0207
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0100	0.0000	0.0000	0.001[illegible]
.0059	.0918	.1504	0.0000	.0025	.1 [illegible]	.54[illegible]	0.0000	1.3420	[illegible]
.9674	.7610	.2301	.6727	.2301	1.0432	.4691	.1062	3.7598	99.057[illegible]

THE VECTOR q IS

7.198
3.641
3.008
8.419
1.295
5.783
4.025
2.442
25.464
176.984

THE VECTOR m IS

7.576
4.235
1.553
4.161
0.936
6.040
1.8 97
0.002
9.181
107.180

THE VECTOR f IS

-0.478
-0.594
1.455
4.258
0.959
3.743
2.128
2.440
16.283
69.804

" THE CHARACTERISTIC ROOT OF THE MATRIX A IS", .60027

The multipliers for standardizing the system are

1.8673 for sector 1
2.2939 for sector 2
0.8288 for sector 3
0.7447 for sector 4
0.9482 for sector 5
0.8821 for sector 6
0.7308 for sector 7
0.0013 for sector 8
0.5984 for sector 9
1.0344 for sector 10

The max. rate of profit is 0.8659

Do you want to enter your vector of labor from the terminal?
(Answer yes or no)
no

THE NATIONAL INCOME FOR VARYING VALUES OF W IS

1.00
2.42
5.98
14.86
37.09
92.72
231.94
580.32
1452.29
3636.00

THE PRICE MATRIX FOR VARYING VALUES OF W IS

.01000	.01000	.01000	.01000	.01000	.01000	.01000	.01000	.01000	.01000	W=1
[illegible]	.01031	.00837	.00808	.00700	.00930	.01117	.00906	.00730	[illegible]	W=
.00770	.01033	.00742	.00796	.00670	.00902	.01130	.00605	.00615	[illegible]	W=
.00716	.01025	.00693	.00733	.00620	.00864	.01128	.00617	.00565	[illegible]	W=
.00687	.01020	.00670	.00657	.00609	.00847	.01113	.00583	.00546	[illegible]	W=
.00674	.01016	.00660	.00678	.00593	.00839	.01104	.00567	.00540	[illegible]	W=
.00659	.01014	.00656	.00670	.00590	.00836	.01099	.00561	.00537	[illegible]	W=
.00657	.01013	.00654	.00668	.00589	.00835	.01097	.00559	.00537	[illegible]	W=
.00657	.01013	.00654	.00667	.00589	.00835	.01097	.00559	.00537	[illegible]	W=
.00657	.01013	.00654	.00667	.00589	.00834	.01097	.00559	.00537	.01131	W=

Do you want to standardize your system ?
yes

Do you want to have the results of the physical system printed?
Answer yes or no

THE STANDARD SYSTEM IS:

YES

THE MATRIX Q IS

4.5299	.3383	.2570	1.9565	.0349	.1536	.1474	.0012	.2488	0.7191
.2957	2.3244	.3308	.1152	.1394	1.0545	.4374	.0001	.2265	0.0394
.0091	.0235	.1970	.1288	.0645	.0431	.0438	.0000	.1350	0.7191
.1921	.0017	.0007	1.5564	0.0000	0.0000	0.0000	0.0000	.0442	2.1490
.0013	.2405	.0020	.0006	.2480	.0026	.0324	0.0000	.0137	0.5374
.0018	.1911	.0633	.0354	.0120	2.3515	.0165	.0000	1.0505	1.4770
.0013	.0718	.0007	0.0000	.0248	.0009	.6636	.0009	.0046	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0019
.0094	.2105	.1247	0.0000	.0023	.1693	.4018	0.0000	.0031	7.4277
1.6197	1.7456	.1907	.5144	.2181	.0208	.3362	.0001	2.2447	107.4229

THE VECTOR q IS

13.440
8.352
2.453
6.438
1.797
8.629
2.942
0.003
15.235
151.930

THE VECTOR p IS

8.068
5.014
1.497
3.865
1.079
5.180
1.766
0.002
9.148
115.213

THE VECTOR f IS

5.373
3.339
0.997
2.573
0.718
3.449
1.176
0.001
6.091
76.716

```
" THE CHARACTERISTIC ROOT OF THE MATRIX A IS ",.00029
The multipliers for standardizing the system are
  1.0000  for sector   1
  1.0000  for sector   2
  1.0000  for sector   3
  1.0000  for sector   4
  1.0000  for sector   5
  1.0000  for sector   6
  1.0000  for sector   7
  1.0000  for sector   8
  1.0000  for sector   9
  1.0000  for sector  10
 The max. rate of profit is  0.6659
THE NATIONAL INCOME FOR VARYING VALUES OF W IS :   (The Standard Commodity)
        1.00
        2.51
        6.29
       15.73
       39.35
       98.44
      246.28
      616.19
     1541.73
     3857.92
THE PRICE MATRIX FOR VARYING VALUES OF W  IS
.00995 .00995 .00995 .00996 .00996 .00996 .00996 .00996 .00995 .00995 v=1.
.00832 .00994 .00807 .00856 .00752 .00904 .01077 .00777 .00710 .01042 v= .
.00732 .00982 .00706 .00757 .00637 .00848 .01082 .00651 .00595 .01066 v= .
.00676 .00970 .00655 .00693 .00596 .00816 .01056 .00583 .00534 .01071 v= .
.00648 .00961 .00631 .00657 .00556 .00799 .01050 .00549 .00515 .01072 v= .
.00635 .00957 .00621 .00639 .00553 .00791 .01040 .00534 .00502 .01081 v= .
.00630 .00955 .00617 .00631 .00555 .00787 .01035 .00528 .00506 .01083 v= .
.00628 .00954 .00616 .00628 .00555 .00786 .01033 .00526 .00505 .01083 v= .
.00628 .00954 .00616 .00628 .00555 .00786 .01033 .00525 .00505 .01083 v= .
.00628 .00954 .00616 .00623 .00554 .00786 .01032 .00525 .00505 .01083 v= .
Do you want to execute the program again,with new data?
(Answer yes or no)
no
*go
```

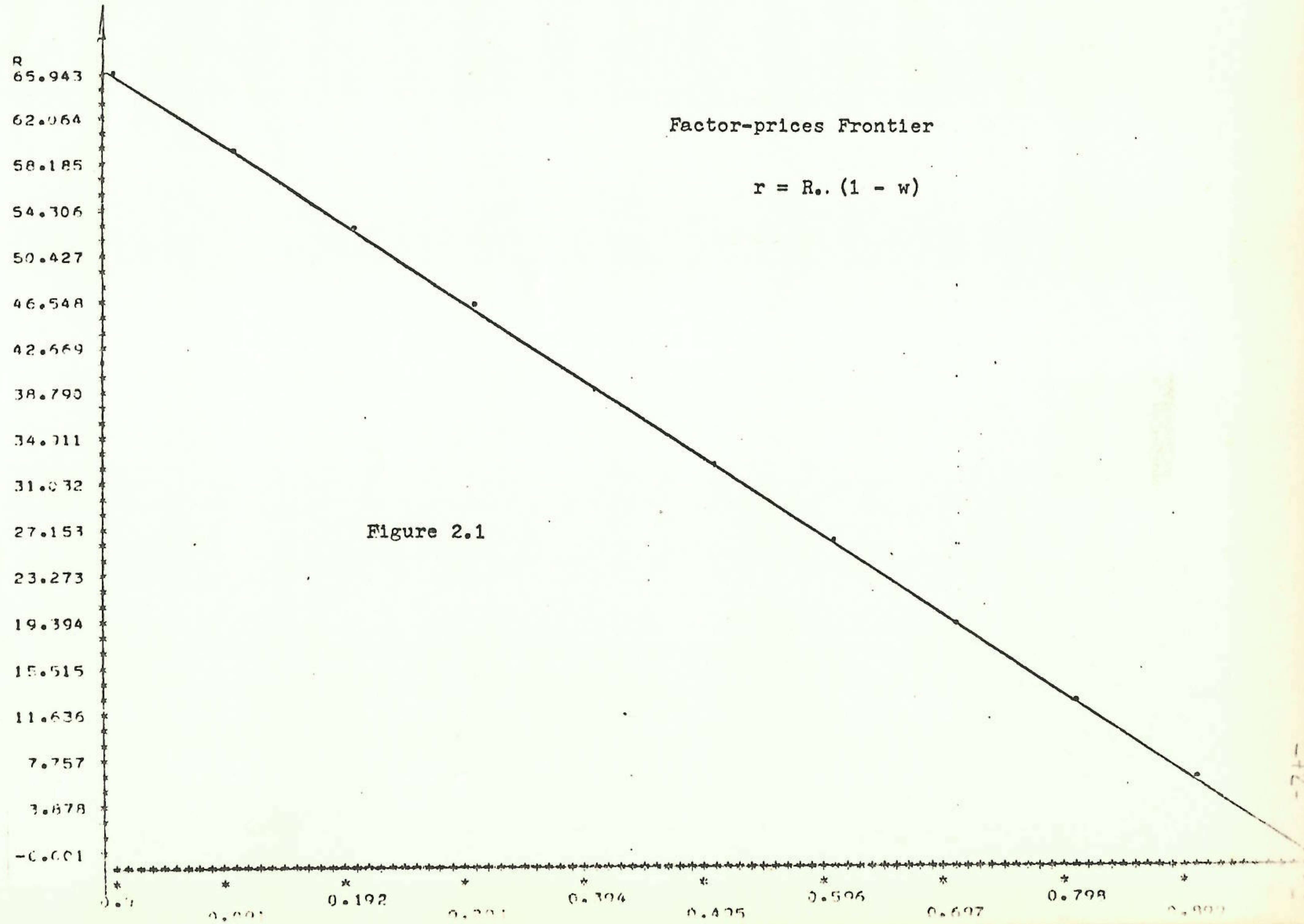

Factor-prices Frontier
r = R.. (1 - w)
R
65.943
62.064
58.185
54.306
50.427
46.548
42.669
38.790
34.911
31.032
27.153
23.273
19.394
15.515
11.636
7.757
3.878
-0.001
0.192
0.394
0.495
0.596
0.697
0.798

Figure 2.1

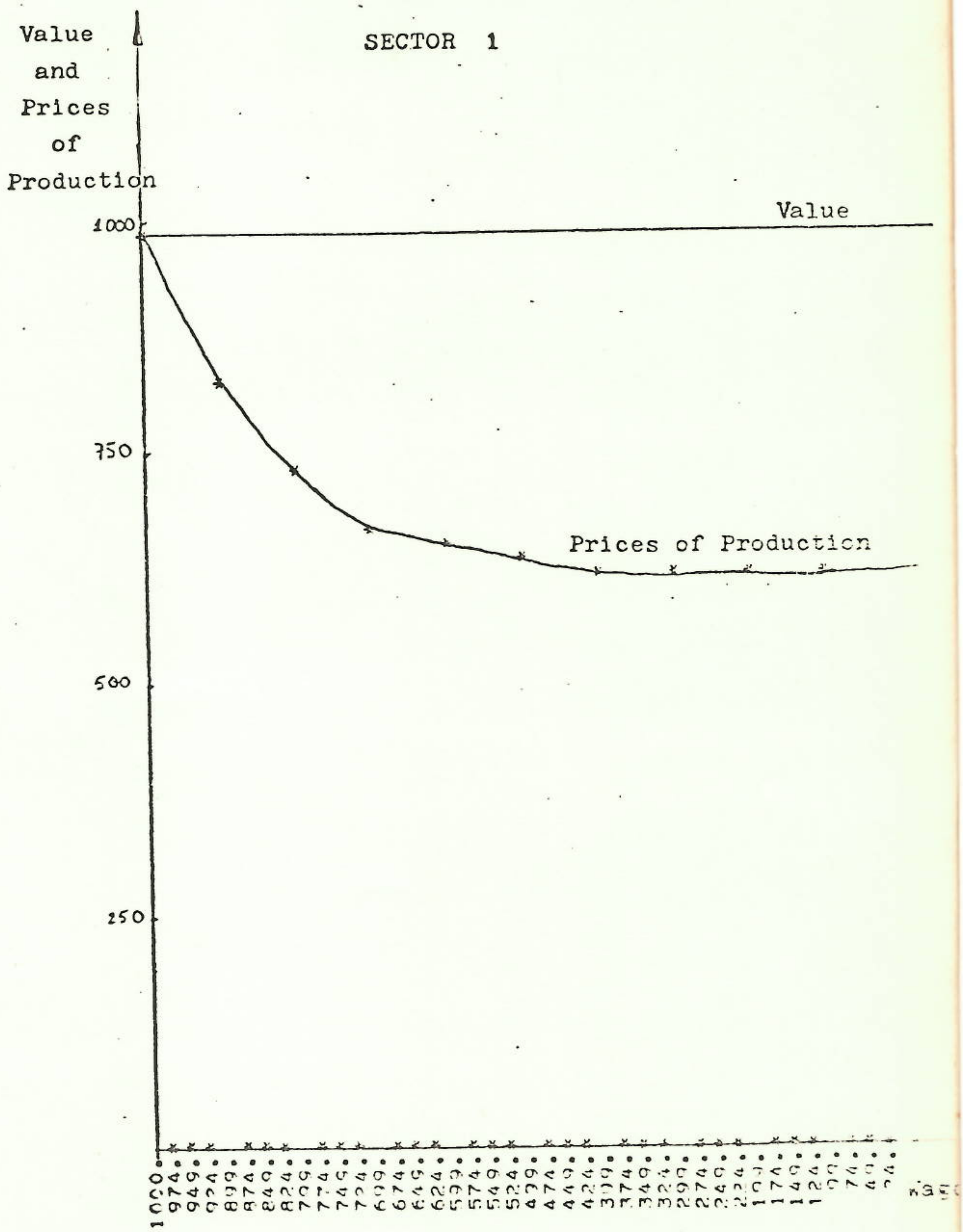
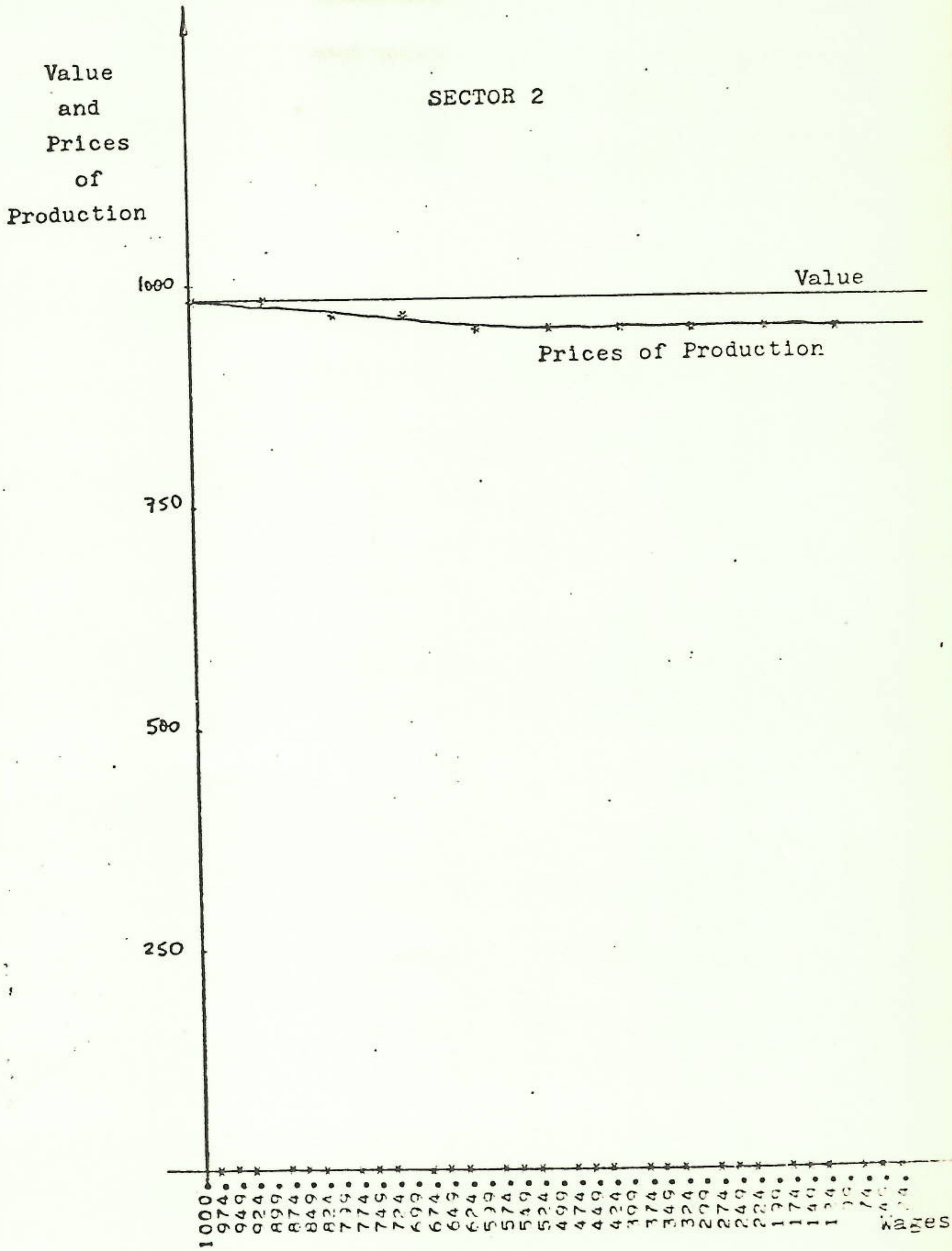


FIGURE 2.2

LINE 3 TIME SERIES PROCESSOR VERSION .6 NOV..1974 IBM 370 76.346



TIME SERIES PROCESSOR VERSIC 1.6 NOV..1974 IBM 370 75.346 7:26

LINE 4

FIGURE 2.3

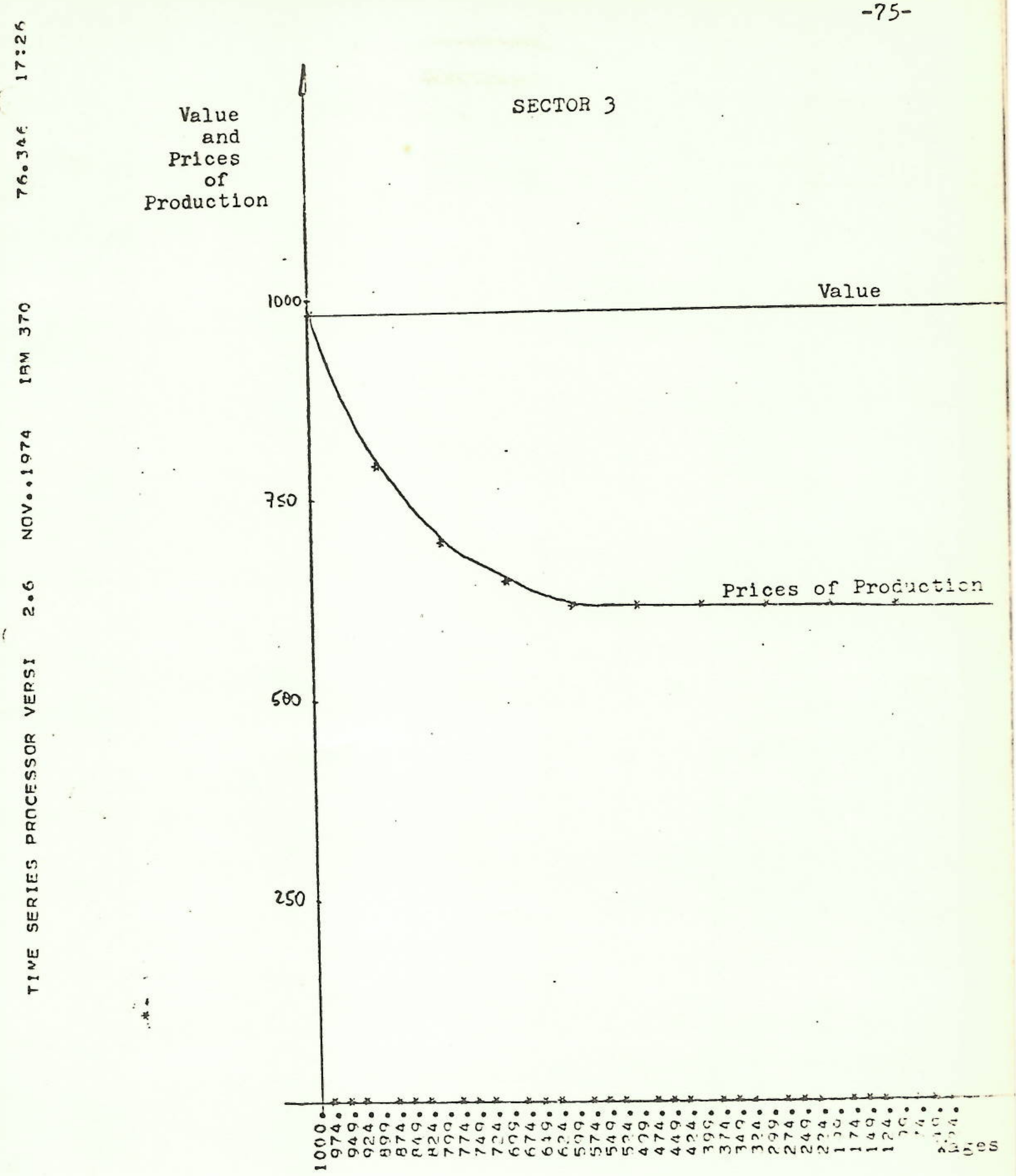

Value
and
Prices
of
Production
SECTOR 3
1000
750
500
250
Value
Prices of Production
TIME SERIES PROCESSOR VERSI 2.6 NOV..1974 IBM 370 76.346 17:26
LINE

FIGURE 2.4

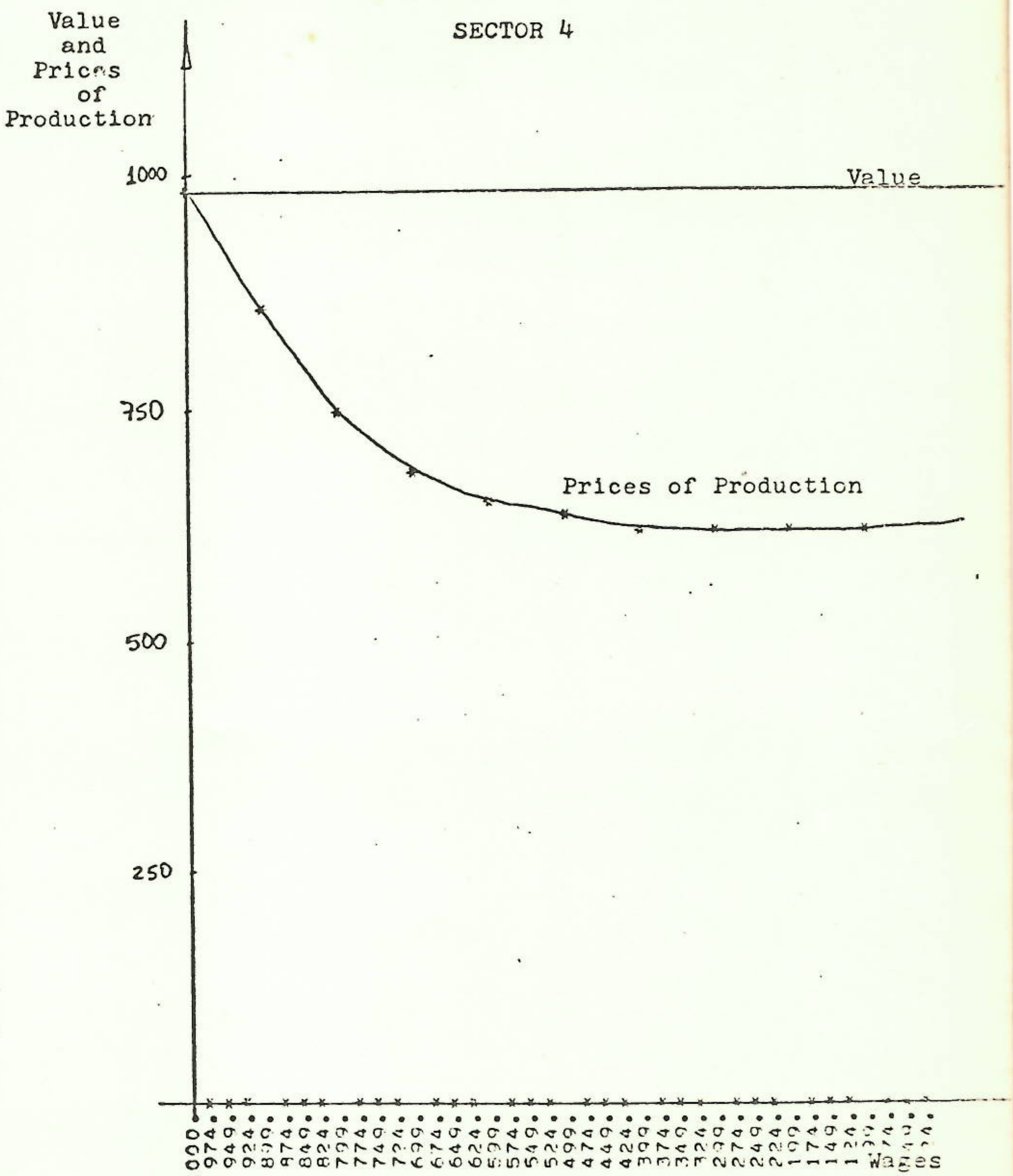

Value and Prices of Production
SECTOR 4
1000
750
500
250
Value
Prices of Production
Wages

FIGURE 2.5

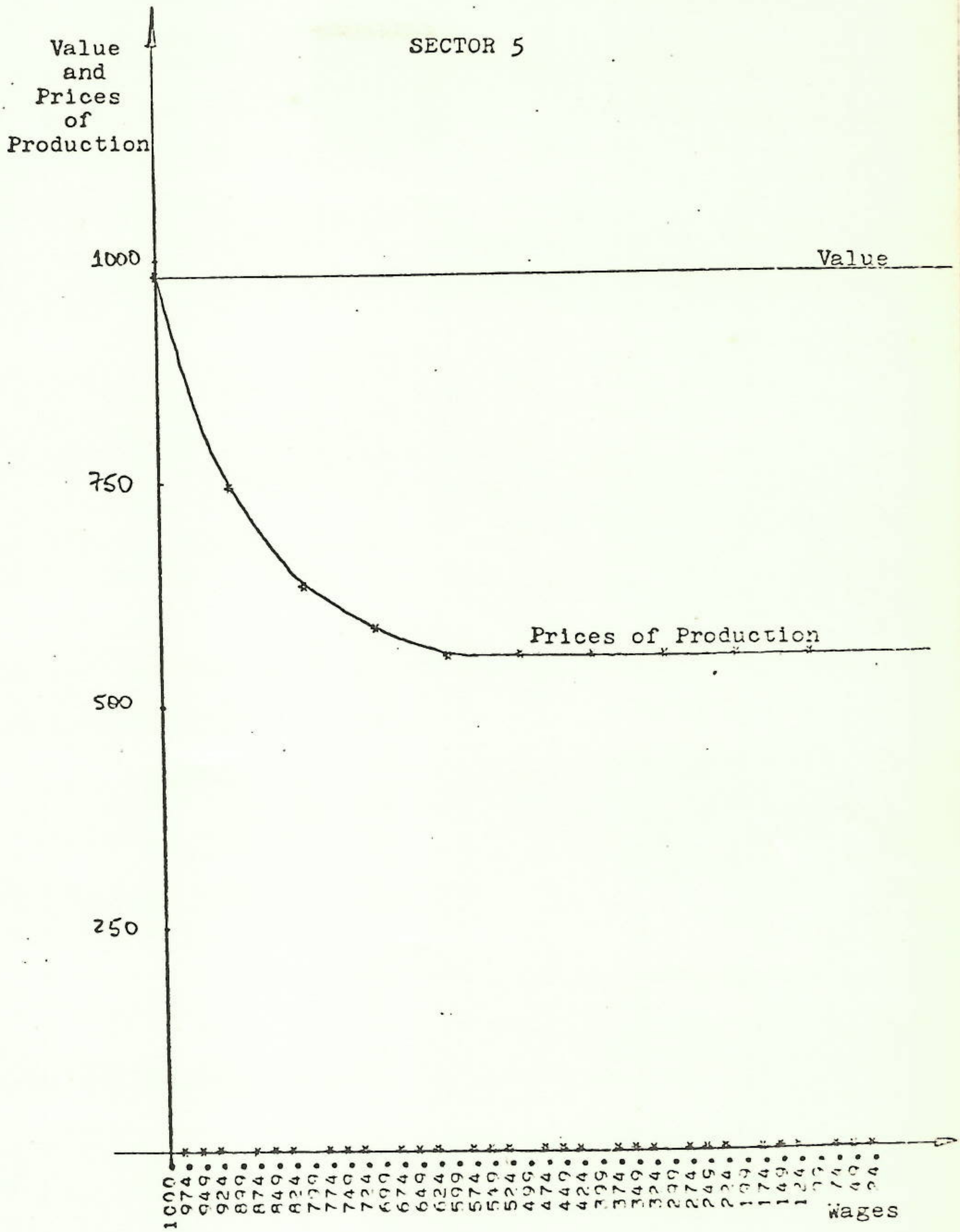

SECTOR 5
Value
and
Prices
of
Production
1000
750
500
250
Value
Prices of Production
Wages

FIGURE 2.6

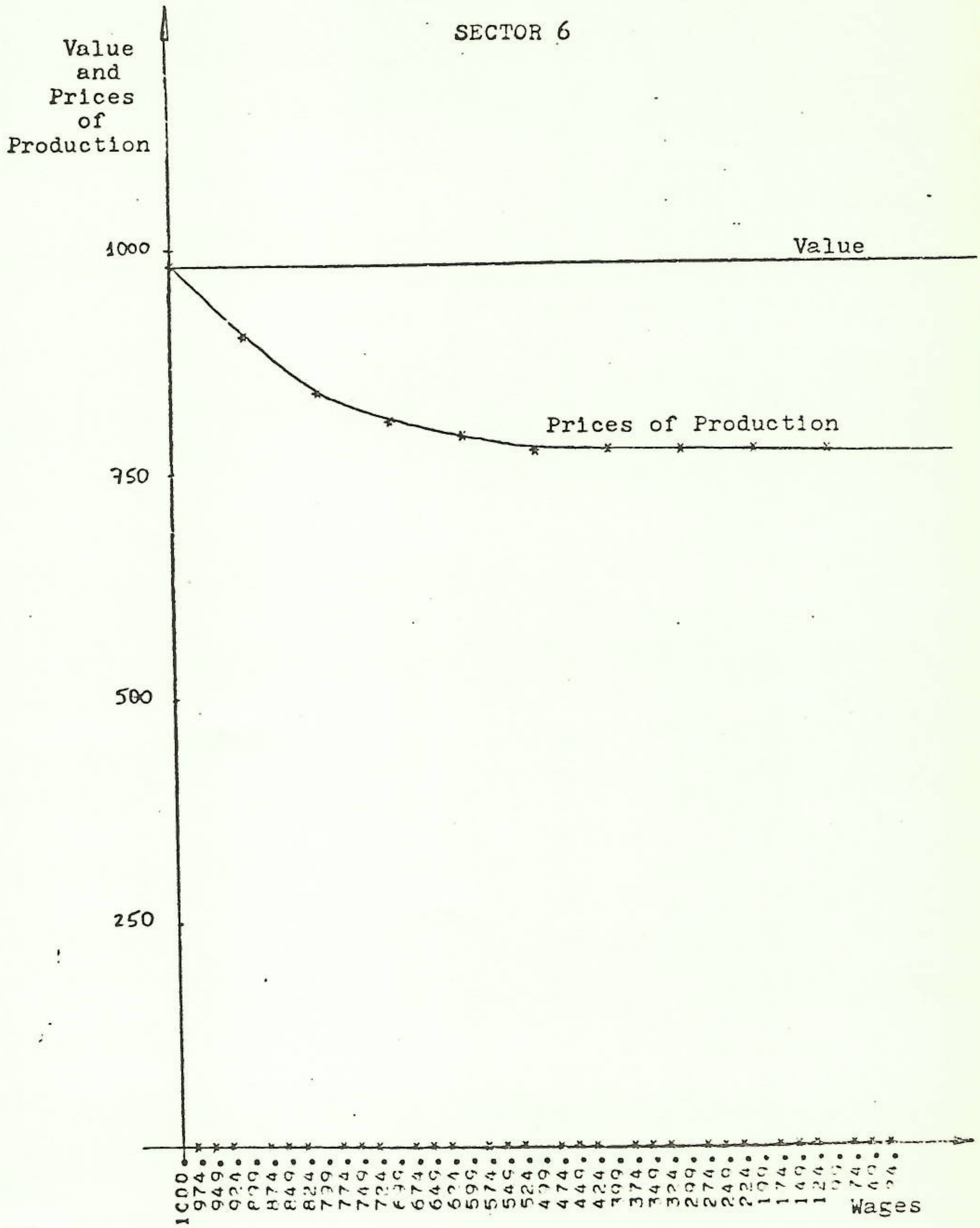

SECTOR 6
Value and Prices of Production
1000
750
500
250
Value
Prices of Production
Wages

FIGURE 2.7

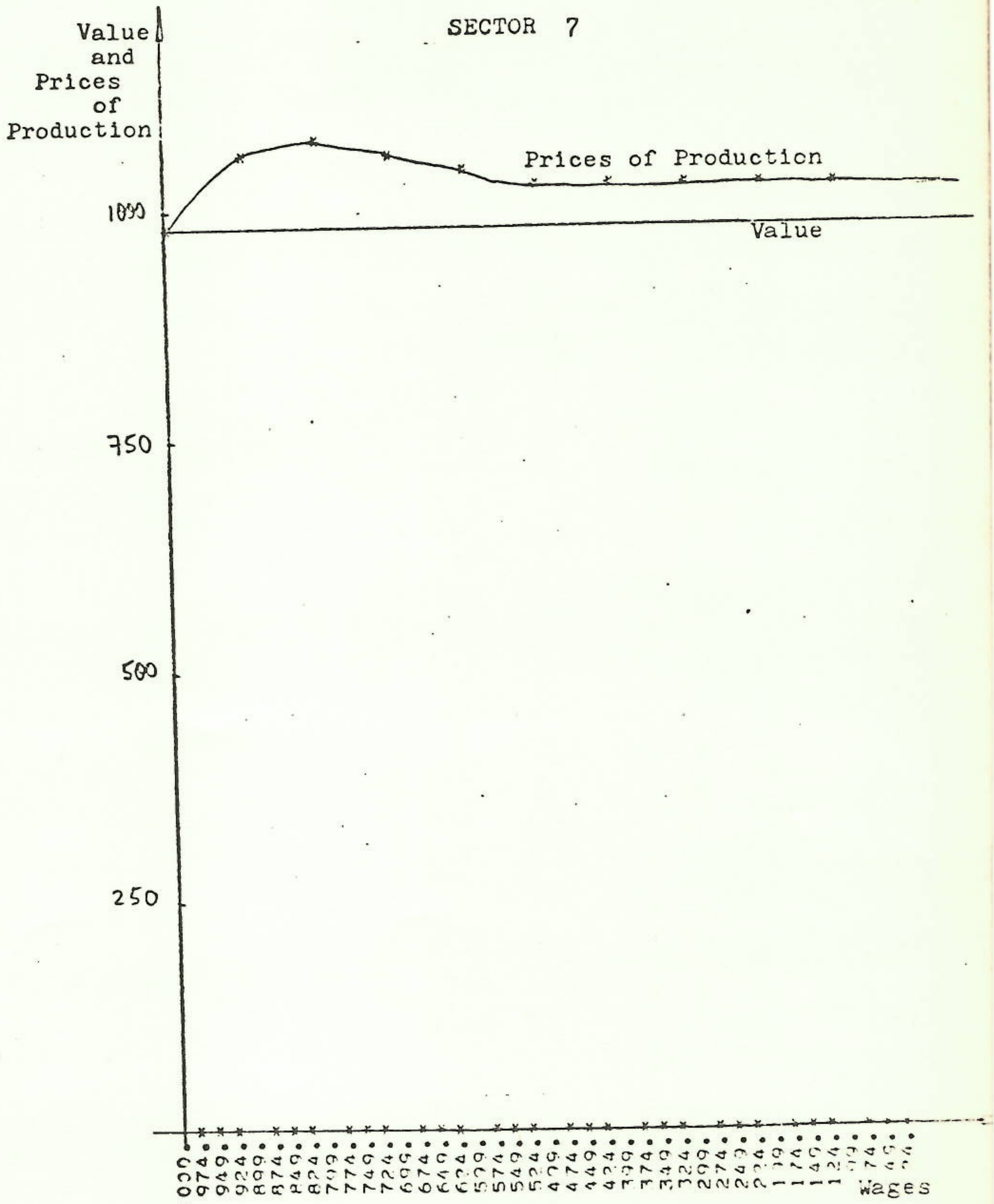

SECTOR 7
Value and Prices of Production
Prices of Production
Value
1000
750
500
250
Wages
1000. 974. 949. 924. 899. 874. 849. 824. 799. 774. 749. 724. 699. 674. 649. 624. 599. 574. 549. 524. 499. 474. 449. 424. 399. 374. 349. 324. 299. 274. 249. 224. 199. 174. 149. 124. 99. 74. 49. 24.

FIGURE 2.8

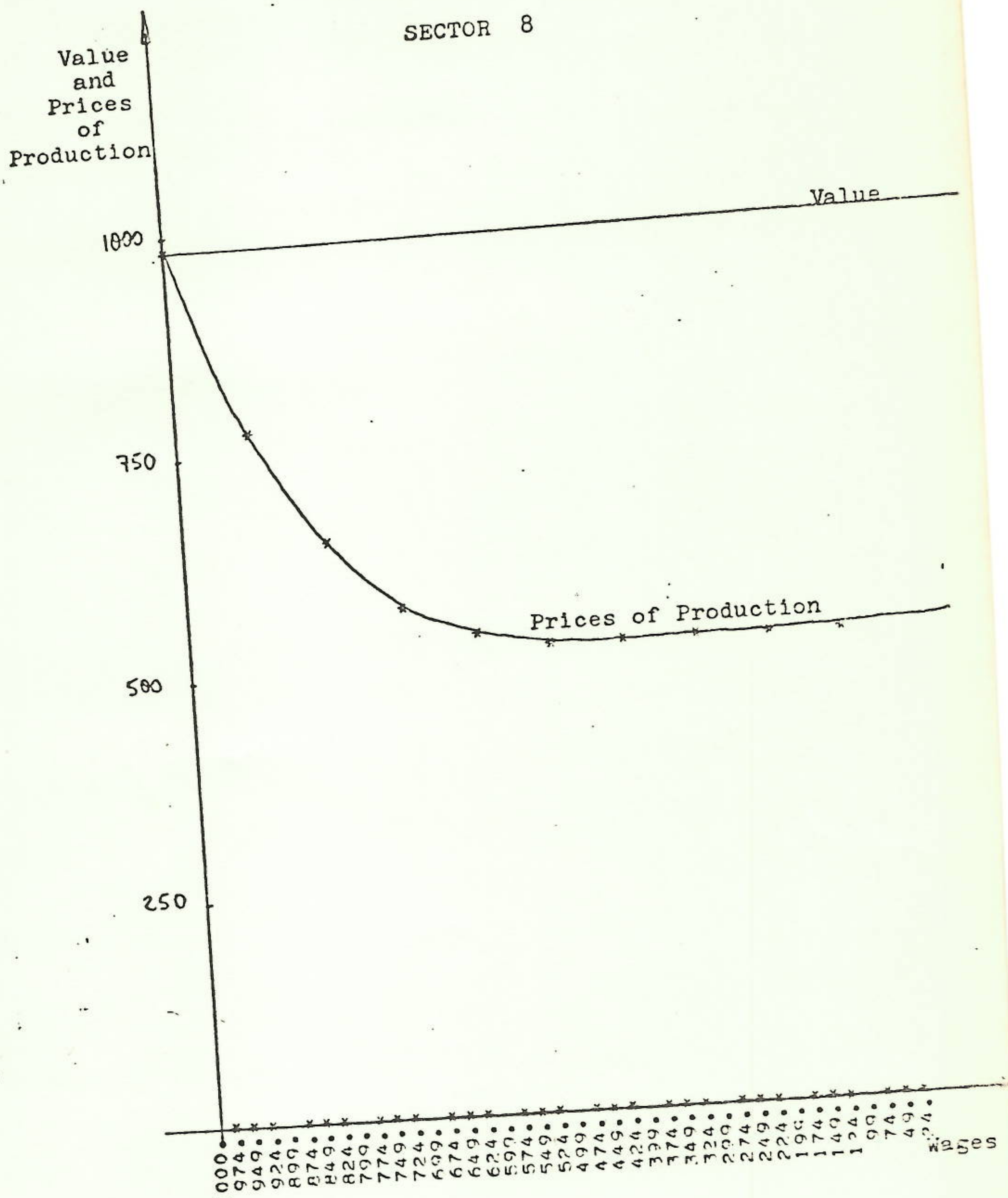

SECTOR 8
Value and Prices of Production
1000
750
500
250
Value
Prices of Production

FIGURE 2.9

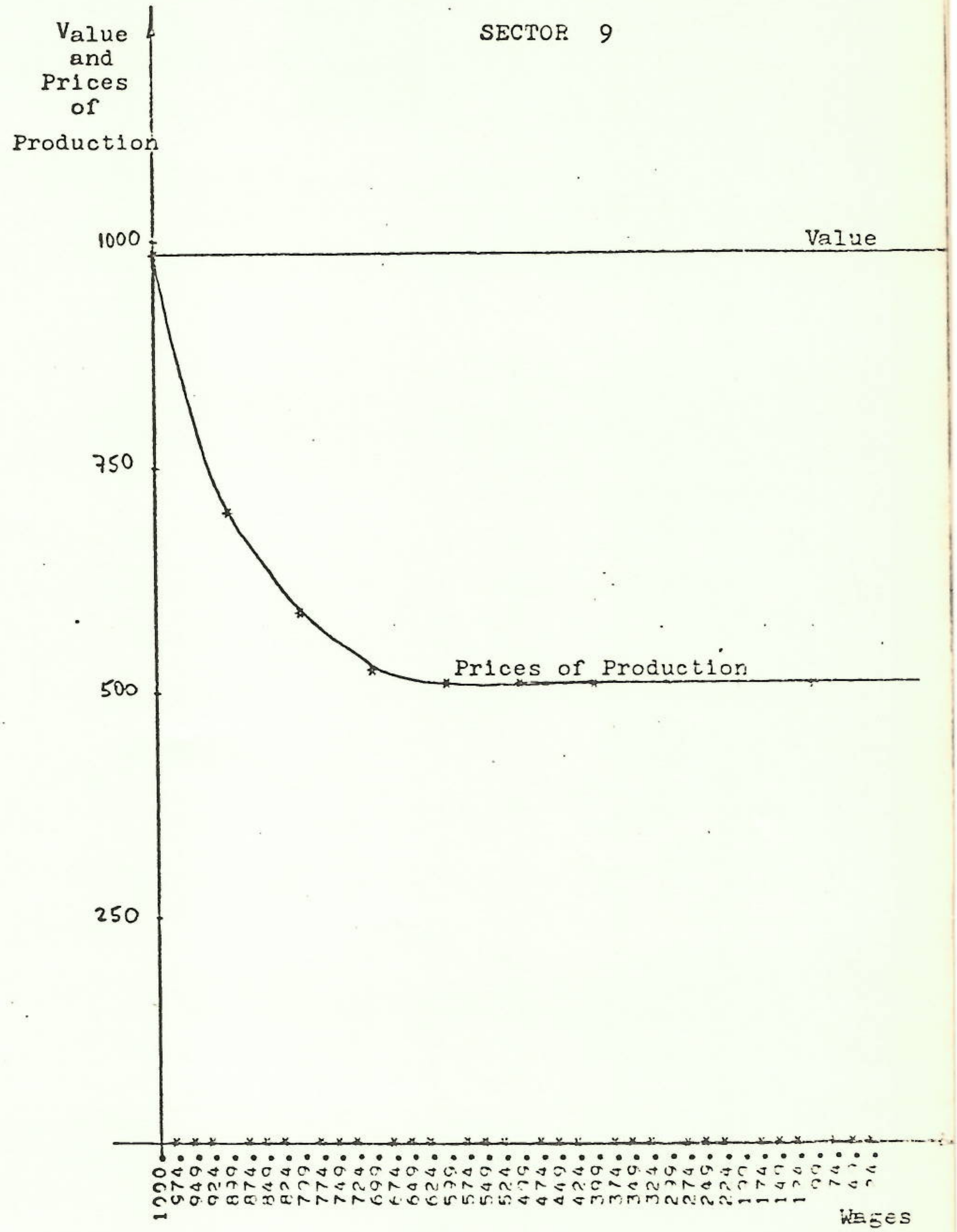

SECTOR 9
Value and Prices of Production
1000
750
500
250
Value
Prices of Production
Wages

FIGURE 2.10

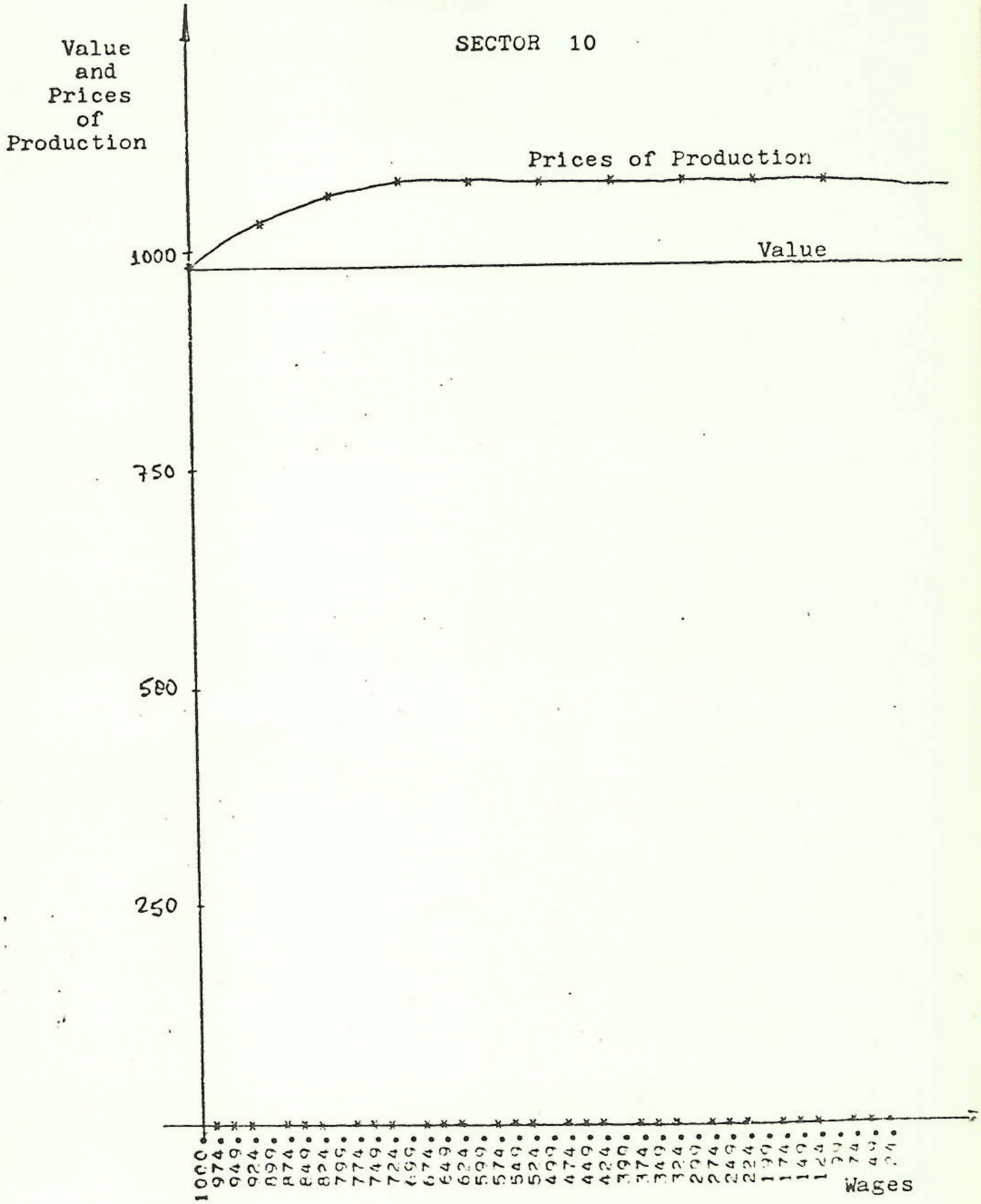

SECTOR 10
Value
and
Prices
of
Production
Prices of Production
Value
1000
750
500
250
Wages

FIGURE 2.11

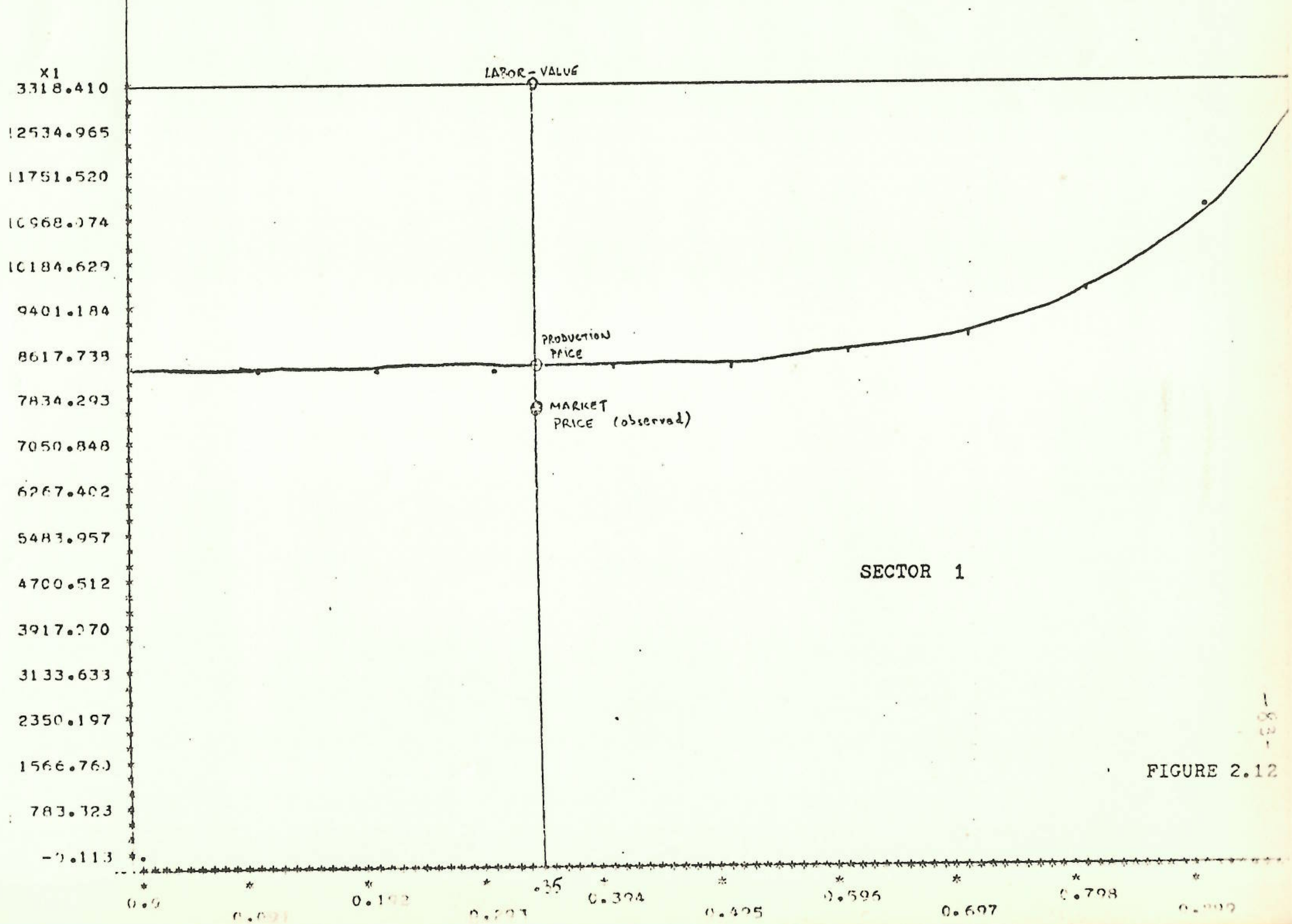

X1
3318.410
12534.965
11751.520
10968.074
10184.629
9401.184
8617.739
7834.293
7050.848
6267.402
5483.957
4700.512
3917.070
3133.633
2350.197
1566.760
783.323
LABOR - VALUE
PRODUCTION PRICE
MARKET PRICE (observed)
.35
0.192
0.304
0.425
0.596
0.697
0.798
SECTOR 1

FIGURE 2.12

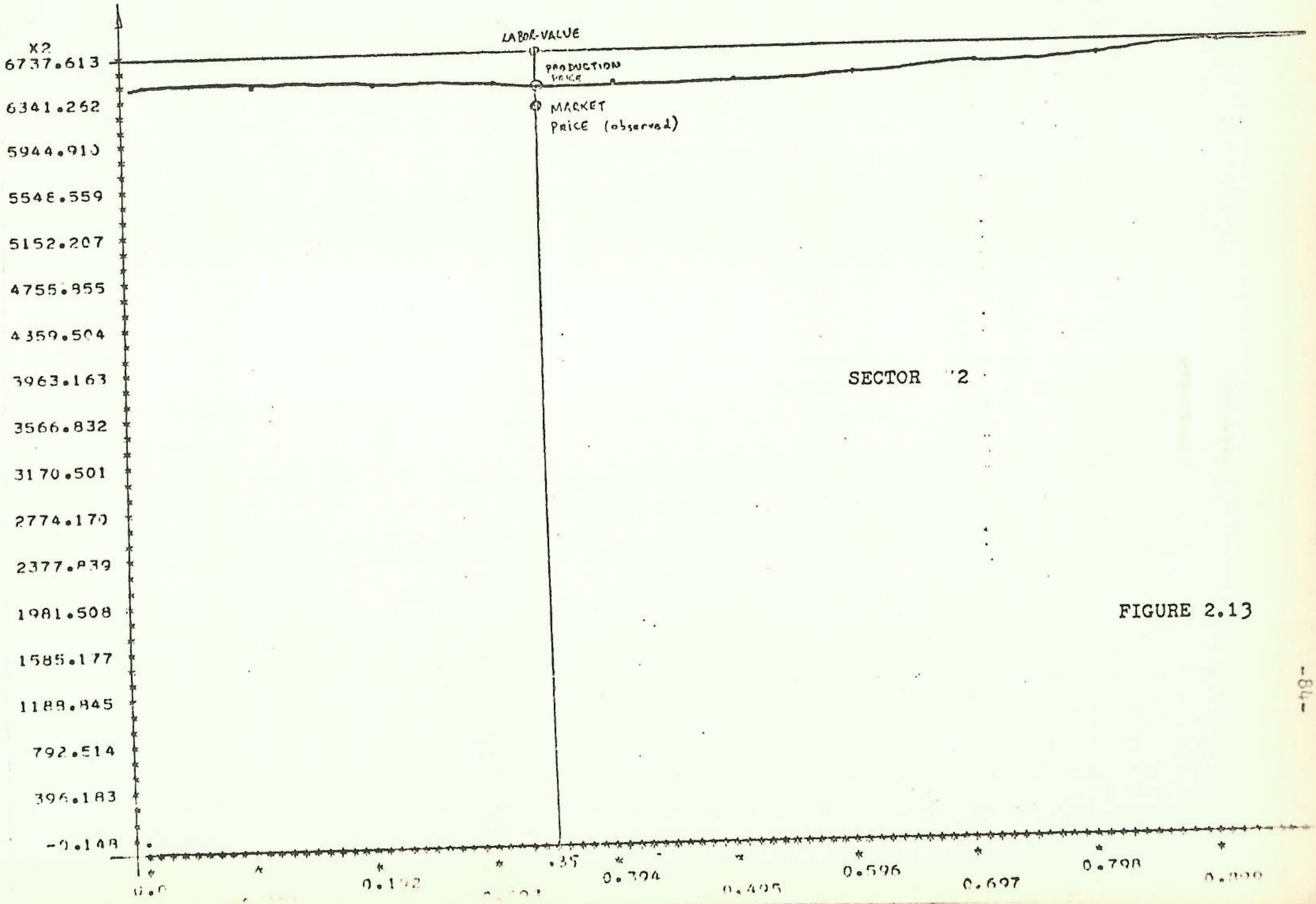

X2
6737.613
6341.262
5944.910
5548.559
5152.207
4755.855
4359.504
3963.163
3566.832
3170.501
2774.170
2377.839
1981.508
1585.177
1188.845
792.514
396.183
-0.148
LABOR-VALUE
PRODUCTION PRICE
MARKET PRICE (observed)
.35
0.0
0.192
0.394
0.405
0.596
0.697
0.798
SECTOR 2

FIGURE 2.13

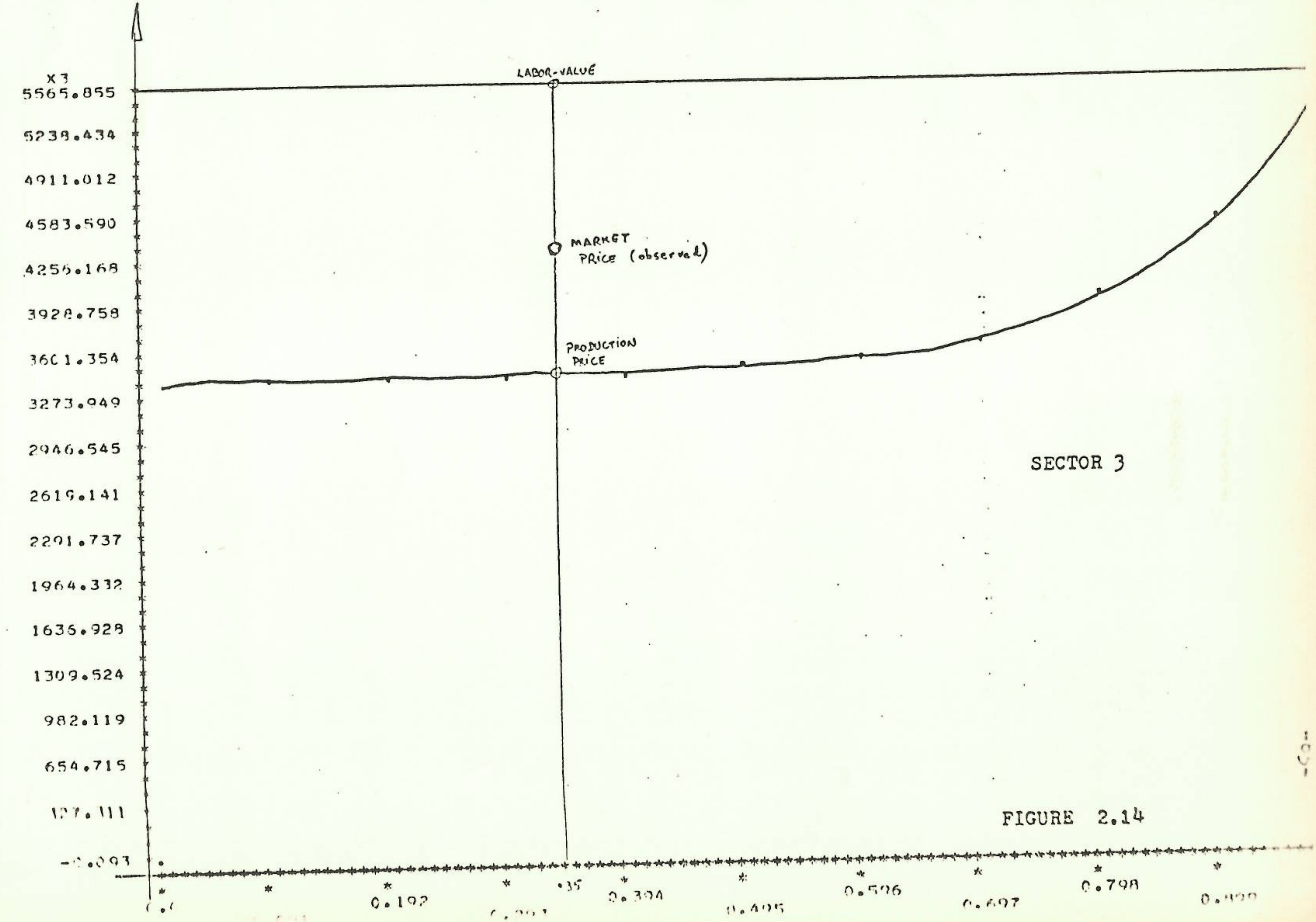

X3
5565.855
5238.434
4911.012
4583.590
4256.168
3928.758
3601.354
3273.949
2946.545
2619.141
2291.737
1964.332
1636.928
1309.524
982.119
654.715
LABOR-VALUE
MARKET PRICE (observed)
PRODUCTION PRICE
SECTOR 3
0.192
.35
0.394
0.596
0.697
0.798

FIGURE 2.14

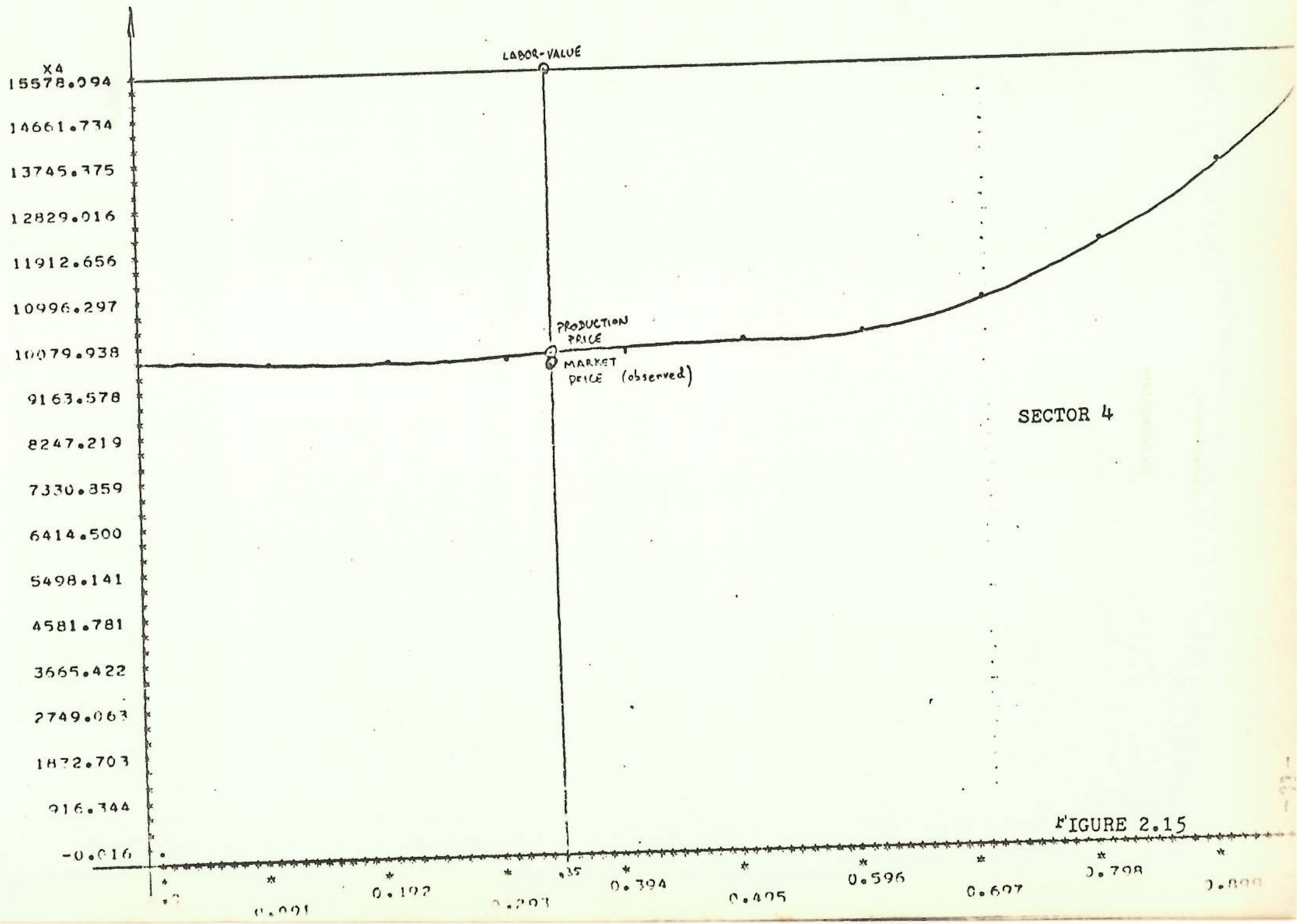

X4
15578.094
14661.734
13745.375
12829.016
11912.656
10996.297
10079.938
9163.578
8247.219
7330.359
6414.500
5498.141
4581.781
3665.422
2749.063
1872.703
916.344
-0.016
LABOR-VALUE
PRODUCTION PRICE
MARKET PRICE (observed)
SECTOR 4
0.192
.35
0.394
0.495
0.596
0.697
0.798

FIGURE 2.15

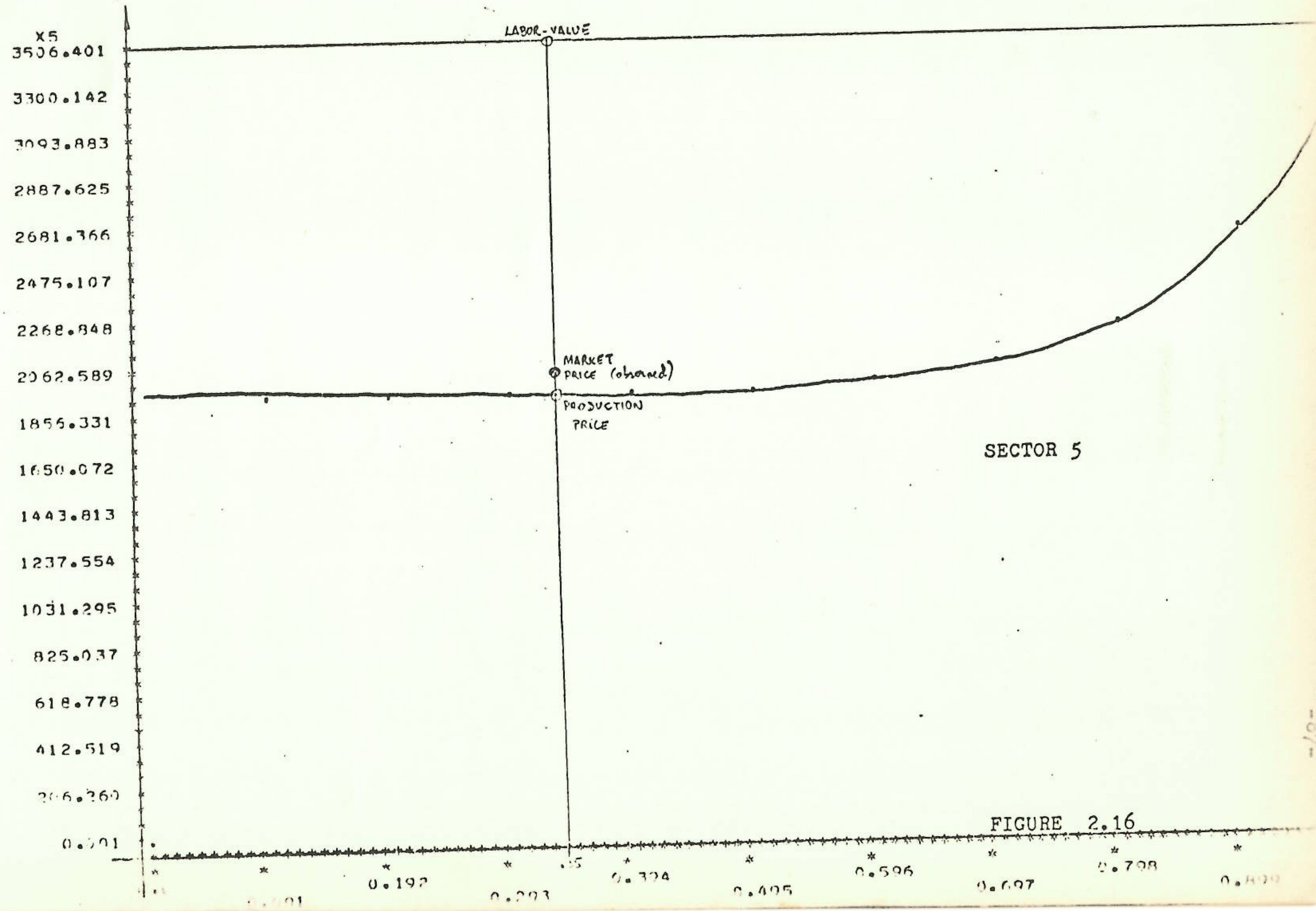

X5
3506.401
3300.142
3093.883
2887.625
2681.366
2475.107
2268.848
2062.589
1856.331
1650.072
1443.813
1237.554
1031.295
825.037
618.778
412.519
206.260
0.001
LABOR-VALUE
MARKET PRICE (observed)
PRODUCTION PRICE
SECTOR 5
.5
0.192
0.293
0.324
0.495
0.596
0.697
0.798

FIGURE 2.16

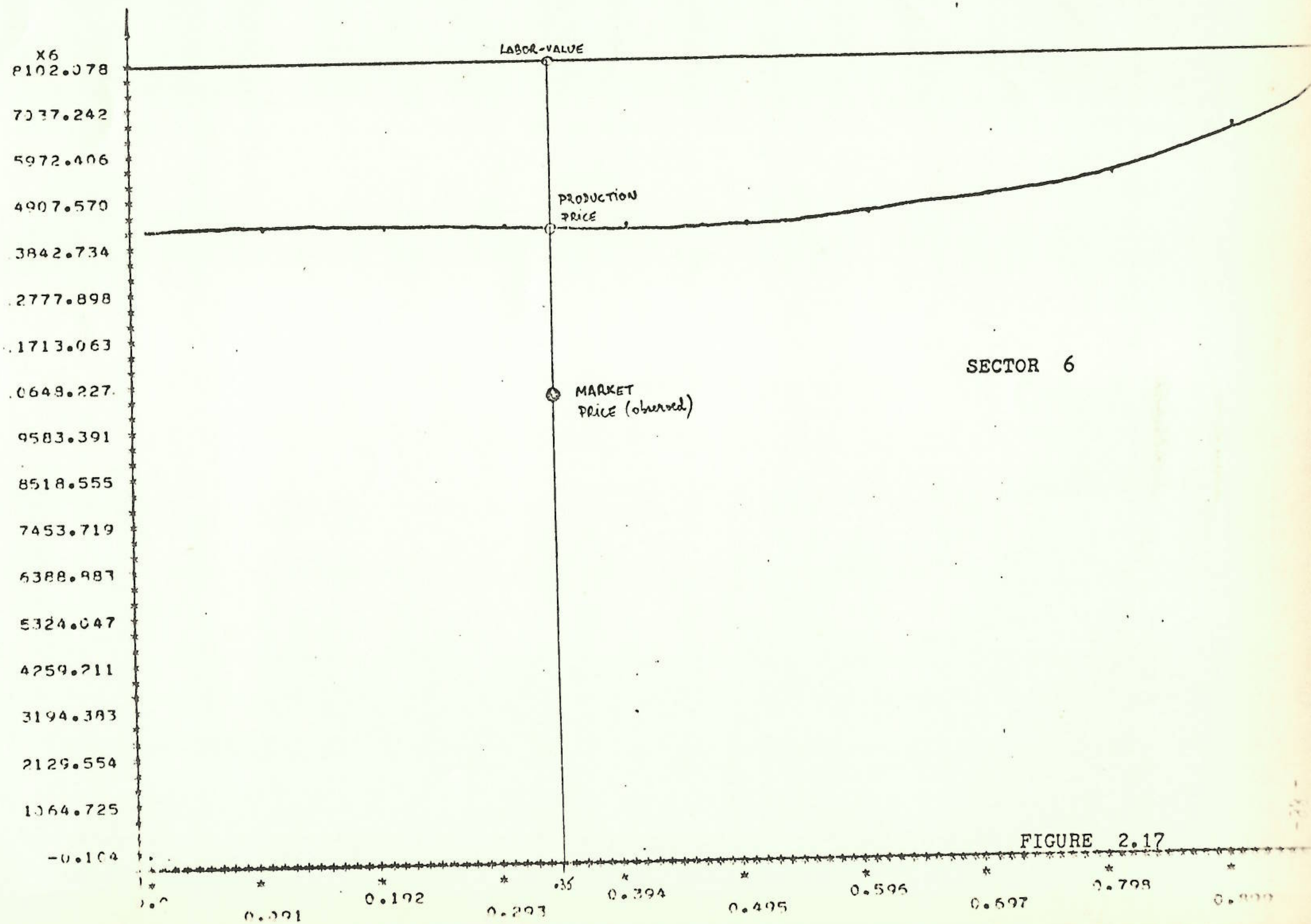

X6
8102.078
7037.242
5972.406
4907.570
3842.734
2777.898
1713.063
0648.227
9583.391
8518.555
7453.719
6388.983
5324.047
4259.211
3194.383
2129.554
1064.725
-0.104
LABOR-VALUE
PRODUCTION PRICE
MARKET PRICE (observed)
SECTOR 6
0.192
.35
0.394
0.495
0.596
0.697

FIGURE 2.17

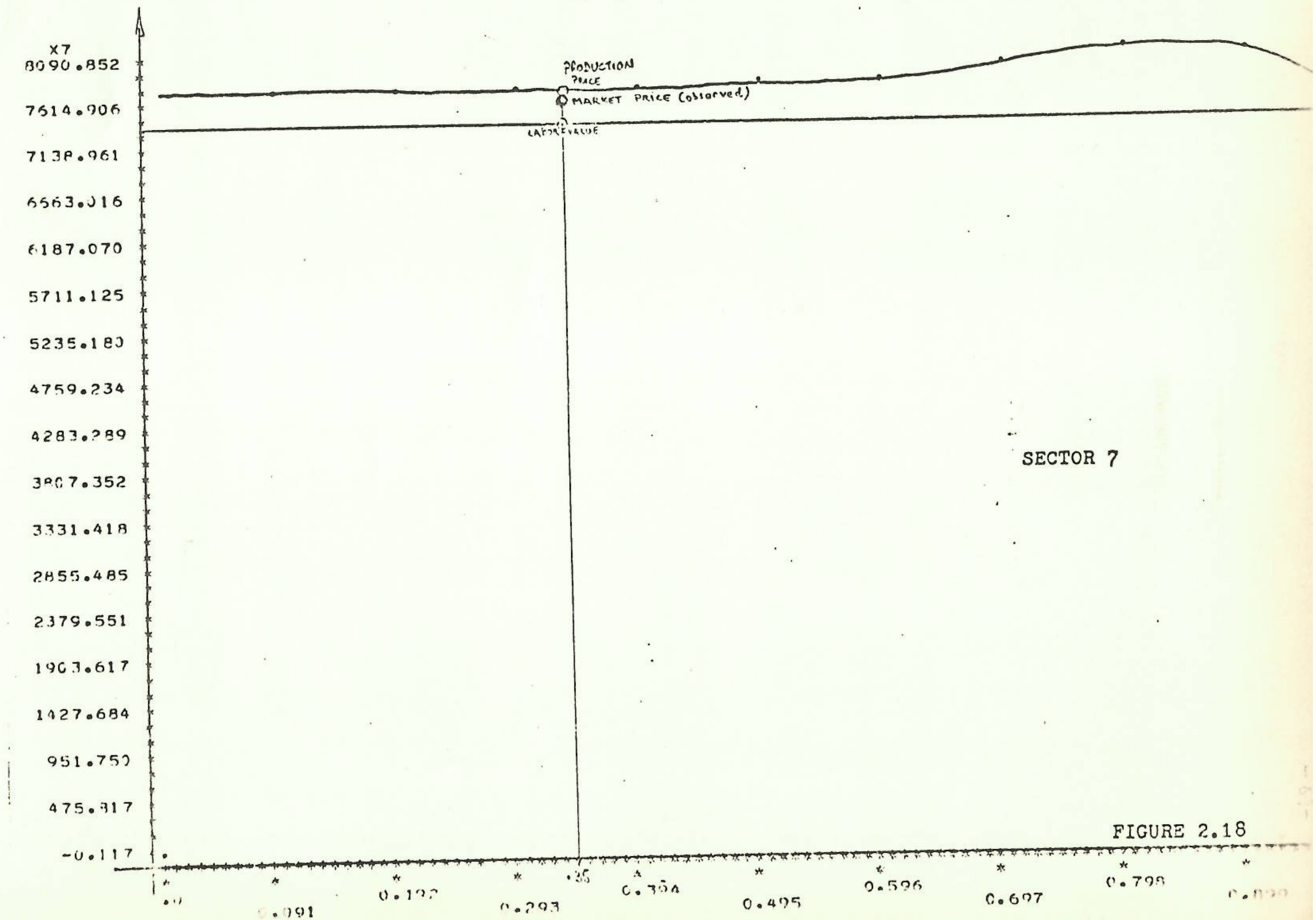

X7
8090.852
7614.906
7138.961
6663.016
6187.070
5711.125
5235.180
4759.234
4283.289
3807.352
3331.418
2855.485
2379.551
1903.617
1427.684
951.750
475.817
-0.117
PRODUCTION PRICE
MARKET PRICE (observed)
LABOR VALUE
SECTOR 7
0.091
0.192
0.293
.35
0.394
0.495
0.596
0.697
0.798

FIGURE 2.18

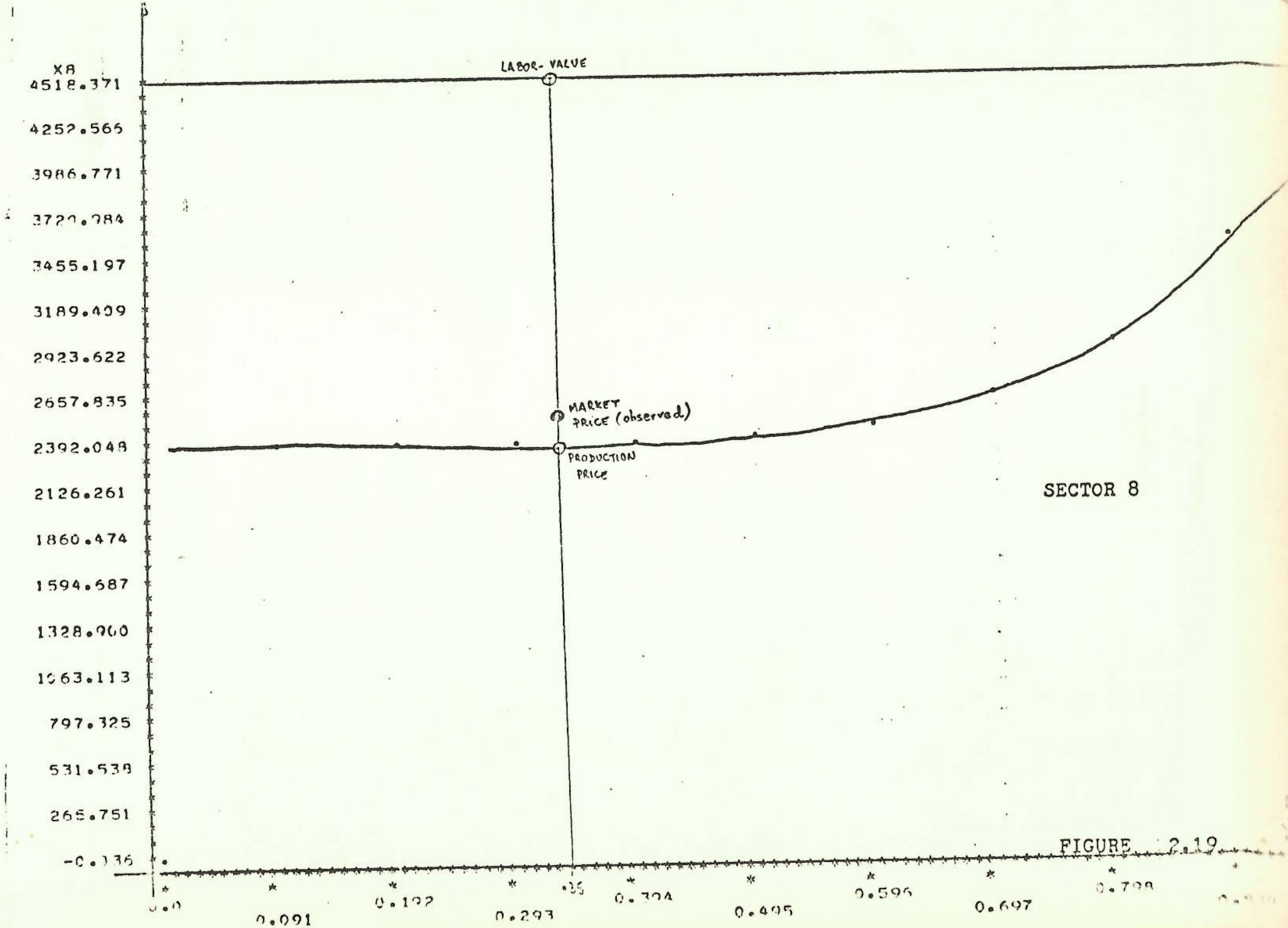

XR
4518.371
4252.565
3986.771
3720.984
3455.197
3189.409
2923.622
2657.835
2392.048
2126.261
1860.474
1594.687
1328.900
1063.113
797.325
531.538
265.751
-0.136
0.0
0.091
0.192
0.293
.35
0.394
0.495
0.596
0.697
0.798
LABOR- VALUE
MARKET PRICE (observed)
PRODUCTION PRICE
SECTOR 8

FIGURE 2.19

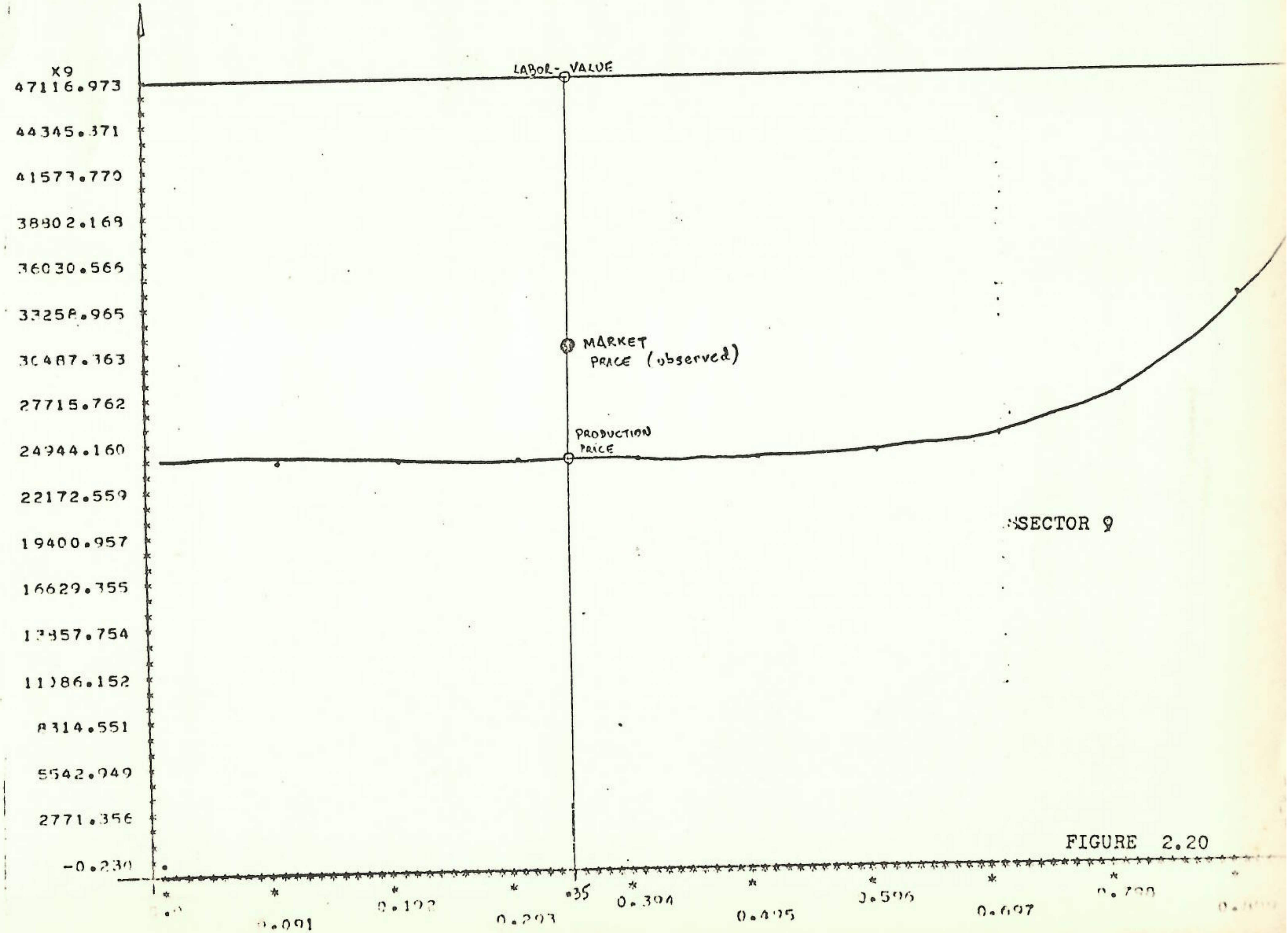
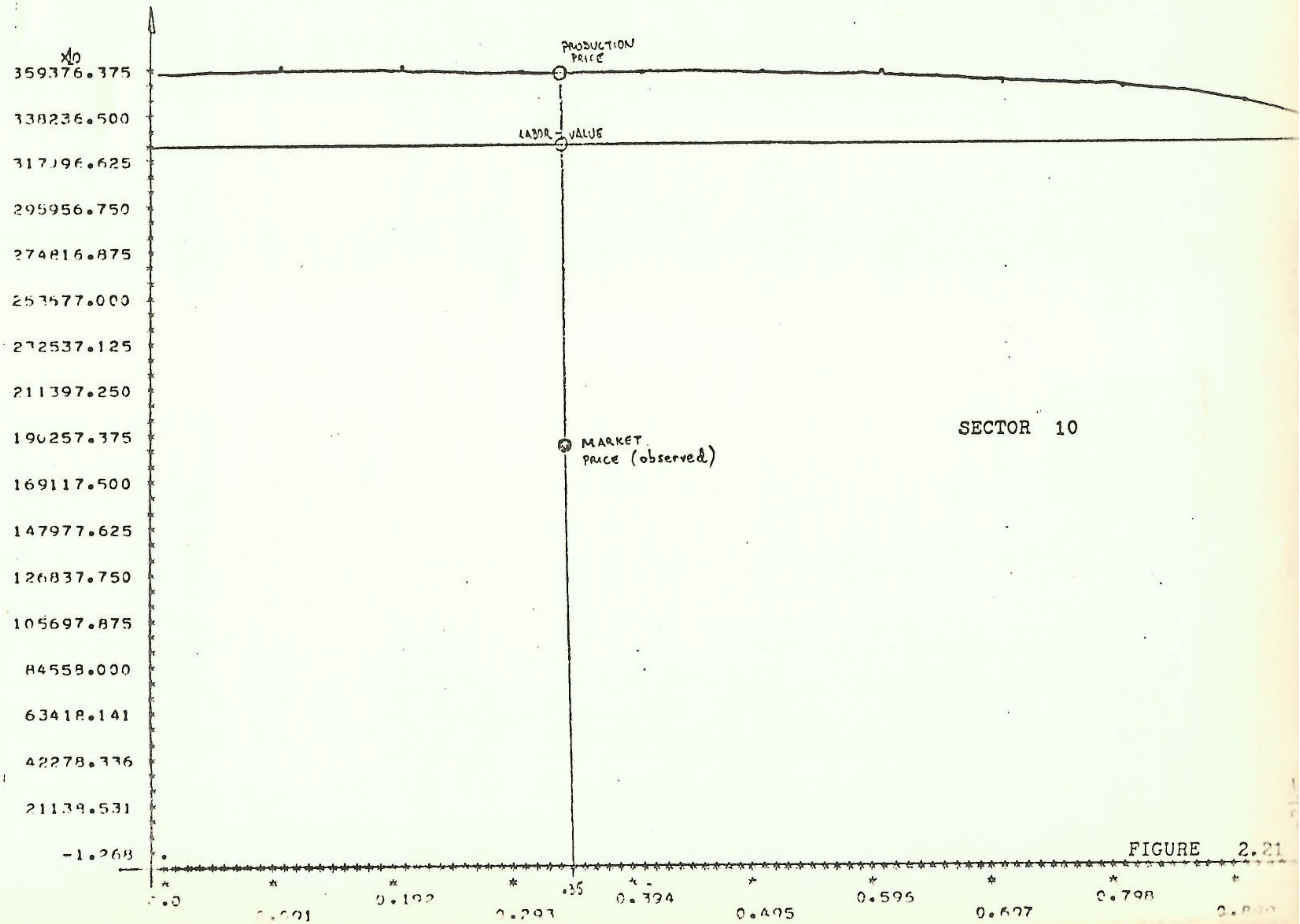

X9
47116.973
44345.371
41573.770
38802.168
36030.566
33258.965
30487.363
27715.762
24944.160
22172.559
19400.957
16629.355
13857.754
11086.152
8314.551
5542.949
2771.356
-0.230
LABOR-VALUE
MARKET PRICE (observed)
PRODUCTION PRICE
SECTOR 9
.35
0.091
0.192
0.293
0.394
0.495
0.596
0.697

FIGURE 2.20



x10
359376.375
338236.500
317096.625
295956.750
274816.875
253677.000
232537.125
211397.250
190257.375
169117.500
147977.625
126837.750
105697.875
84558.000
63418.141
42278.336
21139.531
-1.268
PRODUCTION PRICE
LABOR - VALUE
MARKET PRICE (observed)
.35
0.394
0.495
0.595
0.697
0.798
SECTOR 10

FIGURE 2.21

TABLE 2.4

Sector	Production Prices Compared to Value	Market Prices Compared to Value	Diff % points
1	63% below	56% below	- 7
2	96% below	91% below	- 5
3	62% below	78% below	+ 16
4	63% below	63% below	~ 0
5	56% below	56% below	~ 0
6	79% below	57% below	- 22
7	4% above	3% above	- 1
8	53% below	57% below	+ 4
9	51% below	66% below	+ 15
10	9% above	54% below	- 55

As we can see from table 2.4 and figures 2.12 to 2.21, there is a reasonable coherence between production prices and market prices for sectors 1,2,4,5,7 and 8 . Sectors 3, 6 and 9 fall into a group which could be making "windfall" profits or losses - or could be an evidence of some degree of "bottleneckness". On the other hand, sector 10 shows a striking deviation from equilibrium, being indeed, the only sector where the market prices were observed on the opposite side from where they were expected, to be with relation to its labor-value. Probably part of the reason is that most of those sectors aggregated under the "non-discriminated" label (Services, Urban Transport and Utilities) are stagnant and government owned, and even when not stagnant, but yet government owned (as power plants) have their prices at such a level that the assumption of equalization of the rate of profits do not hold. The same thing could be said for sector 9 (Agriculture) which also could not be expected to be equalizing profits with industry. Still we have the "statistical discrepancies" which the table's authors refer to, and which, incidentally have fallen in our sectors 3, 9 and 10; just some with the biggest divergences. An special word should be dedicated to sector 10 (non-discriminated) : it embraces some 65% of the (homogeneized) labor force in our system, leaving just 35% for the 9 remaining sectors. Indeed, this sector is claimed to be "not completely reliable" even by the authors. Therefore we conclude that, although the work was meritorious, asking for more encouragement, this matrix is not already an extremely reliable tool.

2.7$_a$- Some Dynamic Speculations

Even if our objective in this paper is not to deal with dynamic systems (specially because Sraffa's system still asks for a "dynamization"), it would be interesting to advance some speculation from our data, which could be useful for policy-making.

For the distribution of the net product assumed in section 2.6, we have calculated all the eigen values and eigen vectors of the inverse of the Leontief matrix when we subtracted the payments of wages from the net product (or we include the wages as "cost", summing them to matrix A).

Therefore, assuming that all profits are going to be reinvested (or conversely, that a constant proportion of them is going to be reinvested), and therefore treating the dynamic path thus described by Sraffa's model as a "turnpike path", we have that the vector x(t) of total product is given by:

$$x(t) = (-.3805)\begin{bmatrix} .069 \\ .043 \\ .013 \\ .033 \\ .099 \\ .045 \\ .015 \\ 0 \\ .079 \\ .992 \end{bmatrix} e^{(.0095)t} + (.1250)\begin{bmatrix} .897 \\ .291 \\ .023 \\ .015 \\ .031 \\ .032 \\ .007 \\ 0 \\ .071 \\ .728 \end{bmatrix} e^{(.437)t} + (-.1186)\begin{bmatrix} .775 \\ .735 \\ .019 \\ .147 \\ .119 \\ .184 \\ .067 \\ 0 \\ .062 \\ .408 \end{bmatrix} -$$

$$e^{(.485)t} + (.065)\begin{bmatrix} .209 \\ .618 \\ .536 \\ .005 \\ .734 \\ .107 \\ .047 \\ 0 \\ .095 \\ .060 \end{bmatrix} \quad e^{(.813)t} + (-.3002)\begin{bmatrix} .517 \\ .580 \\ 1.023 \\ .010 \\ .293 \\ .415 \\ .005 \\ 0 \\ .850 \\ .026 \end{bmatrix} \quad e^{(862)t} + (-0905)\begin{bmatrix} 199 \\ 226 \\ 188 \\ 005 \\ 065 \\ 389 \\ 002 \\ 0 \\ 1404 \\ 319 \end{bmatrix}$$

$$e^{(.941)t} + (.0576\begin{bmatrix} .097 \\ .973 \\ .002 \\ .179 \\ .182 \\ .838 \\ 1.118 \\ 0 \\ .748 \\ .300 \end{bmatrix} \quad e^{(.641)t} + (-.1975)\begin{bmatrix} .004 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ .004 \\ .001 \\ .001 \end{bmatrix} \quad e^{t} + (.0602)\begin{bmatrix} 1.466 \\ .060 \\ .108 \\ .594 \\ .103 \\ .887 \\ -.649 \\ 0 \\ .469 \\ .143 \end{bmatrix} \quad e^{(.621)t}$$

Solving this model for t=1, we will see that - given the initial position which we have on figures 2.11 to 2.20 (market prices) as initial conditions for t=0 - the sectors 9, 3, 8 and 5 will show a positive growth (ordered by decrescent magnitude of growth rates) while the sectors 6,2,10,4,7 and 1 will show a negative growth. This means that, maintained the present price structure, sectors 6,2,10,4,7 and 1 tend to be incapable of supplying the needs of this self-regenerative system. In the long-run if nothing changes, the system tends to colapse.

2.8 - Conclusion

Needless to say, our work is too rough to be reliable and to suggest some concrete policies and at the same time, to show in detail the bottlenecks and the alternatives we face. Nevertheless, we understand that this should be a first step for us for a better analysis of the brazilian economy with input-output techniques. If we could show that Sraffa's model, more than being just one model more, is <u>the</u> model which maintains the coherence of the labor theory of value and that it is not just a theoretical piece, but that can be applied to practical analysis, we would be fully satisfied.

The "transformation problem" must come out of the books, papers and ideological quarrels to be converted in something estimable and tested. If it is reality, it can be tasted...

Bibliography

- BÖHM-BAWERK,. -- "Karl Marx and the Close of his System" - P. Sweezy ed. 1949

- BORTKIEWICZ, L.. von -- "Value and Price in the Marxian System" - International Economic Paper nº 22

- BRODY, A. -- "Proportions, Prices and Planning - a Mathematical Restatement of the Labor Theory of Value" - Budapeste, 1970
  -- "Three Types of Price Systems" - Economics of Planning, Vol 5 - nº 3 - 1965 - Norway

- CHENERY, H.; CLARK,P. -- "Interindustry Economics" - John Wiley & Sons Inc. NY - 1959.

- DAMÁSIO, J. -- "Principios Para Uma Abordagem Sraffiana ao Problema da Formação do Valor Internacional" - M.Sc. Dissertation- 1976 - Rio de Janeiro

- ENGELS, F. -- "Engels on Capital" - Supplement to Volume III - New World Paperbacks - 1974 p. 100

- GRAHAM, D. A. -- "A Geometrical Exposition of Input-Output Analysis"- American Economic Review - March 1975 - pp 115-126

- JOHANSEN, L. -- "A Note on: Aggregation in Leontief Matrices and the Labor Theory of Value" - Econometrica - April 1961- Vol 29- pp. 221-2

- KÝN, O.; SEKERKA; HEJL, L. -- "A Model for the Planning of Prices" in Fenistein (ed) "Socialism,. Capitalism and Economic Growth"- 1967

- KONUS, AA. -- "Notes to Article by L. Johansen: Labor Theory of Value and Marginal Utilities" - Economics of Planning - Vol 4 nº 3 1964 - Norway

- MARX, K. -- "O Capital" - Volumes I, II, III - New World Paperbacks 1975

- MEEK, R. -- "Is there an 'Historical Transformation Problem?' A Comment" - The Economic Journal - nº 86 - June 1976 - pp 342-7

- LEÃO, A. et al -- "Matriz de Insumo - Produto do Brasil" - Revista Brasileira de Economia - March 1973 - Rio de Janeiro.

- MORISHIMA, M. -- "Marx's Economics - A Dual Labor Theory of Value and Growth" - Cambridge University Press - 1973

-- "Prices, Interest and Profits in a Dynamic Leontief System"- Econometrica nº 26 (1958) pp 358-80

-- and SETON, F. "Aggregation in Leontief Matrices and the Labor Theory of Value" Econometrica - Vol 29 - April 1961- pp 203 -20

-- and CATEPHORES, G.: "The 'Historical Transformation Problem' : a Reply" - The Economic Journal 86 - June 1976 - pp 348-52

- NAPOLEONI , C. -- "Economic Thought of the Twentieth Century" - John Wiley - NY 1972

- RICARDO, D. -- "The Principles of Political Economy and Taxation"- Everyman's Library - London 1973

-- "Works and Correspondence of David Ricardo" X volumes, 1951 - London. P. Sraffa ed.

- ROBINSON, J. -- "Prelude to a Critique of Economic Theory" Oxford Economic Papers, Vol 13 - 1961

- SAMUELSON, P. -- "The Collected Economic Papers of Paul Samuelson" MIT Press - Cambridge - Mass. - 1966

- SEKERKA, B; KÝN, O; HEJL, L. -- "Price Systems Computable from Input-Output Coefficients" in "Contributions to Input-Output Analysis" Carter & Bródy (ed) - 1970

- SETON, F. -- "The 'Transformation Problem '" - The Review of Economic Studies - Vol XXIV nº 65 - June 1957 - pp 149-60

- SRAFFA, P. -- "Production of Commodities by Means of Commodities- Prelude to a Critique of Economic Theory" - Cambridge University Press - 1960