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Non-Boltzmannian distributions observed in small bodies of the Solar System

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Abstract – Previous works have shown that some features of small bodies of the Solar System (mass, diameters, rotation periods, and apparent magnitudes) are relatively well modeled by distributions originally derived from Tsallis statistical mechanics. Some of the observed distributions are fitted in almost the entire range of the variables, different from other distributions. We revisit these previous studies and update them with recent data. Our results corroborate the previous ones, – that distributions based on the *q*-exponential family are usually found in statistical description of these planetary systems –, and reinforce the conjecture that the underlying mechanism that generates these observed data may have a nonextensive nature.

Keywords: asteroids, meteors, meteorites, non-extensive statistical mechanics.

Riassunto – Precedenti lavori hanno dimostrato che alcune proprietà di piccoli corpi del Sistema Solare (massa, diametri, periodi di rotazione e magnitudo apparenti) sono relativamente ben modellate da distribuzioni originariamente derivate dalla meccanica statistica di Tsallis. Alcune delle distribuzioni osservate sono “fittate” in quasi tutto l’intero intervallo delle variabili, diversamente da altre distribuzioni. In questo lavoro rivediamo questi studi precedenti e li aggiorniamo con i dati recenti. I nostri risultati confermano quelli ottenuti precedentemente, – le distribuzioni basate sulla famiglia q-esponenziale si trovano di solito nella descrizione statistica di questi sistemi planetari – e rafforzano la congettura che il meccanismo sottostante che genera questi dati osservati possa avere una natura non estensiva.

Parole chiave: testo testo testo testo testo testo testo testo testo testo testo testo

1. Introduction

The solar system was formed approximately 4.6 billion years ago (Allègre et al., 1995). In addition to planets, tiny objects as asteroids, comets, and interplanetary dust were formed. These objects are generally called small bodies of the solar system. Due to its small dimensions and spatial arrangement, a significant part of this population must have maintained chemical and physical properties practically unchanged since the formation of the solar system. Therefore, the study of these bodies can provide us with important clues for understanding the formation of our planetary system as well as the thousands of extrasolar systems already known. An important source of knowledge about these objects can be obtained through the analysis of the statistical distribution of physical parameters such as diameters, rotation periods, mass, among others. One of the main drawbacks of this approach, however, is the fact that the observed distributions can be influenced by the observational bias introduced by instrumental limitations (telescope plus detector), data collection and/or analysis methodology. Another peculiarity of these distributions is due to the physical processes related with its origins, that are associated with gravitational long range interactions, fragmentation, among other complex phenomena. There are plenty of examples for which it is observed that exponential or exponential-like distributions do not properly fit the observed data of strongly interacting systems, or the fitting is limited to a narrow range of values, while, alternatively, q-exponential, or q-exponential-like distributions are able to describe them in a wide range of values, and frequently in the entire range of the independent variable. These q-distributions usually give better results than simple power-laws. The exponential (or Gaussian, or other exponential-like) distributions are connected to Boltzmann-Gibbs statistical mechanics, and the q-exponential (or q-Gaussian, or other q-exponential-like) distributions are connected to nonextensive statistical mechanics (Tsallis, 2009). These peculiarities have motivated us to test q-exponential-like distributions to model population data on asteroids, meteors, and meteorites.

The objective of this work is to revisit and update some applications of q-distributions in the study of population data of small bodies of the solar system. Particularly, regarding the paper on rotation periods of asteroids and diameters of the near-Earth Asteroids (NEA), (Betzler & Borges, 2012), we have added the analysis of the rotation periods of the Flora clan and Jupiter Trojans, and the absolute magnitudes of 19,935 NEA contained in the 2019 June version of the Minor Planet Center Orbit Database (MPCORB). We have considered the distribution of apparent magnitudes of Orionids and Geminids meteor showers observed in 2012, 2014, 2016, and 2018 (a larger period than that analyzed in Betzler & Borges, 2015). Finally, we have analyzed the meteorites found in the Sahara desert, complementing a recent analysis of samples found on Brazil, Canada, China, Russia, USA, and Antarctic (Betzler & Borges, 2020).

2- q-*Distributions and its connection with nonextensive statistical mechanics*

The exponential and the Gaussian distributions are associated with the Boltzmann-Gibbs statistical mechanics. More specifically, they can be obtained by a Lagrange optimization procedure of the Boltzmann-Gibbs entropy (see, for instance, Balian, 1991),

$$S = -\,\mathrm{k} \sum_{\mathrm{i}=1}^{\mathrm{W}} p_{\mathrm{i}} \ln p_{\mathrm{i}}, \qquad (1)$$

p_{i} is the probability of the i-th state of the ensemble of W total states, k is the Boltzmann constant) with the constraints of finite momenta: the first moment (the average value) for the exponential distribution, the second moment (the variance) for the Gaussian distribution, and the zero-order moment (the normalization constraint) for all cases. The foundation building block of nonextensive statistical mechanics is the definition of the nonadditive entropy $\mathrm{S_q}$ (Tsallis, 1988),

$$S_{\mathrm{q}} = \mathrm{k}\,\frac{1-\sum_{\mathrm{i}=1}^{\mathrm{W}} p_{\mathrm{i}}^{q}}{q-1}. \qquad (2)$$

It is called nonadditive because the entropy of a system composed by two statistically independent sub-systems A and B, for which the joint probability $p_{\mathrm{i,j}}^{\mathrm{A \cup B}}$ is the product of the individual, separate, probabilities of finding system A in state i and system B in state j, is not the simple sum of entropies of each separate sub-system, but an expression that also depends on the product of the entropies of the sub-systems and the entropic parameter q. Whenever q = 1, all formalism recovers the usual one. A similar maximization of $\mathrm{S_q}$ under appropriate constraints (there are details in this procedure, particularly the differences between the discrete probabilities and the continuous probability densities, the generalization of the moments, and the use of escort probabilities, that won't be considered here because they are secondary for our purposes; the interested reader may see Tsallis (2009) and references therein), yield the q-exponential distribution,

$$p(x) = A_q \exp_q(-\beta_q x) \qquad (3)$$

and the q-Gaussian distribution,

$$p(x) = \boldsymbol{A}_q \exp_q(-\boldsymbol{\beta}_q x^2). \qquad (4)$$

The q-exponential function of a real variable u is defined as

$$\exp_q(u) = [1 + (1-q)u]^{\frac{1}{1-q}} \qquad (5)$$

if $[1 + (1 - q)u] > 0$, and $\exp_q(u) = 0$ otherwise. The parameters A_q and $\boldsymbol{A}_q$ are obtained by the normalization condition. In the present work, these parameters, as well as β_q and $\boldsymbol{\beta}_q$ are considered as fitting parameters, and thus their analytic expressions are not relevant for our purposes. The interested reader may see details in Tsallis (2009) and references therein. Equation (4) is not normalizable for $q \geq 3$, so q-Gaussians must obey the constraint $q < 3$. The ordinary exponential and Gaussian distributions are generalized by equations (3) and (4) respectively, i.e., the exponential and Gaussian distributions are recovered for the case $q = 1$. The q-exponential and the q-Gaussian distributions are asymptotically power-laws ($x >> 1$, $q > 1$). Note the negative argument of the q-exponentials in equations (3) and (4), i.e., variable u in equation (5) is replaced by $u = -\beta_q x$, or $u = -\boldsymbol{\beta}_q x^2$, respectively. Equation (3) can be further generalized by substituting the independent variable x by a power, x^α ($x \geq 0$), and the resulting expression is the stretched q-exponential. The stretching parameter $0 < \alpha < 1$ stretches the distribution, $\alpha > 1$ compresses it. The particular value $\alpha = 1$ is the q-exponential distribution, ($\alpha = 1$, $q = 1$) recovers, of course, the exponential distribution. It is possible to extend the validity of the stretched q-exponential to the entire real domain, instead of being limited to $x \geq 0$, by replacing the independent variable x in equation (3) by $|x|^\alpha$. The particular case $\alpha = 2$ recovers the q-Gaussian distribution (the absolute value in the replacement is obviously not necessary for this particular case).

3. *The data*

We have analyzed the distribution of rotation periods and diameters of asteroids, apparent magnitudes of meteors and mass of meteorites. These data are available in public databases. The rotation periods were obtained from the lightcurve derived data available in the Planetary Database System (PDS). In this study, we analyzed versions 7 (V7, 2005), with 1971 periods, and V11 (2010) with 4310 periods. The periods of each version of these files are classified according to the quality code defined by Harris and Young (1983). Only periods with $Rel \geq 2$ (*Rel* for reliability) were considered for analysis, which means that these data are 20% outlier, resulting in 1621 entries in V7 and 3567 asteroids for V11.

The diameters of NEA were calculated from their absolute H magnitudes available in the MPCORB- "Minor Planet Center Orbit Database", using the equation (Bowell *et al.*, 1989):

$$D = 1329\,\frac{10^{-H/5}}{\sqrt{p_v}}. \tag{6}$$

Three versions of the compilation of absolute magnitudes of NEAs were used corresponding to the 2001 October (similar to the version used by Stuart (2001)), 2010 October and 2019 June. The albedo of the population of NEAs was assumed to be $p_v = 0.14 \pm 0.02$. This value was estimated by Stuart and Binzel (2004) taking into account the wide variety of taxonomic types present among these asteroids (Binzel et al., 2004).

The cumulative distributions of meteor showers were obtained from meteorological counts, by visual magnitude range, available by the International Meteor Organization (IMO) (VMDB - Visual Meteor Database, http://www.imo.net/data/visual/). In this analysis, meteors with apparent magnitudes between -6 and 6 spaced with one magnitude interval, were considered. Differently from the one applied by Betzler & Borges (2015), here we do not exclude data from observers who reported that the local sky had a limit magnitude $lm \geq 5$ 5. With this, we want to verify the impact that observational bias would have on the results obtained in our previous study. For the current work, we consider the meteor showers of Geminids (GEM) and Orionids (ORI). The choice of these showers is associated with the variety of physical and dynamic characteristics of each meteoroid stream. We have analyzed meteors observed from the year 2012 to 2018 spaced with consecutive two years interval to verify the occurrence of temporal variation in the q-distribution parameters that models the observed distribution of magnitudes of the meteors. The observed distribution of meteorite masses was extracted from the "Meteoritical Bulletin Database" (http://www.lpi.usra.edu/meteor/metbull.php) on February 24, 2020. We only consider meteorites with names approved by "Meteorite Nomenclature Committee"of the "Meteoritical Society ", to ensure the accuracy of the physical and mineralogical parameters associated with each meteorite in the sample. In this part of the study, we have performed the analysis of the mass distribution of meteorites found in the Sahara desert (942 meteorites) in order to establish the action of the gathering bias, as suggested by Betzler & Borges (2020).

All analyzed parameters were organized to form a decreasing cumulative distribution. This observed cumulative distribution was modeled with the q-distributions (Eq. 3 or 4). Their parameters were obtained by an optimization procedure using the non-linear generalized reduced gradient for the line search, and the optimum values of distribution parameters are those that minimize the Pearson chi-square coefficient. This test was also used to assess the goodness of the fit of the q-distributions in the observational data. The similarity between different observed distributions of magnitudes of meteors in section 4.3 was carried out using the Kruskal-Wallis H-test (Kruskal & Wallis 1952).

4. *Processing and analysis*

4.1. *Rotation periods of asteroids*

It was shown by Betzler & Borges (2012) that the rotation periods p of asteroids can be modeled by a q-Gaussian (Fig. 1). This q-distribution has a better goodness of fit if compared to distributions of exponential nature such as the Maxwellian distribution (Harris & Burns, 1979), in addition to being valid for the entire domain of the data without the need of separately consider the diameter ranges. We infer a temporal variation of the derived 2005 and 2010 values of q and $\mathbf{B_q}$ parameters of the q-Gaussian. It is highly unlikely that the nature of the entire distribution has been modified in five years – the time lapse between the V7 and V11 datasets. The most reasonable hypothesis for the difference is the action of observational bias associated with the determination of the rotation periods. Warner and Harris (2010) demonstrated that the rotation periods estimated with greater accuracy are associated with objects with p ≤ 8h and amplitudes A ≥ 0.3 mag. Thus, it is concluded that the difference between the transition points may be caused by observational bias. Additionally, sample V11 was separated in terms of the taxonomic complexes C, S and X, as shown in Fig. 3 of Betzler & Borges (2012). Each complex have 503, 663 and 321 asteroids, respectively, and are adequately described by q-Gaussians with confidence levels of 95% or higher.

The entropic parameter q for each taxonomic complex, the Jupiter Trojans and the members of the Flora clan asteroids are proportional to the size N of the samples (Fig. 2 and 3).

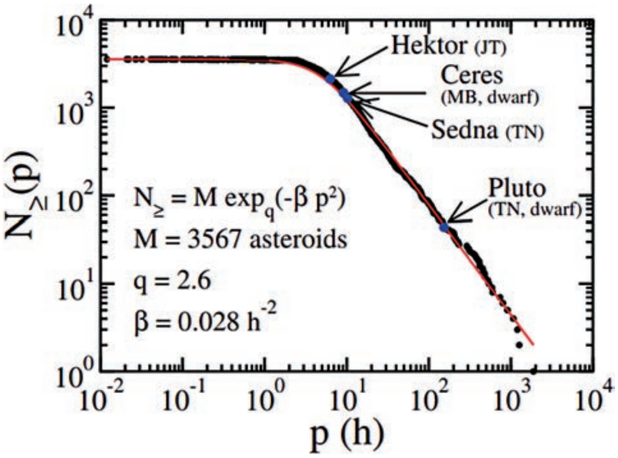

Fig. 1. Fitting of a q-Gaussian (solid red line) for the distribution of rotation periods of asteroids (version v11 of the lightcurve derived data). Some asteroids of different dynamic types are identified: the Jupiter Trojan (JT) Hektor, and the Transnetunian Objects (TN) Sedna and Pluto.

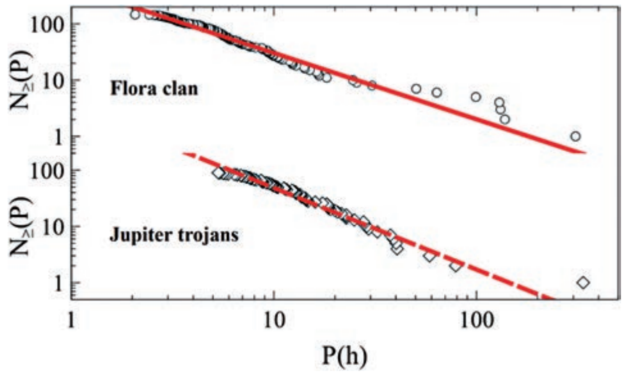

Fig. 2. Cumulative decreasing distributions of the rotation periods of 146 asteroids from the Flora clan (circles) and 90 Jupiter Trojans asteroids (diamonds). The adjustment of a power law of the type $N_{\geq}(P) = aP^{b}$ is represented by the red lines: a = 455 ±6, b = -1.178 ±0.007 (continuous), and a = (1.37 ±0.03) × 10^3, b = -1.452±0.008 (dashed). Considering b = 1/1-q , the values of b imply q = 1.849± 0.005 and 1.689±0.004 for the asteroids of the Flora clan and the Jupiter Trojans. Data obtained from Kryszczyńska *et al.* (2012) and Mottola *et al.* (2011).

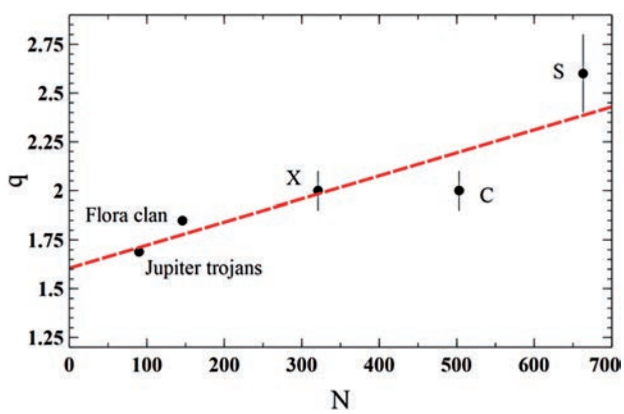

Fig. 3. Relationship between the parameter q and the sample size N. The linear adjustment (dashed line) has angular coefficients (1.2±0.2) × 10^{-3} and linear 1.60± 0.08.

This hypothesis was accepted with a confidence level of 95% using Pearson's chi square test. For small samples, $q \approx 1.60$ indicating that usual exponential distributions are not appropriate to describe the observed distributions regardless of the sample size.

4.2. *Diameters of asteroids*

The adjustment of a q-exponential to the diameters of NEAs for the years 2001, 2010 and 2019 is satisfactorily good for the entire sample domain with a level of confidence greater than 95% (Fig. 4). The parameters of the q-exponentials are $q = 1.3$ and $\beta_q = 1.5$ km^{-1} (2001), $q = 1.3$ and $\beta_q = 3.0$ km^{-1} (2010) and $q = 1.3$ and $\beta_q = 4.5$ km^{-1} (2019). Ordinary exponentials ($q = 1$) are inadequate in representing the entire sample, as shown in Figure 2 of Betzler & Borges (2012). Since the value of q is the same for all samples, we can speculate that this parameter reflects some real physical process. The value of $q = 1.3$ (different from the unit) is an indication that not only collisional (short range like) processes are involved in the formation of these objects. Other processes may be present, such as the YORP effect (Yaryovsky-O'Keefe-Radzievskii-Paddack, Rubincam, 2000), which may lead to a reduction in the rotation period up to the breaking limit.

From Figure 4, it can be seen that the three distributions are practically identical in the region of the power law, but differ at the point of transition to the quasi-plane region. This point of transition is obtained from the equation (compare with Fig. 1 of Betzler & Borges 2012):

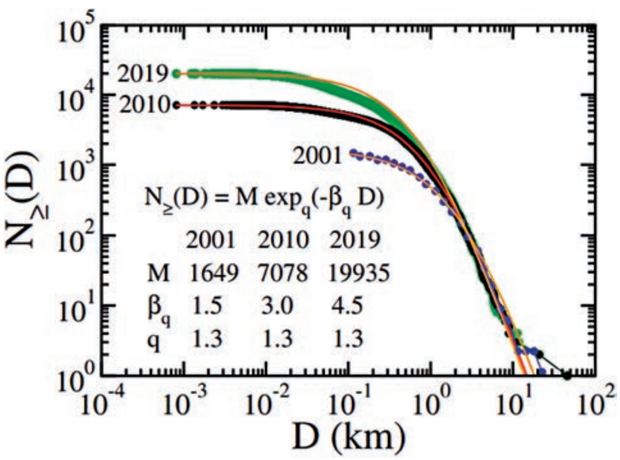

Fig. 4. Cumulative decreasing distribution of the diameters of NEAs known in 2001, 2010 and 2019. Fittings with a q-exponential distribution are represented by solid lines.

$$X^* = \frac{1}{[(1-q)\beta_q]^{\frac{1}{u}}}, \tag{7}$$

in which the parameter μ defines the type of q-distribution. Specifically, μ = 1 is a q-exponential and μ = 2 is a q-Gaussian.

The transition point corresponds to the lower limit of the diameter at which a sample could be considered as complete (see Betzler & Borges, 2012). In 2001, this value was estimated to be D* = 2.20 km, and 1.16 km in 2010. Considering the values of q and β_q obtained in 2019 (Fig. 4), D* = 0.74 km. A simple linear regression of the relationship between the year and D* suggests that all NEAs with diameters greater than or equal to 1 km must have been discovered by the second half of 2013.

According to the q-exponential distribution, adjusted with the parameters q and β_q of 2019, the number of asteroids with diameters equal to or greater than 1 km is 1155 objects. This value is slightly higher than that suggested by Betzler & Borges (2012) (994 asteroids), but still reasonable when compared to more current estimates (≈ 1000 asteroids, Morbidelli *et al*. 2020).

4.3. *Distribution of magnitudes/mass of meteors*

The relation between the magnitude m and meteor mass M can be expressed by an exponential function (Jacchia, Verniani & Briggs 1965, see also Sotolongo-Costa *et al*. 2008):

$$M = M_0 e^{-\gamma m}, \tag{8}$$

$M_0 = M(m = 0)$ and γ is a constant that relates the apparent magnitude and the mass of the meteoroid. The probability densities for the magnitude distribution and the mass distributions must obey

$$p(m)\mathrm{d}m = p(M)\mathrm{d}M, \tag{9}$$

integrating between m and $-\infty$, we obtain the inverse cumulative distribution

$$N_{\geq}(m) = N_t[\exp_q(-\beta_m e^{-\gamma m})]^{2-q}, \tag{10}$$

with and $\beta_m = \beta\, m_0$ and β_m is a is a parameter to be adjusted. A power of a q-exponential can be easily rewritten as another q-exponential with a different q' index:

$$N_{\geq}(m) = N_t \exp_{q'}(-\beta'_m e^{-\gamma m}), \tag{11}$$

with $\frac{1}{1-q'} = \frac{2-q}{q-1}$ and β'_m=(2-q)β_m.

The cumulative inverse absolute and relative distributions of the apparent magnitude/mass of the meteors associated with the 10 meteor showers can be modeled by Equation 11, with a confidence level greater than 95% (see Betzler & Borges, 2015). The observed relative distributions are time-independent and uncorrelated with the type of meteoroid stream progenitor bodies (asteroids or comets). This result suggests that there may be variations in the numerical quantity of meteors in a meteor shower, as in the case of an outburst, but the relative distribution of meteors by magnitude range remains constant. The mean parameter $q' = 2.3 \pm 0.3$ suggests that the distributions were generated under the same physical/observational condition.

Approximately 2.4% of meteors in a shower can be considered as telescopic. Given their low brightness, these meteors can be properly observed with the support of optical instruments such as wide-angle telescopes or binoculars.

The absolute and relative distributions of GEM and ORI showers are well adjusted by equation 19, with a confidence level greater than 95%. The relative distributions does not have temporal variation and there is a similarity between these meteors showers, as suggested by the H-test.

Combining all the adjustments of the GEM and ORI showers, we obtained an average $q' = 2.1 \pm 0.4$ (1 σ),

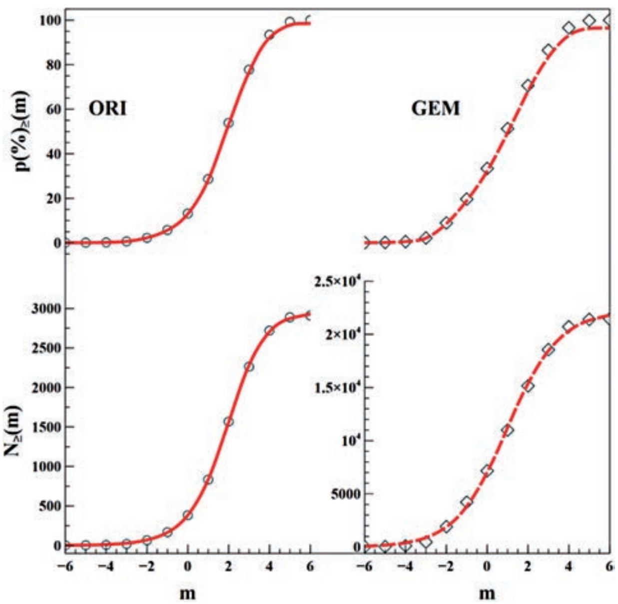

Fig. 5. Absolute ($N_{\geq}(m)$) and relative percentage (100% x $N_{\geq}(m)/N_t$) distributions of the apparent magnitudes m of meteor showers GEM (diamonds) and ORI (circles) observed in 2012. The red lines represent the adjustments of the equation 11, with N_t=22,243, β_q=2.07, γ = 0.77 and q' = 1.94 (dashed) and N_t=2,953, β_q=11.12, γ = 1.94, and q' = 2.34 (continuous).

which is consistent with that found by Betzler & Borges (2015). The distribution of this parameter follows a Gaussian distribution inferred by the Shapiro-Wilk test, with a confidence level greater than 95%. The random variation of this parameter may be the result of observational processes (variation of the number of observers with different environmental conditions), but it reflects that all meteor showers were generated under similar conditions.

The percentage of telescopic meteors ($m > 6$) is 3 ± 1%. Despite presenting a higher absolute value, the current estimate is similar to that proposed in our previous work. Probably the exclusion of observers by the limit magnitude range is of secondary importance in the study of the apparent magnitude distributions of meteors when the number of these observers is reduced. For example, we found that 25% of observers made their magnitude estimates at sites with a limit magnitude less than 5.5 in the 2012 observational session of ORI meteor shower.

4.4. *Meteorite mass distribution*

The mass distribution of meteorites has been recently considered (Betzler & Borges, 2020). The sample consisted of specimens presumably from the same meteoroid (Sutter's Mill, Whitecourte and Kosice), meteorites of different mineralogical types and fall/finding sites on the Earth's surface. The observed mass distributions are well adjusted by q-exponentials or their stretched version, the stretched q-exponential. This distribution is more suitable for modeling samples that are apparently incomplete in the domain of large masses, a fact observed in the Košice meteorite specimens, in part of the analyzed mineralogical types (pallasites, mesosiderites and iron IAB) and in all national samples (Brazil, Canada, China, Russia, and USA). Here we extend the study by analyzing the distribution of meteorites found in the Sahara desert. Similarly to other desert regions on our planet, the Sahara has become a propitious place for meteorite collection due to the absence of vegetation and the color difference between meteorites and the surrounding material of the environment (Bischoff & Geiger, 1995). Initially, the 942 meteorites were modeled by a stretched q-exponential. We have found a stretching parameter very close to the unit, which makes it different from the average value found for the national samples (0.53 ± 0.08), and implying that a stretched q-exponential is more adequate to model the observed distribution. In fact, the fit with a q-

exponential is slightly higher or obtained by its stretched version (Fig. 6).

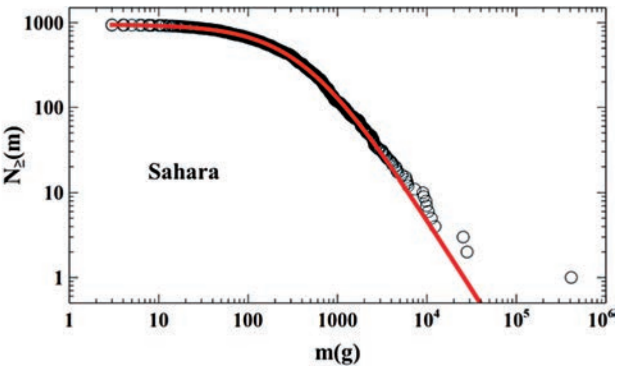

Fig. 6. Absolute distributions $N_{\geq}(m)$ of the mass of 942 meteorites found in the Sahara desert until February 24, 2020. The solid red line represents the fitting of a q-exponential with $N_t = 954$, $\beta_q = 3.9 \times 10^{-3}$ g^{-1}, and $q = 1.60$.

The β_q value is approximately the inverse of the median mass, and thus we can conclude that the Saharan sample has a mass distribution close to that presented by the Russian collection in June 2019 ($\beta_q = 4.20 \times 10^{-3}$g^{-1}). An analysis of the mineralogical composition reveals that 91% of the meteorites in the Saharan collection can be classified as ordinary chondrites. The Russian sample shows 57% of meteorites of the same type. The share of these rocky meteorites is extremely high compared to the almost 50% of siderites found in the meteorite collections in Brazil and Canada. This implies that the mineralogical composition of the sample does not have a direct correlation with the type of q-distribution that is more adequate to model the dataset.

5. Conclusions

We have modeled population data of asteroids, meteors, and meteorites using q-exponential-like distributions. These distributions derived from Tsallis' statistical mechanics are observed to be valid alternatives in the statistical study of population data of small bodies in the solar system. The adjustments of these distributions show high adherence to observational data when compared to power laws traditionally used in this field, with the great advantage of modeling the entire spectrum of variation of the analyzed parameter. This feature makes it possible to assess the influence of observational bias in the data by comparing samples obtained at different times or observational conditions.

We hope similar studies can be applied to other solar system datasets, and the formulation of physical models able to explain the generation of observed distributions are welcome.

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