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MODEL PREDICTIVE CONTROL STRATEGIES FOR TRACKING DYNAMIC TARGET REFERENCES

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Model Predictive Control Strategies for Tracking Dynamic Target References

Trabalho apresentado ao Programa de Pós-Graduação em Engenharia Elétrica da Universidade Federal da Bahia como requisito parcial para obtenção do grau de Doutorado em Engenharia Elétrica.

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Resumo

Controladores preditivos baseados em modelo (MPC) são amplamente utilizados para controlar sistemas com características comuns a muitas aplicações industriais e acadêmicas. A lei de controle de um MPC é formulada pela minimização de uma função objetivo que considera o erro entre saídas previstas estimadas e referências futuras ao longo de um horizonte de tempo especificado. Essas saídas previstas podem ser definidas usando um modelo nominal paramétrico ou modelo baseado em dados. Tipicamente, nas abordagens lineares, o processo de minimização gera uma sequência ótima de entradas através do princípio de horizonte deslizante, incorporando naturalmente as restrições do sistema em sua solução. Um problema conhecido com MPC com restrições é a perda de factibilidade quando há mudança de referências. Isso acontece devido ao horizonte finito escolhido e às restrições impostas pelo controlador que podem levar o otimizador a um alvo inatingível, dependendo das condições iniciais do problema, o que pode causar em insucesso do seguimento de referência. Este trabalho apresenta contribuições para estratégias de MPC aplicadas a referências variantes no tempo. Será visto que o MPC apresenta uma função custo conflitante em tais condições de tal forma que uma simples modificação pode proporcionar uma flexibilidade à sintonia do MPC, evitando assim problemas indesejados de ganhos elevados do controlador causados por sintonias agressivas. Inicialmente, uma abordagem filtrada do DMC é avaliada, na qual o filtro de predição mantém sensibilidade a distúrbios de alta frequência, enquanto a sintonia do MPC melhora o rastreamento de referências e sua eficácia é avaliada em simulação e experimento do controle de temperatura de um sensor termorresistivo. Em seguida, uma abordagem de função custo modificada em um MPC baseado em receptância foi abordado com vista a reduzir o valor nominal da função custo em condições de referências variantes no tempo. A modelagem baseada em matriz de receptância propõe-se a trazer uma simplicidade na etapa de identificação de sistemas complexos, de tal forma que parâmetros do modelo fenomenológico de um sistema de múltiplos corpos não precisa ser necessariamente conhecido. Assim, resultados em simulação foram avaliados para um sistema subatuado, mostrando sua eficácia para seguimento de referências periódicas. Por fim, este trabalho também trata do problema do MPC com garantia de estabilidade robusta por meio do uso de referências artificiais e da reformulação analítica do alvo desejado. A eficiência dessa abordagem é validada por análises numéricas de simulação, e sua aplicabilidade prática é demonstrada no controle de trajetória de um veículo terrestre não tripulado (UGV).

Palavras-chave: Controle Preditivo Baseado em Modelo; Sistemas Restritos; Seguimento de trajetória.

Abstract

Model predictive controllers (MPC) are widely used to control systems with characteristics common to many industrial and academic applications. The control law of an MPC is formulated by minimizing an objective function that considers the error between estimated predicted outputs and future references over a specified time horizon. These predicted outputs can be defined using either a nominal parametric model or a data-based model. Typically, in linear approaches, the minimization process generates an optimal sequence of inputs through the receding horizon principle, naturally incorporating system constraints into the solution. A well-known issue with constrained MPC is the loss of feasibility when reference changes occur. This happens due to the finite horizon and the constraints imposed by the controller, which may lead the optimizer to an unreachable target depending on the initial conditions of the problem, potentially causing failure in reference tracking. This work presents contributions to MPC strategies applied to time-varying references. It will be shown that MPC exhibits a conflicting cost function under such conditions, such that a simple modification can provide greater tuning flexibility, thus avoiding undesirable issues like excessively high controller gains caused by aggressive tuning. Initially, a filtered DMC approach is evaluated, where the prediction filter maintains sensitivity to high-frequency disturbances while MPC tuning improves reference tracking. Its effectiveness is assessed through simulations and experiments on temperature control of a thermoresistive sensor. Next, a modified cost function approach is proposed for a receptance-based MPC to reduce the nominal value of the cost function under time-varying reference conditions. Additionally, the receptance matrix-based modeling aims to simplify the identification stage of complex systems, eliminating the need for prior knowledge of the phenomenological model parameters of a multibody system. Simulation results were evaluated for an underactuated system, demonstrating its effectiveness in tracking periodic references. Finally, this work also addresses the issue of MPC with robust stability guarantees through the use of artificial references and analytical reformulation of the desired target. The efficiency of this approach is validated through numerical simulation analysis, and its practical applicability is demonstrated in trajectory control of an unmanned ground vehicle (UGV).

Keywords: Model Predictive Control; Constrained Systems; Trajectory Tracking.

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List of Acronyms

AMPC	Approximated Model Predictive Control
CARIMA	Controlled Autoregressive Integrated Moving Average
DMC	Dynamic Matrix Control
DOF	Degree of Freedom
ESO	Extended State Observer
FDMC	Filtered Dynamic Matrix Control
GPC	Generalized Predictive Control
iLQR	iterative Linear Quadractic Regulator
IBVS	Image-based Visual Servoing
LQR	Linear Quadractic Regulator
LQG	Linear Quadractic Gaussian Regulator
MIMO	Multiple-Input Multiple-Output
minRPI	Minimal Robust Positively Invariant Set
MPC	Model Predictive Control
QP	Quadratic Programming
QDMC	Quadractic Dynamic Matrix Control
RMPC	Robust Model Predictive Control
RNN-MPC	Recurrent Neural Network Model Predictive Control
SISO	Single-Input Single-Output
SMC	Sliding Mode Control
PVC	polyvinyl chloride
UAV	Unmanned Aerial Vehicle
UGV	Unmanned Ground Vehicle
UVMS	Underwater Vehicle-Manipulator System
RMS	Root-Means Square

SLQ	Sequential Linear Quadractic
SSMPC	State-Space Model Predictive Control
NMPC	Nonlinear Model Predictive Control
NN-MPC	Neural Network-based Model Predictive Control
GP-NMPC	Gaussian Process-based Nonlinear Model Predictive Control

List of Symbols

$x[i\|k]$	Prediction value of $x[i]$ at a given discrete instant k, with $i > k$.
A	Matrix with appropriate dimensions.
$\mathbb{R}^n$	Vector space of real numbers with dimension n.
$v^\top$	Transpose of the vector v.
$\|\|v\|\|_Q$	Weighted Euclidean norm of the vector v: $\|\|v\|\|_Q \triangleq \sqrt{v^\top Q v}$.
$\mathbf{0}_{n,m}$	Matrix of zeros with dimensions $n \times m$.
$\mathbf{I}$	Eye Matrix with appropriate dimensions.
$\mathbf{I}_n$	Eye matrix with dimensions $n \times n$.
$\check{x}[k]$	Filtered signal of $x[k]$.
$g_j^{l,r}$	Step response coefficients of the output from an input r to the output l.
$\Delta x[k]$	Increment operator of a signal $x[k]$: $\Delta x[k] \triangleq x[k] - x[k-1]$.
z	Forward-shift operator.
z^{-1}	Backward-shift operator.
$F(z)$	Transfer matrix in z.
s	LaPlace Transform operator.
$F(s)$	Transfer function.
$\oplus$	Minkowski sum.
$\ominus$	Pontryagin difference.

Contents

1 Introduction

The Model Predictive Control (MPC) is a set of control strategies contained in the family of optimal controllers that has been proposed in the 1970s and has been widely applied in both industry application and academy research (CAMACHO; ALBA, 2013). Although MPC defines an entire family of control strategies, most of the algorithms are solved online from a finite horizon optimal control problem, using the current state of the controlled process as the initial state (MAYNE, 2000). The optimization is then solved at each time sample, respecting the receding horizon principle and therefore only the first element of the solution is applied as input of the process at each sampling instant. This receding horizon principle is the common element of the MPC algorithms.

The MPC has been attracting attention from many researchers from the past decades due to many advantages when compared to other methods (CAMACHO; ALBA, 2013), whilst many points will be addressed within this chapter:

- It is attractive to tune with a limited knowledge of control systems since the concepts are very intuitive;
- It can be applied to a wide range of dynamic systems: from simple dynamic systems to plants with non-minimum phases, long delay times or unstable behaviors;
- It is natural to deal with Multiple-Input Multiple-Output (MIMO) systems;
- The impact of delays in the system is dealt easily within the prediction phase;
- Measurable disturbances can be handled in the control law in a feed-forward control that is consequence of its formulation;
- Constraint satisfaction is simple and can be included by proper modification of the optimization problem.
- The consideration of future references is possible and its usage is opportunely explored in practical applications;
- It can be applied to control systems described by nonlinear model (MAYNE, 2000).

The strategy first demonstrated its potential in industrial applications in the late 1970s (RICHALET et al., 1978), utilizing an impulse response for a multivariable model. It heuristically searched for a sequence of inputs to drive the future outputs of the internal model as close as possible to the reference trajectory. The results were tested in a distillation column at an oil refinery and a polyvinyl chloride (PVC) plant.

The Dynamic Matrix Control (DMC) is presented as an MPC variant based on the convolutional model, firstly proposed by (CUTLER; RAMAKER, 1980). Such model bring simplicity to the modelling and prediction phase of the strategy, thus it was widely appraised by the process control industry since its initial formulation, as cited in (QIN; BADGWELL, 2003). The application using Quadratic Programming is known as Quadractic Dynamic Matrix Control (QDMC) (GARCIA; MORSHEDI, 1986) and is highly accepted due to its satisfactory efficiency for constrained system and due to advance of Quadratic Programming (QP) algorithms over the years. The vast application of the DMC comes to the fact that its modelling phase, as stated in (XU et al., 2020), depends only from a data-based convolutional model. The convolutional model is constructed based on its step response which brings simplicity and justifies the wide application on industrial applications. Moreover, practical applications of the DMC has been explored in recent works such as (KLOPOT et al., 2018a; ZHANG; HU; GAO, 2022; FERNANDES et al., 2020a). The grown academic interest on data-driven approaches comes to the William's fundamental Lema and the behavioral system theory as stated in (WILLEMS et al., 2005; MARKOVSKY; DÖRFLER, 2021).

As the DMC depends basically from its data-based modelling from an experimental basis, its application might be limited since its prediction does not handle disturbance rejections of systems with integrating and unstable modes. Thus, (SANTOS; NORMEY-RICO, 2023) presents a Generalized Dynamic Matrix control that allows the DMC to handle with unstable processes.

The implicit integral action proposed in various MPC strategies may result in an unsatisfactory response when considering reference tracking in the presence of time-varying objectives (SATOH et al., 2019). The design of predictive controllers with implicit integral action has a fundamental role in time-varying references since the prediction model and the cost function are established under the premise of tracking constaRge nt references over time. The Filtered Dynamic Matrix Control (FDMC) deals with the balance between robustness and disturbance rejection without compromise the nominal performance (LIMA; NORMEY-RICO; SANTOS, 2016). Therefore, the open-loop gains could be reduced without losing the efficiency on the reference tracking issue in a scenario without uncertainties. Such additional degree of freedom is particularly important in the case of aggressive tuning due to reduced reference tracking error objectives.

The Generalized Predictive Control (GPC) was firstly introduced in (CLARKE; MOHTADI; TUFFS, 1987), which is presented by a simple formulation based on the Controlled Autoregressive Integrated Moving Average (CARIMA) plant model and is capable to handle with open-loop unstable processes, non-minimum phase plants, and systems with unknown dead time . Apart from the original DMC (CUTLER; RAMAKER, 1980), the GPC introduces the concept of using a control horizon different from the prediction horizon, which makes the explicit and QP problem much lighter if a sufficient small horizon is chosen. Recent works focused on practical applications have been proposed in this type of MPC as in (ZHANG et al., 2022; JIANG et al., 2023).

Despite the many advantages provided by these strategies, some drawbacks might be highlighted. The first drawback is concerned about the computational effort to achieve the optimal control law. For linear, unconstrained formulation, the calculation of the input is easy to implement and usually is simplified as a state/output feedback, which requires little computation. This issue becomes more relevant when the change of the process dynamic needs to be considered during the prediction formulation. Therefore, the prediction computation needs to be calculated online at each sampling time, in contrast to the prior condition when the prediction matrices can be computed offline. Additionally, the formulation considering constraints in the system variables demands way more computational power due to inequalities and equality (for stabilizing conditions, which will be assessed further) conditions to the optimization problem. Fortunately, standard QP problem achieves the optimal solutions fast enough for the majority of the problems (CAMACHO; ALBA, 2013), thus the analysis of computational effort should be done when applying to fast systems.

It has been cited that MPC strategies are suitable to control MIMO systems naturally and consider the dead-time impact of the systems easily by simply including its effect in the various predictors available in the literature. Moreover, its capability to naturally deal with constraints is an advantage when comparing to other classical controllers and infinite horizon strategies. The control law for MPC can be computed offline when constraints are not considered. The same applies to the infinite horizon case, such as Linear Quadractic Regulator (LQR) and Linear Quadractic Gaussian Regulator (LQG) strategies, which compute the infinite horizon control function offline. In contrast, restricted MPC requires periodic solving of a finite horizon constrained optimization using a QP problem (MAYNE et al., 2000). Depending on the system dynamics complexity and computational resource limitations, reduced horizon values may be necessary to enable online solving within the control loop time window.

Despite the aforementioned advantages, stability is not guaranteed due to the finite horizon principle since the optimal finite horizon action is not optimal in the infinite horizon sense (MAYNE et al., 2000). Anyway, for linear unrestricted systems, there exists a finite control with proper tuning configuration (sufficient long horizons, mainly) such as the MPC is stabilizing.

However such condition rarely represents the reality of controlled systems since hardware restrictions, natural limitations of the process, and/or non-linearities are commonly present in the control problems, which could lead the MPC to not ensure stabilizing control laws for a given target. For this matter, additional stabilizing ingredients have been used (KOUVARITAKIS; ROSSITER; CHANG, 1992; ROSSITER; KOUVARITAKIS; RICE, 1998). Terminal equality constraints (NICOLAO; MAGNI; SCATTOLINI, 1996; MAGNI; SEPULCHRE, 1997) and constraints relaxation for ensuring stability and feasibility (KERRIGAN; MACIEJOWSKI, 2000) are also proposed. Moreover, (MAYNE, 2000) presents sufficient conditions that guarantee the stabilizing properties of constrained MPC by appropriately utilizing additional terminal cost functions and constraints, Lyapunov functions, and invariant sets.

As an illustrative case to show the importance of stability guarantees of MPC, (SANTOS et al., 2011) presents two MIMO systems that appears to be similar on first sight - linear and stable open-loop poles - however one of the strategies presents a non-negative transmission zero, having a non-minimum phase, which is not possible to verify by simple inspection. Therefore, the analysis is done by the same GPC configuration and the response for the non-minimum phase system leads to instability in the nominal case, leading to completely different results. This example brings some conclusions, such as: (i) optimality does not guarantee stability in the context of receding horizon control; (ii) it may not be trivial to detect an inappropriate cost function, and; (iii) MPC with stability guarantees are useful when controlling systems with complex dynamics such as open-loop unstable processes, non-minimum phases and natural constraints.

Thereby, (MAYNE et al., 2000) has settled the foundation of main conditions and ingredients to guarantee asymptotic stability to the system under some assumptions which will be discussed later in this work. In this proposal, stability is achieved by modifying the optimization problem that consists on adding three stabilizing terms, as follows: (i) a terminal cost that penalizes the terminal state (i.e. the value of the state at the end of the prediction horizon); (ii) a terminal constraint that guarantees that the terminal state is within a positively defined invariant set, and; (iii) an implicit stabilizing terminal control law. The overall idea of the stabilizing terms were done from the assumption that the system regulates the output to the origin, although a proper change of coordinates can be successfully done for any other equilibrium condition.

Several MPC strategies consider only the regulation problem, i.e. to steer the system to the origin (MUSKE; RAWLINGS, 1993). Practical applications of MPC often demands non-zero value as the target of the controller (MUSKE, 1997). Therefore a suitable change of coordinates can be done and the problem recurs to original formulation without any change of the original strategy. The change of the steady state reference might lead to loss of feasibility of the strategy since the finite horizon and constraints imposed on the terminal state could lead the optimization problem to an unreachable target and it fails to track the reference (SHEAD; ROSSITER, 2007).

Some preliminary work has been proposed regarding the issue of the loss of feasibility of the MPC for piece-wise constant references, such as reference governors

(GILBERT; KOLMANOVSKY; TAN, 1994; BEMPORAD; CASAVOLA; MOSCA, 1997), interpolation-based control based on a control invariant set as a tracking domain of attraction (BLANCHINI; MIANI, 2000), artificial references in a constrained stable Single-Input Single-Output (SISO) system (ROSSITER; KOUVARITAKIS; GOSSNER, 1996), and dual-mode controller for tracking (CHISCI; ZAPPA, 2003). In (LIMÓN et al., 2005) an MPC for tracking is proposed for constrained general (non-square system) MIMO linear systems which leads the system to steer the states to any admissible target in a feasible solution. The proposed strategy is done based on the addition of artificial variables that defines a virtual equilibrium point based on nominal models, and the inclusion of an offset cost function to the optimization problem that penalizes the deviation between the required target and the artificial steady state. Additionally, terminal boundaries based on the extended concept of the invariant set for tracking is used for stabilizing conditions and then recursive feasibility is ensured for any reference. The properties of the offset cost function are discussed as a role of the steady-state optimizer (FERRAMOSCA et al., 2008). Therefore, the system steers the output to the reference if it is admissible, and leads to the best possible steady state if the reference is not admissible. Optimal performance of the controller in such condition is also discussed and shown that the proposed controller is locally optimal in the neighborhood of the constraints.

Additional contributions on this matter have been done over the recent years, such as extending the proposal to non-linear systems (LIMÓN et al., 2018; SOLOPERTO; KÖHLER; ALLGÖWER, 2022), periodic references (LIMÓN et al., 2015; KÖHLER; MÜLLER; ALLGÖWER, 2020), and distributed MPC for multi-agent systems (KÖHLER; MÜLLER; ALLGÖWER, 2024). Also, a transient performance boundary is set on the closed loop and the solution recovers the infinite horizon solution for sufficiently increased horizons (KÖHLER et al., 2023). Despite the advances regarding the loss of feasibility when reference changes, the MPC for tracking with additional artificial decision variables works under the premise of piecewise-constant references.

The MPC applied to track non-constant references over time is an important subject in mechatronics systems, mainly in robotics systems, that presents underactuated MIMO systems, non-linearities, and the presence of a set of future references is common from its various applications (SCIAVICCO; SICILIANO, 2012, Chapter 8). The next section presents some recent applications of MPC applied to submarine, ground, and aerial robotics, showing the relevance of the MPC on mechatronic systems.

1.1 Motivation - MPC applied to systems with non-constant references

The optimal controller under nonlinear model predictive theory has been widely investigated in off-road mobile robots (FNADI et al., 2021), multiple autonomous nonholonomic robots (GE et al., 2022), ship maneuvering (SANDEEPKUMAR et al., 2022) and underwater autonomous vehicles (YAN et al., 2022). For instance, an interesting MPC trajectory tracking task for a 4-wheel independent steering (4WIS) robot is presented in (LIU et al., 2022). This type of trajectory tracking problem is a typical example of non-constant reference.

The 4WIS robot brings some issues regarding the time delay and speed constraints of the drivers, the MPC controller for trajectory tracking is considered based on the 4WIS robot kinematic model, solving naturally both delay and state constraint challenges. The proposed novel is simulated assuming a reference speed of $3.5m/s$, making the robot follow curved trajectories and comparing its efficiency with classical PID approaches. Complementary, the strategy has been tested in a real experiment, which showed considerable advantages by comparing the performance of both techniques by means of the sum of square residuals of position and angle of attack. Despite, this paper showing promising results, other challenging trajectories could be tested in future works.

Concerning an educational navigation application, (POPESCU et al., 2021) presents a task of a table tennis quad-rotor vehicle with tilting propellers. In this work, aggressive trajectory tracking was achieved by proposing an iterative Linear Quadractic Regulator (iLQR) which achieved a success rate of 40% during real experiments. Therefore, some troubles were found within this novel, such as unwanted vibrations caused by the Unmanned Aerial Vehicle (UAV) plastic material, and limitations on the rate of the motion capture system, which limited the control rate of the iLQR.

In (DAI et al., 2022), a fast tube MPC for controlling an underwater vehicle together with its manipulator system is proposed. Thus, the controller strategy is capable for handling the Underwater Vehicle-Manipulator System (UVMS) for tracking tasks in underwater environment. Robustness properties are guaranteed by adding an Sliding Mode Control (SMC) ingredient to the optimal constrained formulation. This way, the SMC brings the robustness required, solving the issue of the fast MPC approach proposed in this paper, assisting by rejecting model uncertainties and external disturbances during operation. The control tuning of this strategy is done by a linearized model of the system, which can only evaluate local stability of the strategy. Despite that, it shows promising and fast results when compared to the pure MPC formulation, which was reported a loop runtime less than 30 ms for a prediction horizon of 20 steps. Also, the results and comparisons reported were assessed in simulation environment with sinusoidal disturbances applied, being interesting to verify its efficiency under real environment or under non-Gaussian disturbances.

Another hybrid MPC solution for navigation and manipulation tasks is shown in (SLEIMAN et al., 2021), which a whole-body planning MPC framework unifies free-motion missions and manipulation tasks by proposing a multiobjective MPC control law solved by means of an Sequential Linear Quadractic (SLQ) solving approach. In this study, the framework makes use of natural constraints of the SLQ solving to determine any gait sequence or manipulation schedule of the robot. The nonlinear model is based on top of an augmented model that considers the manipulated object dynamics, the robot's center of mass dynamics and finally the full kinematics of the asset, which allows to cover the entire control and task sequencing of the robot in an unified cost function. Despite this strategy covers a wide range of complex missions integrated to locomotion and manipulation tasks, the optimal technique is used only for defining the planning phase of the problem, it still does not cover the low level controllers regarding actuator regulators for trajectory tracking mission, for instance. Although, this paper shows some interesting online system identification techniques to solve uncertainties of the object's model, adding robustness to the novel.

A ballbot balancing stabilization and motion problem is integrated with a manipulator task solving in a single multi-objective cost function in (MINNITI et al., 2019), which solves its optimization by defining a linearized ballbot/manipulator model in each controlling step based on the current state estimation of the system. The optimal technique solves the nonlinear control challenge by means of a Sequential Linear Quadratic MPC (SLQ-MPC), which shows a linear complexity with respect to the optimization of the cost function, proving feasibility of the strategy under laboratory experiments.

Recently, many optimal approaches have been applied to Image-based Visual Servoing (IBVS) tasks using both navigation and manipulation methods. The IBVS general formulation intends to use an image taken from a set of visual sensors as reference for position tracking of the robotic sensor. Despite this technique shows promising results for end-effector position control under the task space, some issues regarding camera uncertainties and unknown dynamics have been tackled by robustness ingredients of the MPC approach. For instance, (JIN et al., 2021) proposes a Gaussian Process-based Nonlinear Model Predictive Control (GP-NMPC) to solve the IBVS task. by using a novel Nonlinear Model Predictive Control (NMPC) to solve a visibility constrained IBVS problem in combination with a Gaussian process estimation to overcome unknown dynamics of the mobile robot. The costly NMPC algorithm performance has also been improved by using a modified iLQR, which also guarantees input constraints applied by the visual servoing, which implies that the object of interest should be within the image throughout the whole prediction. Despite (JIN et al., 2021) presents a promising IBVS approach, the technique has been tested only in simulation so far. Also, it presents some minor limitations due to the lack of uncertainty state propagation under the prediction phase. The author itself proposes the usage of known variances of the model to deal with such limitation, which will result in a nonlinear stochastic optimal control problem. Otherwise, (WU et al., 2021) proposes an Extended State Observer (ESO) based MPC algorithm for a visual servoing application in a robotic manipulator. Hence, the modelling is divided in two parts: i) the first one uses an Recurrent Neural Network Model Predictive Control (RNN-MPC) at the kinematic visual tracking system in order to provide reference signals for the second layer; ii) the second layer takes account of the coupled dynamics of the system, considering the model and camera parametric uncertainties as additional disturbances. Finally an input-to-state stability is evaluate, providing a method to give the maximal admissible uncertainty boundary for the eye-to-hand manipulator configuration. Although this paper shows an interesting solution for the manipulation IBVS challenge, its experimental feasibility validation still has to be done, since the technique has only been tested in simulation with a 2 degrees of freedom manipulator.

On the manipulator problem, the MPC based on linearized manipulator model has been widely investigated in the past years. In (NICOLIS; ALLEVI; ROCCO, 2020), an MPC formulation integrated to an SMC framework for trajectory tracking and impedance control of the end-effector is developed for a redundant manipulator. The SMC-MPC approach is based on a nominal feedback linearized model, that provides robustness guarantees in the task space without getting the state information of velocity and torque from the joint control loop. Besides that, extra sensors should be added for the impedance control of the gripping mechanism to differentiate external forces from residual robot uncertain dynamics.

Despite robotic manipulation has not seen much NMPC applications lately when compared to navigation problem, there is some interesting works exploring nonlinear optimal control solutions directly. An NMPC strategy for a bilateral teleoperation of a manipulator is explored in (PICCINELLI; MURADORE, 2021). This robotics comes in handy in remote tasks that needs to handle with secure and precise manipulation, like surgery application, aerospace and underwater environment missions, which should provide high control reliability and force feedback of the object and environment conditions. Thus, the bilateral strategy is done by a passivity-based control approach that relies on the energy storing principle, using a pair of NMPC algorithms to both i) track the remote manipulator states and ii) provide force feedback of the remote robotic arm to the operator side. Finally, the stability of the proposed solution under time-varying delay is assessed only in simulation tests, with environment conditions being estimated online by means of a recursive least square method.

Since MPC is becoming popular in robotics and mechatronics in past years thanks to the growth of computational power and modern processors, many studies address the predictive approach to low-level/actuator position tracking when the trajectory is already defined by an offline and slow solver. Thus, (FARONI et al., 2019) makes use of an MPC approach to solve a redundant manipulator trajectory planning, combining a fast MPC approach to the ability to deal naturally with a MIMO system with constraints on its inputs and states. The approach for planning is compared to classical techniques, which shows improvements in the feasibility of complex task scaling by preserving geometrical paths when the desired motion is not realizable. This is done since MPC can predict within a time horizon if the set of joint configurations will fall into non-feasible paths, being able to do corrections of the path before the joint inconsistency is reached. Hence, the comparison results were evaluated only in simulation environment, which brings future discussions of the results presented in bench tests with a real manipulator.

A multi-layer Neural Network-based Model Predictive Control (NN-MPC) strategy is used for trajectory tracking of a robotic manipulator under uncertainties and input constraints in (KANG et al., 2021). The novel contribution consists of a two-layer control architecture. While the first neural network layer is responsible to estimate unknown robot dynamics by using an MPC predictive model strategy, the second one makes use of such prediction to solve the optimization problem, presenting a fast approach with feasible time response for robotic tasks. However, such evaluation was done only by testing a 2 Degree of Freedom (DOF) manipulator in a simulation environment. It is expected that the complexity of the problem will raise greatly when a redundant manipulator with more degrees of freedom, having an open question if such strategy could be feasible for a real and redundant manipulator.

Finally, (NUBERT et al., 2020) presents an experimental application of a robust setpoint tracking MPC and an Approximated Model Predictive Control (AMPC) utilizing a multi-staged deep learning novel of a KUKA's LBR4+ manipulator with obstacle avoidance . This paper proposes a robust and stable MPC approach for end-effector position control in face of disturbances and model uncertainties, dealing with every robot control step (i.e. motion planning, path tracking, actuator feedback control) in only one layer by adding an artificial reference in the cost function formulation, as proposed in (LIMÓN et al., 2018), which is an extension of the MPC for tracking (LIMÓN et al., 2008), illustrating an interesting possibility of application of the modified MPC to robotics. Although this strategy provides the robustness required to be applied to experimental evaluations, its response time becomes a challenge to be tackled, having a sampling time of 400 ms due to high computation cost in face of the complex optimal problem, caused by robust guarantees.

To improve computation efficiency, the AMPC proposed by (KARG; LUCIA, 2020) is used to train a novel deep learning based on the robust MPC strategy. Thus, the technique provides a similar solution, but ten times faster than the original one. On the other side, the AMPC approach does not necessarily guarantee mathematically the state constraint required for the robust nonlinear operation. Even so, statistical evaluations were done on top of this option, assuring the safety provided by the Robust Model Predictive Control (RMPC). Hence, these results illustrate the relevance of MPC applications for non-constant references which is the main motivation of this work.

1.2 Objective

The objective of this thesis is to propose novel MPC strategies for tracking general references defined from a dynamic model evolution. These references are dynamic reference signals (KÖHLER; MÜLLER; ALLGÖWER, 2024) or dynamic reference targets. The main motivation is derived from the trajectory tracking specifications for autonomous robots. The following specific objectives are considered in this work:

- To analyze high-gain MPC solutions as a simple alternative for tracking non-constant references.
- To propose alternative MPC solutions based on the cost function modification for dealing with non-constant references.
- To analyze robust MPC solutions to track references defined by dynamic target signals.
- To propose MPC solutions based on QP optimization problems that can be experimentally tested.

1.3 Organization of the text

The following work is organized as follows: Chapter 2 presents the fundamentals of MPC, detailing three classical strategies commonly used in academia and industry: State-Space MPC (SSMPC), DMC, and Generalized Predictive Control (GPC). A discussion on the stability conditions that must be met under certain assumptions is provided, followed by an exploration of the proposed MPC for tracking.

Chapter 3 evaluates the performance of Filtered DMC under time-varying references. The controller's effectiveness in this scenario is demonstrated through both simulation and experimental approaches, focusing on the temperature control of a thermo-resistive sensor.

Chapter 4 introduces a receptance-based GPC and DMC that do not require explicit knowledge of plant parameters. The tracking objective is achieved through a modified cost function designed to ensure nominal zero delay tracking error. Simulation results applied to an underactuated multi-body system are presented to illustrate its potential for practical applications.

Chapter 5 provides a discussion of a robust approach for tracking piecewise-constant reference signals using the artificial target approach to improve recursive feasibility for any target. This way, if the reference is not admissible, the system will converge to the best admissible target determined by an additional offset cost function. The robust property is ensured by strangling the set of constraints based on the worst-case scenario of disturbances to be known during controller design. Finally, a general target modification for any RMPC controller that follows the same concept of set shrinkage in order to ensure offset free trajectory tracking of a time-varying reference. To illustrate the main transient and steady states of the modified approach, a simulation and experimental analysis will be presented on top of a skid steering Unmanned Ground Vehicle (UGV) while tracking a lemniscate trajectory.

Finally, Chapter 6 presents the final conclusions of the work and outlines open topics for future research and list the publications from the result of the research conducted from this thesis.

2 Model Predictive Control

This section provides the technical foundations for various MPC algorithms. While the initial strategies are well-known to researchers in optimal control, some concepts are fundamental for developing the theory around stability guarantees and modifications of the cost function to assess feasibility when controllers are subject to time-varying references. Such modifications are necessary because nominal asymptotic stability conditions are typically satisfied under the premise of a terminal set that depends on the desired reference, ensuring the terminal state is contained within this set. Thus, changing the reference might cause issues on feasibility of the optimizer and then loss of tracking might occur.

Therefore, this chapter will resume the common concepts behind the various MPC algorithms, showing the main concepts in common behind the strategies. This evaluation will be followed by a discussion on the stability of MPC, primarily based on the work of (MAYNE et al., 2000). Finally, a discussion on MPC for tracking, as proposed by (LIMÓN et al., 2008) and (FERRAMOSCA et al., 2008), will be conducted. Additionally, constructive aspects of the MPC will be conducted in the Annex A of this work, focusing on the State-Space Model Predictive Control (SSMPC), DMC, and GPC algorithms.

2.1 Open-loop MPC statement

Consider a nonlinear time-invariant state-space description given by

$$x[k+1] = f(x[k], u[k], w[k]), \tag{2.1}$$
$$y[k] = h(x[k]), \tag{2.2}$$

where $f(.)$ denotes the state evolution over time k and has an equilibrium point at the origin (i.e. $f(0,0,0) = 0$), $x[k]$ is the state vector, $u[k]$ is the control signal vector, $y[k]$ is the output and $w[k]$ is the disturbance vector. If the system is linear, the system dynamic is

$$x[k+1] = Ax[k] + Bu[k] + w[k], \tag{2.3}$$
$$y[k] = Cx[k]. \tag{2.4}$$

In the following control statements to be presented in this work, consider that the system must satisfy the conditions

$$u[k] \in \mathbb{U}, \tag{2.5}$$
$$x[k] \in \mathbb{X}, \tag{2.6}$$

where $\mathbb{U}$ is a convex, compact subset of $\mathbb{R}^m$ and $\mathbb{X}$ is a convex, closed subset of $\mathbb{R}^n$. Also consider that $y \in \mathbb{R}^p$ and could be subject to a hard output constraint set $\mathbb{Y}$, that is

$$y[k] \in \mathbb{Y}. \tag{2.7}$$

Therefore, from now on, the system is considered to have m inputs, n states and p outputs. It is assumed that these sets contain the origin in its interior.

The first strategies of state-space MPC assumes that the main goal is to regulate, i.e. to steer the state to the origin or to a given (constant) equilibrium state x_r, in which $y_r = h(x_r)$. For the sake of simplicity, the second case can be simplified to the first one if a proper change of coordinates is applied to the problem. Asymptotic constant references scenarios are proposed by (LIMÓN et al., 2005), which modifies the classical formulation and will be discussed at the end of this section. For now, consider that a sequence of reference is known for the whole time frame and the controller disposes the information required for any future time window.

For a given optimal input sequence defined $\mathbf{u}(k) = \{u[k|k], u[k+1|k], \ldots, u[k+N-1|k]\}$[1] defined by solving the optimization problem $\mathcal{P}_N(x[k])$ within a finite prediction horizon N

$$\mathcal{P}_N(x[k]) : \min_{\mathbf{u}} \{J(x[k], \mathbf{u}(k)) | \mathbf{u}(k) \in \mathcal{U}(x[k])\}, \tag{2.8}$$

in which the cost function

$$J(x[k], \mathbf{u}(k)) = \sum_{i=0}^{N-1} L(x[k+i|k], u[k+i|k]) + F(x[k+N|k]), \tag{2.9}$$

where the stage cost function $L(.,.)$ and $F(.)$ are properly chosen combining together as the objective to be minimized and $L(0,0) = 0$. To consider the operational constraints of the system to be controlled, the problem may be formulated as follows

$$u[k+i|k] \in \mathbb{U}, i = 0, 1, \ldots, N-1, \tag{2.10}$$
$$x[k+i|k] \in \mathbb{X}, i = 0, 1, \ldots, N. \tag{2.11}$$

[1] The notation $f(k+i|k)$ denotes the future value of f in instant $k+i$ calculated at a given time k.

Additionally, the state at the receding horizon (instant $k+N$ at a given time k) is subject to a terminal set defined as

$$x[k+N|k] \in \mathcal{X}_f \subset \mathbb{X}, \tag{2.12}$$

and the set $\mathcal{U}(x[k])$ combines the conditions (2.10), (2.11) and (2.12) together for the sake of mathematical simplification of the definition of $\mathcal{P}_N(x[k])$.

Thus, the receding horizon principle states that only the first element of $\mathbf{u}(k)$ is applied to the system at time k, restarting the whole optimization process at the next instant $k+1$. This way, the implicit model predictive control law is defined:

$$u[k] = \kappa_N(x) = u[k|k], \tag{2.13}$$

and the solving step of (2.8) is repeated at each sampling time. According to (MAYNE, 2000), if $f(.)$ and $J(.)$ are time-invariant, the problem $\mathcal{P}_N$ is also time invariant and by consequence the optimization stage only depends of the initial state $x[k]$ at any time k.

With the initial statements of the model predictive controller being disposed, the various strategies differs mainly on the definition of the stage cost function, which could be designed for various objectives, either considering the error between the future outputs and references, or considering economic properties of the system in terms of energy consumption and supply chain fees. For most of derivations, a proper prediction model should be used to evaluate the state and output evolution within a finite horizon N. Therefore, the following sections will explore the most used techniques for state predictions, such as the state space formulation, the impulse/step response of the process and also the usage of the transfer function of the plant for output prediction by proper iterations. In the later sections, the definition of the terminal cost and terminal constraints will be studied in order to evaluate stability (since MPC controllers usually does not guarantee stability as shown by (MAYNE, 2000), the terminal ingredients are necessary to overcome this issue). Moreover, the statements presented until now consider that the control needs to steer the process to a constant reference, thus the final section of this chapter will analyze the tracking properties of the MPC for non-constant references.

2.2 MPC with stability guarantee

Infinite horizon controllers needs only elementary assumptions to guarantee stability. The existence of a stabilizing condition implies that the optimal result leads to bounded responses since the error cannot go to infinite in the best conditions. Such analysis is not possible to be relied for constrained MPC (finite horizon) strategies due to receding horizon principle. Consider that the optimal sequence at a given time k is defined as $u^0[k|k]$,

$u^0[k+1|k]$, ..., $u^0[k+N_u-1|k]$ within a control horizon N_u and the control law is taken by selecting only the first element, that is $u[k] = u^0[k|k]$. The next control input selected upon the next optimal sequence at $k+1$ and the equality $u^0[k+1|k+1] = u[k+1|k]$ is not true for finite horizon N. In fact, Bellman's optimality principle (BELLMAN, 1957) demonstrates that $u[k+1] \to u[k+1|k]$ only if $N = N_u \to \infty$. Therefore, the cost function needs to be properly defined for finite horizons, or the MPC closed loop performance might lead to instability even in the nominal case.

Considering a discrete and time invariant dynamic system described by Equations (2.1) and (2.2), the system constraints are applied to the states and inputs throughout the entire horizon

$$u[k+i|k] \in \mathbb{U}, i = 0, 1, 2, \dots, N-1, \tag{2.14}$$

$$x[k+i|k] \in \mathbb{X}, i = 1, 2, \dots, N, \tag{2.15}$$

such as the constraint sets are identical to the ones at Equation (2.5) and (2.6). Additionally, a terminal constraint is imposed at the end of the prediction horizon

$$\hat{x}[k+N|k] \in \mathcal{X}_f \subset \mathbb{X}, \tag{2.16}$$

in a way that the constraints depicted in (2.1), (2.2) and (2.16) are mapped in terms of the decision variable of the optimization problem $u[k]$ and the composed set $\mathcal{U}(x[k])$ such as $u[k] \in \mathcal{U}(x[k])$.

The new optimization problem within a finite horizon is defined by

$$\mathcal{P}_N(x[k]) = \min_{\mathbf{u}[k]} (V(x[k], \mathbf{u}[k]) | \mathbf{u}[k] \in \mathcal{U}(x[k])). \tag{2.17}$$

The new cost function is

$$V(x[k], \mathbf{u}[k]) = \sum_{i=0}^{N-1} L(x[k+i|k], u[k+i|k]) + F(x[k+N|k]), \tag{2.18}$$

with $L(.,.)$ the regular stage cost function, and $F(.)$ a known and determined terminal cost function which is used to guarantee stability in the MPC framework.

Following some usual simplifications to reduce formalism from the discussion of (MAYNE et al., 2000), consider that the cost function must follow the condition $L(x, u) \geq c \cdot ||(x, u)||^p$ with $p \geq 1$ and $c > 0$. The condition could consider only the stage cost for the output $L(h(x), u) \geq c \cdot ||(h(x), u)||^p$ if the pair (f, h) is detectable with $L(0, 0) = 0$. Thereby, the controlled system is stable if the following four conditions are satisfied.

1. $\mathcal{X}_f \subset \mathbb{X}$, and $\mathcal{X}_f$ is closed and contains the origin;

2. $\kappa_f(x) \in \mathbb{U}, \forall x \in \mathbb{X}$;

3. $f(x, \kappa_f(x)) \in \mathcal{X}_f, \forall x \in \mathbb{X}$;

4. $[F(f(x, \kappa_f(x)) - F(x) + L(x, \kappa_f(x)))] \leq 0, \forall x \in \mathbb{X}$.

The conditions 1. and 2. implies that the system constraints are respected in terms of the terminal state and the local controller law $\kappa_f(x)$. The statement in 3. uses the state update function $f(.,.)$ in Equation (2.1) such as the terminal state constraint is an invariant positively definite set, such as every state successor in $\mathcal{X}_f$ at instant k stays in the terminal set at the next instant $k+1$. Finally, 4. implies that the terminal cost $F(.)$ is a suitable Lyapunov function that describes the trajectory of the terminal state.

If assumptions 1., 2. and 3. are achieved by the proposed controller, it is guaranteed that the terminal set is positively invariant and the recursive feasibility applies. This means that $\mathcal{P}(x[k])$ presents a feasible solution for $x[k]$, then a solution for $\mathcal{P}(x[k+1])$ exists. Also, the last assumption implies that the cost function $V(x[k], [u][k])$ is a Lyapunov control function as well. If the four assumptions are met for a given initial state $x[0]$, then the system is stable for a feasible solution and then guarantees stability. The set of possible initial conditions that returns feasible solutions is defined as attraction domain, which will be discussed in the next section.

2.3 MPC with artificial reference for asymptotic constant references

This MPC for tracking is done by adding an artificial steady state and input into the cost function description which are treated as additional decision variables of the optimization problem to be solved. The artificial variables are added by means of an offset cost function that penalizes the error between the virtual steady state and the desired target. Then, the stage cost now should consider the deviation between the predicted outputs (or state evolution) and the artificial values. Furthermore, the strategy consider extended stabilizing terminal conditions by adding a terminal constraint in both terminal state and the new ingredients. This way, the controller is able to steer the system to any admissible target. Moreover, the system will not lose feasibility if the required reference is not reachable. In such case, the controller will drive the output to the closest admissible steady state, defined by the minimization of the offset cost function.

To describe the controller, consider an undisturbed linear system described from (2.1) and (2.2) (if the system is linear, $x[k+1] = Ax[k] + Bu[k]$ and $y[k] = Cx[k] + Du[k]$). Assuming that the system is stabilizable and it is subject to hard constraints as disposed in Equations (2.5) and (2.6).

Assumption 1 *The linear system (or pair (A, B)) is stabilizable.*

The main objective of the MPC for tracking is to define a control law that is a function of the current state reading or estimation and the desired target, in which should steer the state (or output) as close as possible to the reference considering the constraints imposed by the controller.

2.3.1 Characterization of the steady states and inputs

For any given output target y_r imposed by the controller interface, there is a pair of vectors x_s and u_s associated to the desired reference that must attend the following condition:

$$(A - \mathbf{I}_n)x_s + Bu_s = \mathbf{0}_{n,1}, \tag{2.19}$$

$$Cx_s + Du_s = y_r. \tag{2.20}$$

If Assumption 1 is met, then the solution for the pair $z_s = \begin{bmatrix} x_s^\top & u_s^\top \end{bmatrix}^\top \in \mathbb{R}^{n+m}$ can be parameterized in terms of the vector $\theta \in \mathbb{R}^m$ with suitable size such as:

$$z_s = \begin{bmatrix} M_x \\ M_u \end{bmatrix} \theta = M_\theta \theta, \tag{2.21}$$

$$y_r = N_\theta \theta. \tag{2.22}$$

The solution for this problem could be defined on basis of any solution of the null space of $\begin{bmatrix} (A - \mathbf{I}_n) & B \end{bmatrix}$, thus the choice of θ defines any solution for z_s. Moreover, for the non-empty, compact and convex set $\mathcal{Z} = \mathbb{X} \times \mathbb{U}$, the parameter θ must respect the constraints:

$$M_\theta \theta \in \mathcal{Z}. \tag{2.23}$$

Defining the sets $\mathcal{X}_s = Proj_x(\mathcal{Z})$ and $\mathcal{U}_s = Proj_u(\mathcal{Z})$, the set of admissible states and inputs (x_s, u_s) is the interior of $\mathcal{X}_s \times \mathcal{U}_s$. Consider the projection operation $Proj_x(\mathcal{Z}) = \{x \in \mathbb{R}^n | \exists u \in \mathbb{R}^{(n+m)}, (x, u) \in \mathcal{Z}\}$ and the same applies to $Proj_u(\mathcal{Z})$.

2.3.2 Invariant set for tracking

Considering that the controlled system is subject by the following control law

$$u[k] = K(x[k] - x_s[k]) + u_s = Kx[k] + L\theta, \tag{2.24}$$

such as

$$L = \begin{bmatrix} -K & \mathbf{I}_m \end{bmatrix} M_\theta. \tag{2.25}$$

If the eigenvalues of the matrix $(A + BK)$ are inside the unitary circle, there is sufficient condition that the system state x is steered to the steady state x_s with the steady state input u_s, which can be found by any solution z_s. Although, (SANTOS, 2018) presents a trivial parametrization for M_θ, such as

$$M_x = (\mathbf{I}_n - A - BK)^{-1} B, \tag{2.26}$$

$$M_u = \mathbf{I} + KM_x, \tag{2.27}$$

$$N_\theta = CM_x + DM_u. \tag{2.28}$$

Notice that if the trivial definition of M_θ in (2.26) and (2.27) is used, then $L = I_m$. Therefore, a modified invariant set for tracking can be now defined in terms of the artificial variables. The invariant set for tracking proposed here is the set of the initial states and steady states/inputs that can be stabilized using the proposed control law in an admissible way.

For this matter, define the augmented state for tracking $q[k] = \begin{bmatrix} x^\top & \theta^\top \end{bmatrix}^\top$. The evolution of the augmented state is then defined by the autonomous system

$$q[k+1] = A_q q[k], \tag{2.29}$$

such as its dynamic is determined by A_q

$$A_q = \begin{bmatrix} A + BK & BL \\ 0 & \mathbf{I}_{n_\theta} \end{bmatrix}. \tag{2.30}$$

Consider the set W_λ as a convex polyhedron defined for the scalar λ contained in the set $(0, 1]$.

$$W_\lambda = \{q = (x, \theta) : (x, Kx + L\theta) \in \mathcal{Z}, M_\theta \theta \in \lambda \mathcal{Z}\}. \tag{2.31}$$

A set Ω^q is an admissible invariant set for tracking if

$$q \in \Omega^q \Rightarrow A_q q[k] \in \Omega^q. \tag{2.32}$$

Considering that $\Omega^q \subseteq W_1$. In theory, the maximal admissible invariant set for tracking $\mathcal{O}^q_\infty$ is then defined based on the set W_1 for any value and evolution of the extended state q, that is $\mathcal{O}^q_\infty = \{q : A^i_q q[k] \in W_1, \forall i \geq 0\}$. Since A_w has unitary eigenvalues, $\mathcal{O}^q_\infty$ might not be described by a finite number of constraints, thus the approximation $\mathcal{O}^q_{\infty,\lambda}$

$$\mathcal{O}^q_{\infty,\lambda} = \{q : A^i_q q[k] \in W_\lambda, \forall i \geq 0\}, \tag{2.33}$$

is a reliable polyhedral approximation of the maximal invariant set if λ is sufficiently close to one (LIMÓN et al., 2008).

2.3.3 MPC for tracking formulation

The proposed MPC for tracking by (LIMÓN et al., 2008) is given based on three ingredients on the optimization formulation: i) artificial steady states as additional decision variables to the optimization problem, which defines; ii) an extended terminal constraint based on the concept of the maximal invariant set for tracking, and; iii) the offset cost function added to the problem which penalizes the deviation between the predicted states and system target, will be properly described in this section. The strategy is defined on the following assumptions from (LIMÓN et al., 2008):

Assumption 2 *(1) Defining $Q \in \mathbb{R}^{n\times n}$, $R \in \mathbb{R}^{m\times m}$ and $T \in \mathbb{R}^{n\times n}$ as positive definite matrices.*
(2) Let $P \in \mathbb{R}^{n\times n}$ be a positive definite matrix such that $(A+BK)^\top P(A+BK) - P = -Q - K^\top RK$.
(3) The set $\mathcal{X}^q_f \subseteq \mathbb{R}^{n+m}$ is an admissible polyhedral invariant set for tracking for the system (2.1)*, subject to constraints and a gain controller K.*
(4) The offset cost function $J_O : \mathbb{R}^p \to \mathbb{R}$ is a convex, positive definite and sub-differentiable function and $J_O(0) = 0$ (FERRAMOSCA et al., 2008).

The modified cost function of the MPC for tracking is calculated for a given state reading x and state target x_t at instant k. The vector of future inputs $\mathbf{u}(k)$ and artificial variables $\theta(k)$ are added to the function according to (2.21).

$$\begin{aligned} J_N(x[k], y_t[k], \mathbf{u}(k), \theta(k)) = &\sum_{i=0}^{N-1} (||x[i] - x_s[k]||^2_Q + ||u[i] - u_s[k]||^2_R) + \dots \\ &\dots + ||x[N] - x_s[k]||^2_P + ||y_s[k] - y_t[k]||^2_T, \end{aligned} \tag{2.34}$$

The cost function now has a modified stage cost function weighted by the tuning parameters Q and R as prediction error and control effort respectively, and the terminal cost function $J_t(.) = ||x[N] - x_s[k]||^2_P$ also has a modification by adding the steady state. Finally the offset cost function $J_o(.) = ||y_s[k] - y_t[k]||^2_T$ is defined, which penalizes the deviation between the artificial target and the desired reference using the weighting factor

T. The minimum value J_N^0 of the proposed cost function is defined in terms of the optimal input sequence $\mathbf{u}^0 = [u^0[k|k] \dots u^0[N-1|k]]$ (assuming that the control horizon is equal to the prediction horizon for notation simplicity) and optimal value of the artificial variable θ^*.

$$\begin{aligned}
\underset{\mathbf{u}(k),\theta(k)}{\text{minimize}} \quad & J_N(x[k], y_t[k], \mathbf{u}(k), \theta(k)), && (2.35a)\\
\text{subject to} \quad & x[0] = x, && (2.35b)\\
& x[k+1] = Ax[k] + Bu[k], && (2.35c)\\
& z_s[j] \in \mathcal{Z}_s, \forall j = 0, \dots, N-1, && (2.35d)\\
& z_s = M_\theta \theta, && (2.35e)\\
& (x[N], \theta) \in \mathcal{X}_f^q. && (2.35f)
\end{aligned}$$

Since the region $\mathcal{X}_f^q$ is a polyhedron that can be written by a finite set of inequalities in terms of the decision variables, the optimization problem stated in (2.35a)-(2.35f) is a standard QP problem which can be solved by any known specialized library or algorithm such as (ANDERSSON et al., 2019). Despite the algorithm is more heavy in terms of computational effort when compared to standard MPC (CAMACHO; ALBA, 2013), it still belongs to the class of multi parametric solvers and its application can be applied to systems with fast response times. The control law for this strategy is achieved by applying the receding horizon principle, that is

$$\kappa_N(x, x_t) = u[k|k]^0. \tag{2.36}$$

Respecting the Assumption 1 and Assumption 2, and for any feasible initial state $x[0]$ inside the invariant set for tracking, the controller proposed is stable for any target and respect the constraints over time. If the target $y_t = Cx_t$ is admissible, i.e. $x_t \in \mathcal{X}_f^w$, then the states converges to x_t, such that $\lim_{k\to\infty} ||y[k] - y_t|| = 0$. If the reference is not admissible, then the system converges to be the closest reachable steady state $z_s^0 = \begin{bmatrix} x_s^{0\top} & u_s^{0\top} \end{bmatrix}^\top$ possible, determined by the offset cost function, where

$$y_s^0 = \arg \min_{y_s \in \mathcal{X}_f^q} J_O(y_s - y_t), \tag{2.37}$$

such as $y_s = Cx_s$ Moreover, it is stated in (LIMÓN et al., 2008) that the proposed controller has a larger domain of attraction when compared to the standard MPC. For that, consider an MPC control law with the same ingredients, for instance, the weighting factors Q, R, P, K (these two last are used for stability guarantees as per imposed in (MAYNE et al., 2000)), and horizon N and maximal invariant set in terms of the desired target $\mathcal{O}_\infty(x_t)$. Additionally, consider the terminal constraint as $\mathcal{O}_{\infty,\lambda}$ with λ close to one.

Since $\mathcal{O}_\infty(x_t) \subseteq Proj_x(\mathcal{O}_{\infty,\lambda})$, the MPC for tracking has a larger domain of attraction when compared to the standard MPC. The optimality of the controller is then discussed in (LIMÓN et al., 2008), which shows that optimality is not guaranteed, but its loss could be arbitrarily reduced with large values of the matrix T (ALVARADO, 2007), resulting the local optimal control law defined by P, Q and R. Besides, (FERRAMOSCA et al., 2008) proves that the closed loop evolution is locally optimal in the neighbourhood of the external reference since it provides the best possible effort in terms of the cost function of the MPC. Even though, the closed loop performance recovers an infinite horizon optimal solution for an increasing horizon of the strategy (KÖHLER et al., 2023).

2.4 Conclusion

This chapter provided the theoretical basis for the upcoming MPC analysis in the next chapters. Firstly, the general idea for most of the MPC family was set as a control law that relies on the minimization of a given objective (usually recurring to a QP problem) for a given current state of the system and decision variables to be set in order to achieve such objective of the problem. Then, a set of optimal inputs are given, and the control signal is applied by means of a receding horizon strategy, which takes only the first element of the optimal sequence, restarting the whole process in the next control loop in order to achieve success in regulation and disturbance rejection even with modelling mismatches. The analysis for stability guarantees on MPC were discussed for constrained systems and the fundamental conditions for achieving it were set based on the work done in (MAYNE et al., 2000).

Finally, a modification of the standard MPC was presented based on the work of (LIMÓN et al., 2008) and (FERRAMOSCA et al., 2008) to assess feasibility of the MPC for tracking changes of references, since the conditions for feasibility and stability relies on the concept of a terminal set that depends on the constant reference applied to the controller. For that, the optimizer is modified to add artificial variables to the QP problem together with an offset cost function and the addition of an invariant set for tracking, allowing the controller to be feasible for any target, converging the output to its reference, if admissible, or to the best reachable steady state possible, if the target is not admissible.

3 Filtered DMC applied to systems with time-varying references

This chapter will analyze the filtered DMC contributions when the reference varies over time such that non-constant references can be considered. The additional filter imposes an extra degree of freedom, which helps alleviate the effects of high-frequency disturbances caused by measurement and modeling uncertainties. The discussions and results of this chapter were published in (PEREIRA; SANTOS, 2023).

The predictive control algorithms such as DMC and its variants are useful in practice since a data-based convolutional method could be used in its formulation (XU et al., 2020). It is only necessary the step response data to describe the prediction model, which simplifies the model characterization of industrial processes. Moreover, recent works on data-based MPC have been demonstrated in practical application such as (KLOPOT et al., 2018b; ZHANG; HU; GAO, 2022; FERNANDES et al., 2020b; PECCIN et al., 2022; SHI et al., 2022). The whole foundation of MPC with prediction models based on experimental data have been receiving constant attention from academic community, which is motivated by the William's fundamental Lema (WILLEMS et al., 2005) and the behavioral system theory (MARKOVSKY; DÖRFLER, 2021).

MPC strategies allows to consider future references naturally in its optimization problem, as seen in Section 2. Moreover, the implicit integral action proposed in various MPC strategies may result in an unsatisfactory response when considering reference tracking in the presence of time-varying objectives (SATOH et al., 2019). The design of predictive controllers with implicit integral action has a fundamental role in time-varying references since the prediction model and the cost function are established under the premise of tracking constant references over time.

This way, controllers with elevated open-loop gains reduces the tracking error under time-variant references, with the drawback of reducing its robustness margin and the high-frequency noise attenuation might be severely compromised.

The filtered DMC (FDMC) deals with the balance between robustness and disturbance rejection without compromise the nominal performance (SANTOS; NORMEY-RICO, 2023). Therefore, the open-loop gains could be reduced without losing the efficiency on the reference tracking issue in a scenario without uncertainties. Such additional degree of freedom is particularly important in the case of aggressive tuning due to reduced reference tracking error objectives.

This section proposes the filtered DMC to deal with the problem of output tracking under the condition of time-varying references. The FDMC allows to deal with constraints and to optimize the tracking performance based on step response model at the same time that an additional filter (LIMA; NORMEY-RICO; SANTOS, 2016) gives more freedom on the context of robust tuning, which gives controller configurations with better nominal performance.

Simulation results and a preliminary experimental analysis are given based on a temperature control of a termoresistive sensor, which is shown to illustrate the main benefits of the filtered approach in the context established.

3.1 Filtered DMC

The filtered DMC was firstly proposed as an alternative to improve the balance between robustness and regulatory performance. Later, it was demonstrated that such strategy could be used to stabilize the convolutional prediction model for open-loop unstable processes, overcoming the main disadvantage of original DMC formulation.

In a simplified manner, the filtered DMC makes use of a filtered version of $\eta_l(k)$, represented by $\check{\eta}_l(k) = \mathcal{Z}^{-1}\{F_l(z)\mathcal{Z}\{\eta_l(k)\}\}$ and $F_l(z)$ is the transfer function of an appropriate prediction filter.

The scope of this discussion is limited only to stable processes, such that only the compromise between regulatory performance and robustness is discussed in a isolated way. Therefore, the filtered output prediction of the output l is calculated as

$$\hat{y}_l[k+j|k] = \underbrace{\sum_{r=1}^{m}\sum_{i=1}^{j} g_i^{l,r}\Delta u_r[k+j-i]}_{\text{Forced Response}} + \underbrace{\sum_{r=1}^{m}\sum_{i=1}^{\tilde{N}_{l,r}} (g_{j+i}^{l,r} - \check{g}_i^{l,r})\Delta u_r[k-i] + \check{y}_l[k]}_{\text{Free Response}}, \tag{3.1}$$

with the increment of the input r defined as $\Delta u_r[k] = u_r[k] - u_r[k-1]$ and also considering $\check{g}_i^{l,r}$ and $\check{y}_l(k)$ as the filtered values of $g_i^{l,r}$ and $y_l[k]$ in (A.28) respectively. A vector form of the filtered DMC prediction can be written by following the modifications applied in (3.1) such as

$$\vec{\mathcal{Y}}_f[k] = \mathbf{G}\boldsymbol{\Delta}\mathbf{u}[k] + \tilde{\mathcal{G}}\Delta\mathcal{U}[k] + \vec{\check{Y}}[k]. \tag{3.2}$$

In this case, the optimal input increment sequence is defined by

$$
\begin{aligned}
\underset{\Delta\mathbf{u}}{\text{minimize}} \quad & \sum_{j=1}^{N} \|y_r[k+j] - \hat{y}[k+j|k]\|_Q^2 + \sum_{j=0}^{N_u-1} \|\Delta u[k+j|k]\|_R^2, \\
\text{subject to} \quad & \vec{\mathcal{Y}}_f[k] = \tilde{\mathcal{G}}\Delta\mathcal{U}[k] + \vec{\check{Y}} + \mathbf{G}\Delta\mathbf{u}[k], \\
& \hat{y}(k+j|k) \in \mathbb{Y}, \qquad j \in \mathbb{N}_{[1,N]}, \\
& u(k+j|k) \in \mathbb{U}, \qquad j \in \mathbb{N}_{[0,N_u-1]}, \\
& \Delta u(k+j|k) \in \Delta\mathbb{U}, \quad j \in \mathbb{N}_{[0,N_u-1]},
\end{aligned} \tag{3.3}
$$

and the FDMC control law follows the receding horizon principle, selecting only the first element of each input increment, repeating the optimization process again in the next loop instant.

3.2 Filtered DMC applied to time-varying references

The efficiency of the FDMC can be directly assessed from the cost function used in the optimization step, as outlined in (3.3). This function can be divided into two terms, allowing the efficiency to be analyzed when the system is subject to time-varying references:

$$
J(k) = \underbrace{\sum_{j=1}^{N} \|y_r(k+j) - \hat{y}(k+j|k)\|_Q^2}_{J_e(k)} + \underbrace{\sum_{j=0}^{N_u-1} \|\Delta u(k+j|k)\|_R^2}_{J_{\Delta u}(k)}. \tag{3.4}
$$

The tuning process for controllers with implicit integral action under systems subjected to time-varying references could be interpreted by the cost function $J(k)$ in two terms: (i) $J_{\Delta u}(k)$ and (ii) $J_e(k)$. The first objective $J_{\Delta u}(k)$ demands the minimization of the control effort, reaching its minimum by the condition $\|\Delta u(k+j|k)\| = 0$, $j \in \mathbb{N}_{[0,N_u-1]}$. Summarizing, the minimium of $J_{\Delta u}(k)$ requires the convergence to a fixed equilibrium point for stable systems. Meanwhile, the second cost function $J_e(k)$ include an objective that requires that the decision variables needs to be chosen in order to guarantee $\|y_r(k+j) - \hat{y}(k+j|k)\| = 0$, $j \in \mathbb{N}_{[1,N_u]}$. Therefore, the cost $J_e(k)$ imposes an objective that requires varying control increments in the presence of changing future references. In summary, both $J_e(k)$ and $J_{\Delta u}(k)$ are conflicting objectives, which does not occur in the steady state during the tracking of a constant reference. As consequence of the Internal Model Principle, in the abscence of a future reference model in the context of internal modes, the reference tracking error will depend on the predictive controller gain, which depends exclusively on the tuning parameters and particularly by the relation between matrix Q and matrix R.

The matrices Q and R are penalizing factors that allows to prioritize conflicting objectives with the same cost. If $Q >> R$, and by consequence $J_e(k) >> J_{\Delta u}(k)$, then the minimization of the future errors will predominate the overall minimization objective. In such case, the DMC may provide sufficient results in varying references, even in the absence of internal models. However, this strategy provides elevated gains, which potentially compromises robust performance and tends to amplify measurement noises to undesired levels. The original DMC does not have any additional degree of freedom to deal with the reduction of the open-loop gain, despite other predictive controllers.

In this matter, the FDMC (LIMA; NORMEY-RICO; SANTOS, 2016) come as an alternative to deal with time-varying references because the filter added in the prediction error allows to modify the robust characteristic without compromising the nominal response of the controller. The robustness of the FDMC is already been detailed in recent works as in (LIMA; NORMEY-RICO; SANTOS, 2016; SANTOS; NORMEY-RICO, 2023).

In the sequence of this discussion, the role of the prediction error filter in the presence of measurement noises will be discussed, which was not discussed in previous works.

3.2.1 Analysis of the prediction error filter in the presence of noises

At this point, it is important to emphasize that the FDMC is a general case of the DMC. If $F_l(z) = 1$, then by definition $\check{\eta}_l(k) = \eta_l(k)$, causing that the FDMC becoming the DMC. This way, the impact of the noise in the FDMC will be analysed directly.

The FDMC cost function can be represented in a vector form based on the standard DMC cost function in (A.35a):

$$J(k) = (\vec{\mathcal{Y}}_f[k] - \vec{\mathcal{W}})^\top \bar{\mathcal{Q}}(\vec{\mathcal{Y}}_f[k] - \vec{\mathcal{W}}) + \Delta\mathbf{u}(k)^\top \bar{\mathcal{R}}\mathbf{u}(k). \tag{3.5}$$

To obtain the analytical solution for the unconstrained case, the substitution $\mathcal{Y}(k) = \mathbf{G}\boldsymbol{\Delta}\mathbf{u}(k) + \tilde{\mathcal{G}}\Delta\mathcal{U}(k) + \vec{\check{Y}}(k)$ (for $\vec{\check{Y}}(k)$ being the filtered version of $\vec{Y}(k)$ defined in (A.33)) is made in (3.5) and the derivative in terms of $\boldsymbol{\Delta}\mathbf{u}(k)$ is done. The optimal solution $\boldsymbol{\Delta}\mathbf{u}^0(k)$ is achieved by equalizing the derivative expression to zero, reaching to:

$$\boldsymbol{\Delta}\mathbf{u}^0[k] = \mathbf{K}\left(\vec{\mathcal{W}} - \tilde{\mathcal{G}}\Delta\mathcal{U}(k) - \vec{\check{Y}}(k)\right), \tag{3.6}$$

with $\mathbf{K} = (\mathbf{G}^\top\bar{\mathcal{Q}}\mathbf{G} + \bar{\mathcal{R}})^{-1}\mathbf{G}^\top\bar{\mathcal{Q}}$. The control law is given by selecting the first element of the sequence of each input with $\mathcal{K} \in \mathbb{R}^{m\times(mN_u)}$

$$\Delta u(k) = \mathcal{K}\left(\vec{\mathcal{W}} - \tilde{\mathcal{G}}\Delta\mathcal{U}(k) - \vec{\check{Y}}(k)\right), \tag{3.7}$$

such as

$$\mathcal{K} = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 1 & 0 & \dots & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & \dots & 1 & 0 & \dots & 0 \end{bmatrix} \mathbf{K}. \tag{3.8}$$

To conclude the analysis of the measurement noise with the robustness filter, consider that the measurement of any output l ($y_l(k)$) is composed by its value without any disturbance ($y_l^*(k)$) and the signal that defines the disturbance itself ($\xi_l(k)$). This way,

$$y_l[k] = y_l^\star[k] + \xi_l[k]. \tag{3.9}$$

and since the discussion is about linear filters ($F_l(z)$), the superposition applies such as

$$\check{y}[k] = \check{y}^\star[k] + \check{\xi}[k], \tag{3.10}$$

being $\check{y}^\star(k)$ the filtered noiseless output and $\check{\xi}[k] = [\check{\xi}_1[k] \ ... \ \check{\xi}_p[k]]$ obtained by applying the prediction filter, that is $\check{\xi}_l[k] = \mathcal{Z}^{-1}\{F_l(z)\mathcal{Z}\{\xi_l[k]\}\}$. To simplify the analysis, consider that the system is SISO. Moreover, the analytical solution for the FDMC control law in a condition without constraints is rewritten by

$$\Delta u(k) = \mathcal{K}\left(\vec{\mathcal{W}} - \tilde{\mathcal{G}}\Delta\mathcal{U}(k) - \mathbb{1}\check{y}^\star(k)\right) - \mathcal{K}\mathbb{1}\check{\xi}(k). \tag{3.11}$$

The FDMC control law presented in Equation (3.11) allows to inspect directly the noise effect in the control increment by means of $-\mathcal{K}\mathbb{1}\check{\xi}(k)$ in the unconstrained scenario. Controllers with elevated gains impacts directly on the amplification effect of the noise related to the input increment.

In the non-filtered case (DMC), $\tilde{\mathcal{G}} \to \mathcal{G}$, $\check{y}^\star(k) \to y^\star(k)$ and $\check{\xi}(k) \to \xi(k)$, so there is no degree of freedom to deal with the effect of $\xi(k)$. In the formulation without the filter, the feedback $-\mathcal{K}\mathbb{1}\xi(k)$ may cause considerable issues in the noise amplifying if $\mathcal{K}$ has elevated values.

On the other hand, in the filtered version, low-pass filters can attenuate high-frequency noises without compromising the medium and low-frequency error estimation modelling. Thus, filters with unitary static gains $F_l(1) = 1$ are selected to not impact on the constant (D.C. level) prediction error, then it maintains the constant disturbance rejection properties of the controller. First-order low-pass filters in form of $F_l(z) = \frac{(1-\alpha)z}{z-\alpha}$ with $\alpha \in (0, 1)$ are simple alternatives that could lead to good results. The discrete bandwidth is directly defined by α introducing a simple parameter that provides an extra degree of freedom to handle output noise without interfering with the MPC gains. In particular,

$\alpha = 0$ would lead to a non-filtered DMC since $F_l(z) = \frac{z}{z} = 1$. Implementation details could be seen in related works as in (LIMA; NORMEY-RICO; SANTOS, 2016).

It is important to mention that filters with strictly small bandwidths could potentially attenuate medium and low frequency information to undesired levels, which could degrade the regulatory performance and compromising the reference tracking when significant modelling errors are present. So the filter tuning should be done following the principle of simplicity with the objective to only reduce the high-frequencies without impairing the medium and low frequency responses.

3.3 Case study - Temperature control of an NTC sensor

The case under study concerns the temperature control of a thermoresistive sensor of the NTC (Negative Temperature Coefficient) type. This is a nonlinear system in which the sensor's temperature is regulated by varying the power dissipated within it. These sensors are used to measure various quantities such as temperature, fluid velocity, and solar radiation. In the configuration with a sensor, the NTC temperature is intentionally varied in such a way that the information about the difference in quantities is used to estimate the desired measurement, providing a time-varying reference in cases of digitally controlled sensors.

The dynamic model relating the dissipated power and the temperature of the NTC sensor can be described as follows:

$$SH(t) + P_s(t) = G_{th}(T_s(t) - T_a(t)) + C_{th}\frac{dT_s(t)}{dt}, \tag{3.12}$$

where $T_s(t)$ and $T_a(t)$ are the temperatures of the sensor and the ambient environment, respectively, $P_s(t)$ represents the power of the sensor, $G_{th}(T_s(t) - T_a(t))$ describes the power dissipated to the surroundings. In this model, G_{th} is the dissipation constant, $SH(t)$ represents the effect of external irradiation, while C_{th} describes the thermal capacity of the sensor. The dissipated power can be defined as $P_s(t) = R_s(t)I_s(t)^2$, where $R_s(t)$ is the sensor's resistance and $I_s(t) = v(t)/R_{en}$ is a current source regulated by the voltage $v(t)$.

For temperature control purposes, $u(t) = v(t)^2 - \bar{v}^2$ and $y(t) = T_s(t) - \bar{T}$ are considered. Simulations were conducted assuming $A = 0.004625\ \Omega$, $B = 3988\ K$, $C_{th} = 22.5\ mJ/K$, $\overline{G}th = 1.5\ mW/K()$, $Ren = 3.3\ K\Omega$ as parameters of an existing system (de J. Souza; SANTOS, 2020). It is assumed that $SH(t) = 0\ mW$ and $T_a = 303\ K$ for simulation purposes. The step response model was obtained around the equilibrium established by $\bar{v}^2 = 40\ V^2$ with steps of $\pm 5\ V^2$ applied to $u(k)$, using the average value of the unit step responses (normalized by the input variation). It should be emphasized that the step response is obtained from (3.12) considering the simulated case.

The parameters of DMC and FDMC are $h = 0.5\ s$ (sampling period), $N = 15$, $N_u = 5$, $\tilde{N} = 150$, $Q = 1$, $R = 10^{-2}$ or $R = 10^{-5}$. Two first-order filters will be tested in the case of FDMC, with $\alpha = 0.6$ and $\alpha = 0.8$, using $F(z) = \frac{z(1-\alpha)}{z-\alpha}$. It's necessary to impose the constraint $1\ V^2 \leq u(t) \leq 100\ V^2$, since $v(t) \leq 10\ V$ is a system constraint, and $1\ V \leq v(t)$ imposes a minimum current that allows resistance measurement without compromising the signal-to-noise ratio (SANTOS; SANTOS, 2022). The reference signal trajectory is described by a fifth-order motion law given by

$$f(t) = f_0 + f_3 t^3 + f_4 t^4 + f_5 t^5, \qquad 0 \leq t < T_f \tag{3.13}$$

$$f(t) = f_0 + f_3 (T_f - t)^3 + f_4 (T_f - t)^4 + f_5 (T_f - t)^5, \quad T_f \leq t < 2T_f, \tag{3.14}$$

considering $f_0 = 1$, $f_3 = 10 f_0/(T_f^3)$, $f_4 = -15 f_0/(T_f^4)$, $f_5 = 6A/(T_f^5)$ e $T_f = 8$ for this problem. Thereby, the discrete reference signal is $y_r(k) = f(kh),\ 0 \leq k < 2T_f/h$ with periodicity defined $y_r(k + N_r) = y_r(k)$ and $N_r = 2T_f/h$.

Comparative results of the DMC without filter with $R = 10^{-2}$ and $R = 10^{-5}$ are presented in Figure 1. It is easy to observe the need for aggressive control tuning ($R = 10^{-5}$) to reduce reference tracking error. The issue with delay estimation error (delay of 0.3 s not considered within DMC formulation) and measurement noise (using Simulink block 10.2 - "Band Limited White Noise" - Power = 10^{-3} - Seed 1) is depicted in Figure 2. In this case, $T_s(t)$ is shown, but $T_\epsilon(t) = T_s(t) + \epsilon(t)$ represents the feedback quantity, where $\epsilon(t)$ denotes the measurement noise. As discussed within this work, aggressive tuning is more sensitive to uncertainties and high-frequency noise, which can be visualized through the input voltage ($v(t)$).

The role of FDMC in the context of time-varying references can be observed in Figures 3 and 4. The aggressive tuning was maintained ($R = 10^{-5}$), but the control signal increment is smoothed by including the filtered error. Thus, the benefits of aggressive tuning can be preserved. Two performance indices will be used to conduct a more detailed analysis as follows:

$$J_{e,sim} = \sum_{k=0}^{200} (y_r[k] - y[k])^2, \tag{3.15}$$

$$J_{\Delta v,sim} = \sum_{k=0}^{200} \Delta v[k]^2. \tag{3.16}$$

These indices were chosen since they are directly established in the optimization criterion of FDMC. When analyzing the results based on Table 1, it is important to note that the filtered DMC allowed for a reduction in the quadratic error ($J_{e,sim}$) and voltage variation ($J_{\Delta v,sim}$) compared to the unfiltered DMC. However, if the low-pass

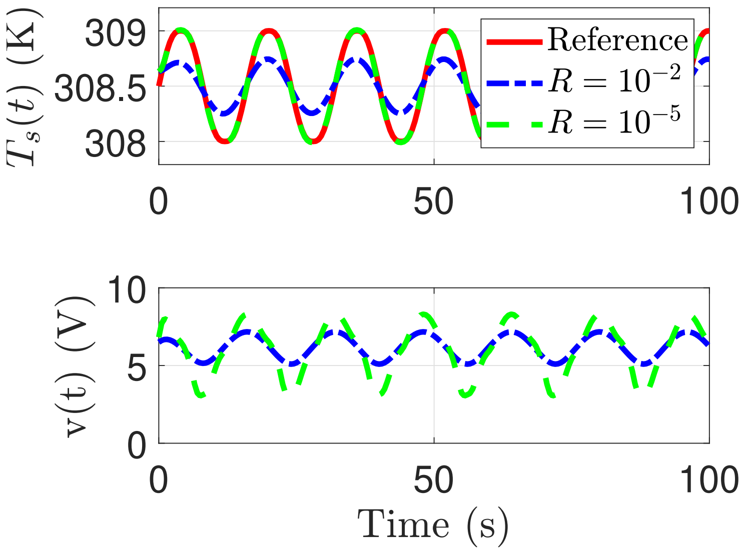

Figure 1 – Output and control signal response for different tuning of the DMC without both noise and modelling error.

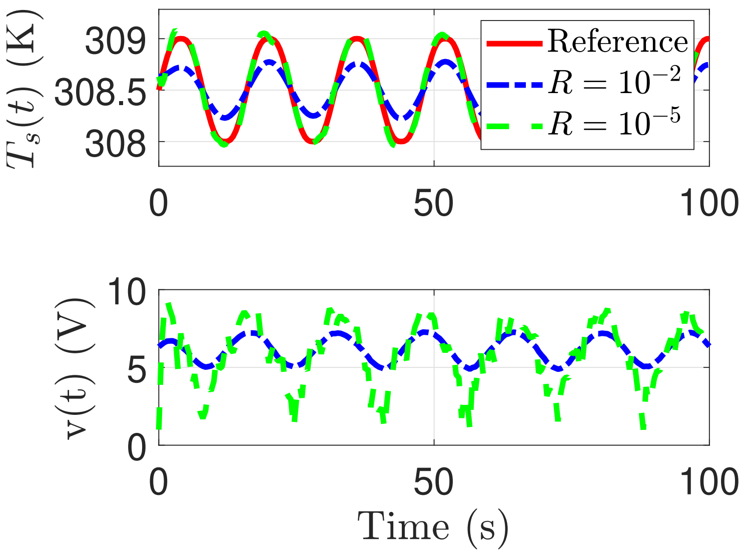

Figure 2 – Output and control signal response for different tuning of the DMC with noise and modelling error.

characteristic of the filter is too pronounced (i.e., the bandwidth is reduced), this tends to degrade regulatory performance. Moreover, modeling errors will be compensated for more slowly when compared to the original formulation. On the other hand, it is relevant to highlight that tuning $\alpha = 0.8$ reduced voltage variation, which may be desirable in certain applications. This result illustrates the balance between regulatory performance and noise attenuation capacity of the proposed controller.

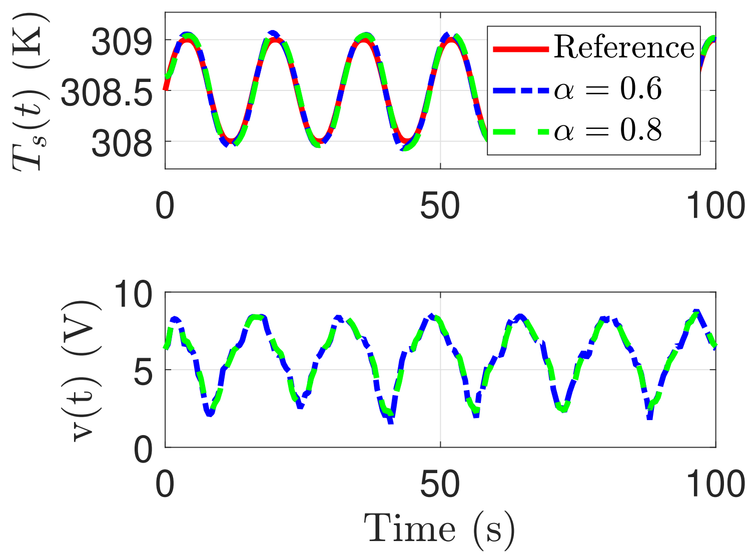

Figure 3 – Output and control signal response for different tuning of the FDMC with noise and modelling error ($R = 10^{-5}$).

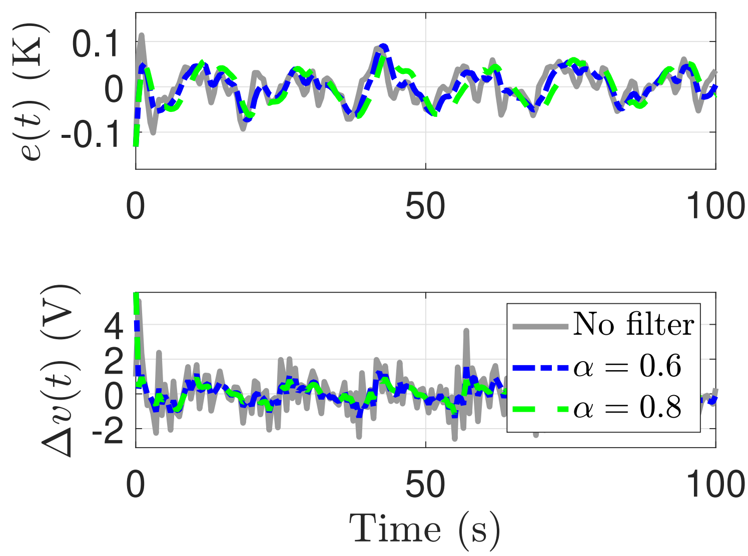

Figure 4 – Error signals and control increment with distinct filter configuration ($R = 10^{-5}$).

3.3.1 Experimental results

The experimental case will be used to compare DMC and FDMC with the previous tunings ($R = 10^{-5}$ and $\alpha = 0.6$) for tracking purpose by varying reference using only the convolutional model. Similar systems have already been experimentally controlled with DMC in works related to fault detection with constant reference (JUNIOR et al., 2013). The experimental setup with the NTC sensor and the voltage-regulated current source is

Table 1 – Performance indices.

Index	Unfiltered	$\alpha = 0.6$	$\alpha = 0.8$
$J_{e,sim}$	0,27	0,24*	0,29
$J_{\Delta v,sim}$	257,1	98,9	80,6*
$J_{e,sim} + J_{\Delta v,sim}$	257,37	99,14	80,89

presented in Figure 5.

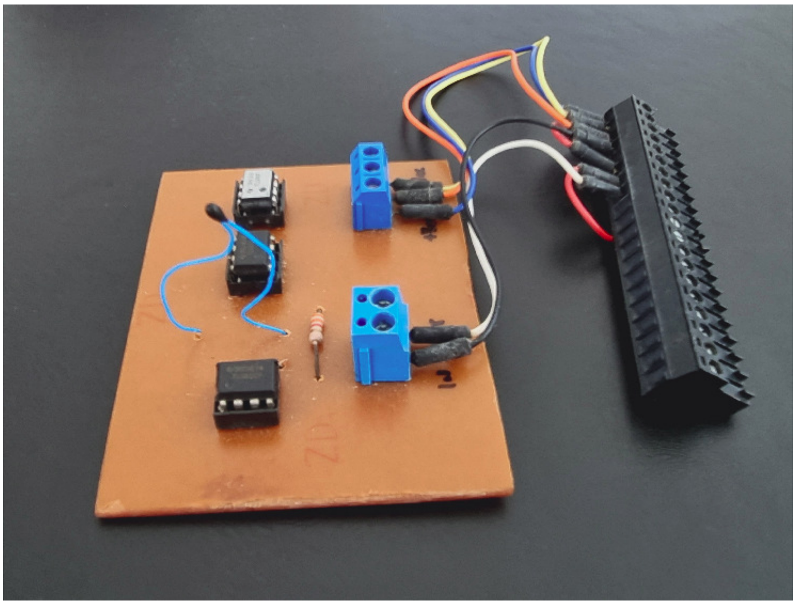

Figure 5 – Experimental control framework for NTC temperature sensor.

To attenuate high-frequency noises without modifying its phase signal, the experimental step response data was filtered by means of an non-causal moving average system of 5 samples

$$y_f[k] = \frac{y[k-2] + y[k-1] + y[k] + y[k+1] + y[k+2]}{5}. \tag{3.17}$$

The estimated coefficients used to implement DMC and FDMC are presented in Figure 6.

Starting from the convolutional model, the experimental results presented in Figure 7 are observed. In these cases, closed-loop control begins after 10 seconds, when the operation mode switches from manual to automatic. It is worth noting that the proposed DMC tuning does not allow for continuous operation due to a process of error amplification from modeling and high-frequency noise. On the other hand, FDMC exhibits a closed-loop experimental behavior that closely resembles the simulated results. The experimental results reinforce the theoretical discussions and observations from simulated cases. The significant advantage of the experimental outcome arises from the natural occurrence

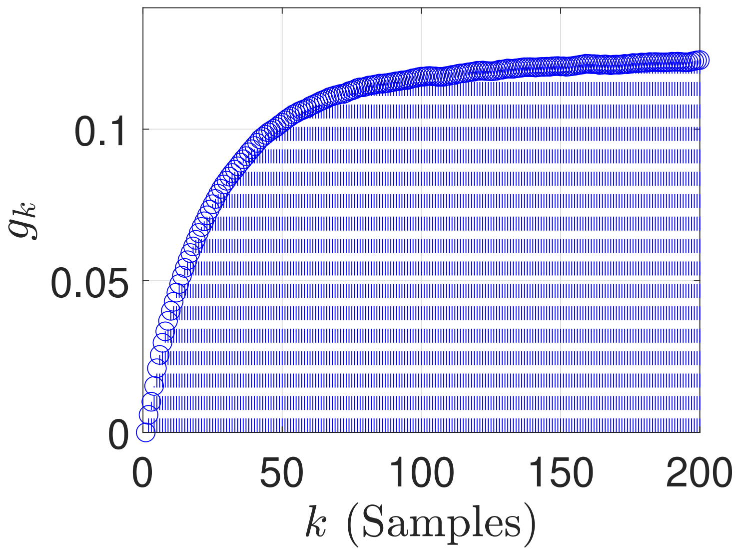

Figure 6 – Estimated coefficients from the step response for the convolutional model.

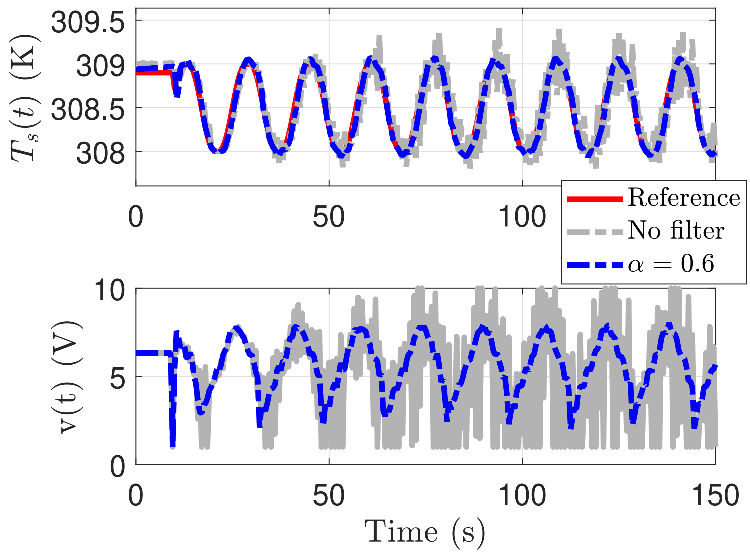

Figure 7 – Experimental results with comparative responses between non-filtered DMC ($\alpha = 0$) and filtered DMC ($\alpha = 0.6$).

of noise effects, modeling errors, and external disturbances during the step response identification, as also observed during the control testing phases.

3.3.2 Conclusion

This chapter provided the discussion regarding the balance between tracking properties of the DMC and robustness compromise, depending on the aggressiveness of the MPC tuning. The filtered FDMC was evaluated as a model predictive control algorithm for tracking time-varying references. The main contribution lies in the utilization of the prediction error filter to balance tunings capable of handling variant references and ensuring sufficient robustness to deal with internal and measurement noise. The role of the prediction error filter is analyzed from the perspective of unconstrained active set solution. The theoretical evaluation is illustrated through simulated and experimental results. Yet, no analysis for unstable cases where evaluated, which poses a potential challenge for open work of this thesis.

Despite the interesting results provided by high-gain controllers in the presence of non-constant references, the tracking performance is sensitive to the design parameters. Furthermore, the tracking objective is achieved if the bandwidth is large enough in comparison to the reference signal. The drawback is the reduced robustness margin which may not be acceptable depending on the control problem. In the next chapter, an alternative solution is proposed to reduce the tracking performance sensitivity in terms of the design parameters.

4 Receptance-based MPC with time-varying references

The analysis and results contained in this chapter are based on the paper published in (PEREIRA; SANTOS; ARAÚJO, 2024), which presents a receptance-based MPC strategy and a modification of the standard cost function to improve reference tracking properties. The receptance is a transfer matrix widely used for second-order MIMO systems and can be obtained experimentally (RAM; MOTTERSHEAD, 2007).

The MPC has been widely used in industrial applications due to its ability to deal with multivariable systems, compensate time delays naturally, handle constraints and even impose performance optimization based on multiple objectives (QIN; BADGWELL, 2003). To illustrate its practical motivation and relevance nowadays, several mechatronic control problems have been illustrated in Chapter 1. Several advantages have been reported by using this set of optimal controllers, such as the ability to deal with non-minimum phase, under-actuated models, and even with systems that has future references disposed to the control designer.

In terms of the tracking problem of the MPC, a trajectory tracking MPC has been proposed for an under-actuated two-link multibody system in (BETTEGA; RICHIEDEI, 2023), which includes an embedded dynamic reference prediction that can be computed from a pair of position and speed or a desired triplet of position, velocity and acceleration. The results from experimental and simulation frameworks have shown the effectiveness of the proposed strategy with periodic reference by considering a linearized second-order model as basis of an augmented state-space model where mass, damping and stiffness must be known, which is not always trivial to determine such parametric model. Thereby, the receptance modelling is a successful alternative for that since it does not require explicit knowledge of the model parameters. The strategy has been early proposed by (MOTTERSHEAD; RAM, 2006; RAM; MOTTERSHEAD, 2007) and then used in relevant applications for active vibration control (TEHRANI; MOTTERSHEAD, 2012; XIANG; ZHEN; LI, 2016). The receptance method presents a natural lower order in its model due to the choice of fewer natural frequencies of interest and the possibility of a model identification framework that is entirely experimental and data-oriented. Therefore, the receptance modelling can be directly used to obtain a system description, and then a suitable prediction model for the DMC and GPC strategies.

This section provides receptance-based MPC controllers for trajectory tracking of a multi-body system, in which its modelling parameters (mass, damping, and stiffness) are not required. This way, the GPC autoregressive model and DMC model explained in Annex A are directly computed from the receptance transfer matrix. In terms of the tracking of the system in a time-varying reference, the limitation of the nominal zero delay during the tracking is analyzed and a modified objective function is proposed to improve the sensitivity of the controller gains in terms of the reference tracking property.

4.1 Problem Statement for multibody control systems

For the model to be used in this chapter, consider a multibody, externally controlled dynamical system (BETTEGA; RICHIEDEI, 2023) defined by

$$\mathbf{M}_\theta(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}}(t) + \mathbf{C}_\theta\dot{\boldsymbol{\theta}}(t) + \mathbf{K}_\theta\boldsymbol{\theta}(t) + \mathbf{d}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \mathbf{g}(\boldsymbol{\theta}) = \mathbf{B}\mathbf{u}(t), \tag{4.1}$$

where $\mathbf{M}_\theta(\boldsymbol{\theta}) \in \mathbb{R}^{n\times n}$, $\mathbf{C}_\theta \in \mathbb{R}^{n\times n}$, and $\mathbf{K}_\theta \in \mathbb{R}^{n\times n}$ are the mass, damping and stiffness matrices, respectively, $\boldsymbol{\theta}(t) \in \mathbb{R}^n$ represents the displacement vector, $\mathbf{d}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \in \mathbb{R}^n$ describes the gyroscopic and centrifugal effects, $\mathbf{g}(\boldsymbol{\theta})$ determines the gravitational forces effect with respect to the second-order multibody model, $\mathbf{u}(t) \in \mathbb{R}^m$ is the vector of input forces, and $\mathbf{B} \in \mathbb{R}^{n\times m}$ defines the control force distribution.

For an equilibrium point given by $\boldsymbol{\theta}_e$, the multibody system can be represented around this equilibrium from a Taylor expansion first-order approximation as follows

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{B}\mathbf{u}(t) + \mathbf{q}(t), \tag{4.2}$$

where $\mathbf{x}(t) = \boldsymbol{\theta}(t) - \theta_e$ is a translated displacement vector, $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K} \in \mathbb{R}^{n\times n}$, are respectively mass/inertia, damping and stiffness matrices of the linearized model, and $\mathbf{q}(t) \in \mathbb{R}^n$ describes external disturbances and the modeling error due to linearization. In most applications such as structural vibrations and multibody dynamics, the triplet $\mathbf{M}, \mathbf{C}, \mathbf{K}$ is symmetric; moreover, it can also be positive (semi) definite (SCIAVICCO; SICILIANO, 2012). It is well known that the case in which $m < n$ is the underactuated system. This case is the most frequent and challenging, for example, due to a cost requirement or physical limitations for actuator placement.

The system receptance can be obtained from (4.2) by recurring to the unilateral Laplace Transform, resulting in

$$\mathbf{X}(s) = \mathbf{H}(s)\mathbf{B}\mathbf{U}(s) + \mathbf{H}(s)\mathbf{Q}(s), \tag{4.3}$$

in which $\mathbf{H}(s) = (\mathbf{M}s^2 + \mathbf{C}s + \mathbf{K})^{-1}$ is the system receptance. In practice, $\mathbf{H}(s)\mathbf{B}$ is obtained experimentally such that the individual values of $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are not necessary (ARAÚJO; SANTOS, 2018; ARAÚJO; SANTOS, 2020; FRANKLIN; ARAÚJO; SANTOS, 2021).

In general, the external disturbance are unknown and unmeasured such that $\mathbf{Q}(s)$ cannot be determined. Hence, the prediction model may be replaced by

$$\mathbf{X}(s) = \mathbf{H}(s)\mathbf{B}\mathbf{U}(s) + \mathbf{H}_\mathbf{q}(s)\mathbf{E}(s), \tag{4.4}$$

where $\mathbf{E}(s)$ is also unknown, but $\mathbf{H_q}(s)$ is a modified disturbance model that can be chosen as a tuning parameter. Actually, $\mathbf{H_q}(s)$ can be designed to achieve a regulatory specification (ÅSTRÖM; WITTENMARK, 2013, Chapter 10) without the exact knowledge of $\mathbf{H}(s)$ as $\mathbf{H_q}(s)$ is defined from the required steady-state performance.

4.2 Receptance-Based MPC with future reference knowledge

This section presents the condition to achieve the tracking of a position reference. Then, a modified strategy to reduce the tracking error sensitivity with respect to the design parameters is proposed. As initial statement, consider the optimization problem in (A.35a)-(A.35d) and (A.61a)-(A.61d) for both standard DMC and GPC strategies. The vector of predicted outputs $\mathcal{Y}_k = \mathcal{Y}_k^{free} + \bar{\mathbf{G}}\Delta\mathbf{u}[k]$ is simply the composition of the free and forced response, such as its free response and prediction parameters are defined for the DMC and GPC as in (A.33) and (A.51) respectively.

For a given periodic output trajectory sequence $\mathcal{W}_k = \{y_r[k|k],\ y_r[k+1|k],\ ...y_r[k+N_{end}|k]\}$ with period N_r, such as $y_r[k] = y_r[k+N_r]$, a standard MPC control law has to achieve three necessary conditions to achieve null tracking error in steady-state:

1. null prediction errors in steady state to avoid issues caused by prediction errors;
2. the constraint should not be active in such condition, which means that the unconstrained solution is equivalent to the optimal constrained solution. This guarantees that the sequence of desired target can be reached despite input and output constraints;
3. the cost function value in steady state should be null, which is the minimum achievable value of (A.35a) and (A.61a), which ensures that the optimal solution drives the output to the desired target.

In practice, condition 1. is not exactly ensured in the presence of time-varying targets due to the uncertainties, but the modeling error may be mitigated by using good descriptions of the dynamic system. Condition 2. is respected if the reference does not push the system to achieve the neighborhood of the constraints in steady-state. Finally, the condition 3. is commonly ensured in the presence of constant references, but the general case with time-varying references deserves a careful attention.

A simple and useful way to achieve an almost ideal tracking with the standard MPC (A.34a) is to use $N = N_u$ and $\lambda_{\min}\{\mathcal{Q}\} \gg \lambda_{\max}\{\mathcal{R}\}$, where $\lambda_{\min}$ and $\lambda_{\max}$ denotes respectively the minimum and the maximum singular value of a matrix. If $\|\hat{y}[k+j|k] - y_r[k+j]\|_Q$ is significantly greater than $\|\Delta u[k+j|k]\|_R^2$, then the optimal solution is mainly driven to reduce the output tracking error. However, this approach is sensitive to the

MPC parameters as depends on tuning definitions, which may cause problems in practice as reduced robustness margins. Another promising alternative is to use a modified MPC using the new cost function $J_m(\Delta\mathbf{u}_k; \mathcal{W}_k, \Delta\mathcal{V}_k, \mathcal{Y}_k^{\text{free}})$ which is given by

$$\begin{aligned}
\underset{\Delta\mathbf{u}}{\text{minimize}} \quad & \sum_{j=1}^{N} \|y_r[k+j] - \hat{y}[k+j|k]\|_Q^2 + \sum_{j=0}^{N_u-1} \|\Delta u[k+j|k] - \Delta\boldsymbol{\nu}[k+j|k]\|_R^2, && (4.5a)\\
\text{subject to} \quad & \hat{y}[k+j|k] \in \mathbb{Y}, \qquad j \in \mathbb{N}_{[1,N]}, && (4.5b)\\
& u[k+j|k] \in \mathbb{U}, \qquad j \in \mathbb{N}_{[0,N_u-1]}, && (4.5c)\\
& \Delta u[k+j|k] \in \Delta\mathbb{U}, \qquad j \in \mathbb{N}_{[0,N_u-1]}, && (4.5d)\\
& \Delta\mathbf{u}[k+j|k] = \Delta\boldsymbol{\nu}_{k+j|k}, \quad j \in \mathbb{N}_{[N_u,N-1]}, && (4.5e)
\end{aligned}$$

where $\Delta\mathcal{V}_k = [\Delta\boldsymbol{\nu}[k|k]^\top \ ... \ \Delta\boldsymbol{\nu}[k+N-1|k]^\top]^\top$ is a target for the future control increments that nominally tracks the desired target $\mathcal{W}_k$ in steady-state. The partial cost defined only by $\|\Delta\mathbf{u}[k+j|k]\|_R^2$ would demand an optimal solution given by $\Delta\mathbf{u}[k+j|k] = \mathbf{0}_{m,1}$, which provides an equilibrium condition. On the other hand, $\|y_r[k+j] - \hat{y}[k+j|k]\|_Q^2$ demands a non-stationary steady-state solution in which the optimal value tends to zero as $k \to \infty$. Indeed, these are conflicting objectives in steady-state. However, if $\Delta\mathbf{u}[k+j|k] = \Delta\boldsymbol{\nu}[k+j|k]$ provides the desired evolution ($y_r[k+j] = \hat{y}[k+j|k]$), then the modified cost function in (4.5a) has non-conflicting elements in steady-state such that $\lim_{k\to\infty} J_m(\Delta\mathbf{u}_k; \mathcal{W}_k, \Delta\mathcal{V}_k, \mathcal{Y}_k^{\text{free}}) = 0$ may be reached.

The optimization problem is similar to the previous case, but the control increment target should be provided previously in order to define a non-conflicting objective in steady-state. Once more, the unconstrained case can be solved from the minimization of the modified cost function, i.e. $J_m(\Delta\mathbf{u}_k; \mathcal{W}_k, \Delta\mathcal{V}_k, \mathcal{Y}_k^{\text{free}}) = (\mathcal{W}_k - \mathcal{Y}_k)^\top \mathcal{Q}(\mathcal{W}_k - \mathcal{Y}_k) + (\Delta\mathbf{u}_k - \Delta\mathcal{V}_k)^\top \mathcal{R}(\Delta\mathbf{u}_k - \Delta\mathcal{V}_k)$ and the optimal explicit (unconstrained) solution is given by

$$\Delta\mathbf{u}_k = (\bar{\mathbf{G}}^\top \mathcal{Q}\bar{\mathbf{G}} + \mathcal{R})^{-1}[\bar{\mathbf{G}}^\top \mathcal{Q}(\mathcal{W}_k - \mathcal{Y}_k^{\text{free}}) + \mathcal{R}\Delta\mathcal{V}_k]. \tag{4.6}$$

In which the term $\bar{\mathbf{G}}$ is computed as follows in (A.33) and (A.51) for DMC and GPC strategies respectively. In this modified strategy, a feed-forward term is naturally included ($\bar{\mathbf{G}}^\top \mathcal{Q}(\mathcal{W}_k - \mathcal{Y}_k^{\text{free}}) + \mathcal{R}\Delta\mathcal{V}_k$) to track the desired position due to the optimality of the proposed solution. This additional implicit and optimal feedforward term provides the tracking property of the modified MPC. The main benefit is to reduce the sensitivity of the tracking error with respect to the design parameters.

However, a concern arises from the computation of $\Delta\mathcal{V}_k$ from a given desired trajectory $\mathcal{W}_k$. The integer N_{end} describes the position checkpoints of the complete trajectory. The trajectory is assumed to be periodic with period N_{end} such that a soft terminal boundary condition is implicitly imposed from this assumption. Indeed, the periodic assumption is just imposed to simplify the definition of the control increment target ($\Delta\mathcal{V}_k$), but this approach is not unique.

The objective of this approach is to define $\Delta\mathcal{V}_k^{\text{aux}} = [\Delta\nu^\top[k-n_b|k]\ \Delta\nu^\top[k-n_b+1|k]\ ...\ \Delta\nu^\top[k+N_{end}-1|k]]^\top$ from $\mathcal{W}_k$ analytically. This problem can be handled with different linear models (state space, convolutional model, transfer function). In this work, any minimal state-space realization $(\Phi, \Gamma, \Psi, \mathbf{0})$ for $\mathbf{P}_\mathcal{C}(z)\frac{z}{z-1}$ can be used such that $\mathbf{P}_\mathcal{C}(z)\frac{z}{z-1} = \Psi(z\mathbf{I}-\Phi)^{-1}\Gamma$ where $\Phi \in \mathbb{R}^{n_\xi \times n_\xi}$, $\Gamma \in \mathbb{R}^{n_\xi \times m}$, and $\Psi \in \mathbb{R}^{n_y \times n_\xi}$ with $n_\xi = n+m$ due to the integral action (control increment).

The control increment target can be obtained from the following unconstrained optimization problem

$$\underset{\Delta\mathcal{V}^{\text{aux}}, \xi}{\text{minimize}} \quad \sum_{j=1}^{N_{end}} \|y_r[k+j] - \mathbf{y}^{\text{aux}}[k+j|k]\|^2 + \|\boldsymbol{\xi}[k|k] - \boldsymbol{\xi}[k+N|k]\|^2, \tag{4.7a}$$

$$\text{subject to} \quad \boldsymbol{\xi}[k+j+1|k] = \Phi\boldsymbol{\xi}[k+j|k] + \Gamma\Delta\boldsymbol{\nu}[k+j|k], \quad j \in \mathbb{N}_{[1,N_{end}-1]}, \tag{4.7b}$$

$$\mathbf{y}^{\text{aux}}[k+j|k] = \Psi\boldsymbol{\xi}[k+j|k], \quad j \in \mathbb{N}_{[1,N_{end}]}, \tag{4.7c}$$

where $\Delta\mathcal{V}^{\text{aux}}$, and $\boldsymbol{\xi}[k|k]$ are free decision variables. The minimization of the partial cost expressed by $\|\boldsymbol{\xi}[k|k] - \boldsymbol{\xi}[k+N|k]\|^2$ can be interpreted as a soft contour condition for periodic trajectories which is an interesting and simple approach from a numerical perspective. The analytical solution is given by

$$[\boldsymbol{\xi}[k|k]^\top\ \Delta\mathcal{V}^{\text{aux}\top}]^\top = \Omega^\dagger\boldsymbol{\theta}_k, \tag{4.8}$$

where † denotes the Moore-Penrose pseudo inverse, $\boldsymbol{\theta}_k = [\mathbf{r}[k]^\top\ ...\ \mathbf{r}[k+N_{end}]^\top\ \mathbf{0}_{n_\xi,1}^\top]^\top$ and

$$\Omega = \begin{bmatrix} \Psi\Phi & \Psi\Gamma & \mathbf{0}_{n_y,m} & ... & \mathbf{0}_{n_y,m} \\ \Psi\Phi^2 & \Psi\Phi\Gamma & \Psi\Gamma & ... & \mathbf{0}_{n_y,m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Psi\Phi^{N_{end}} & \Psi\Phi^{N_{end}-1}\Gamma & \Psi\Phi^{N_{end}-2}\Gamma & ... & \Psi\Gamma \\ \Phi^{N_{end}} - \mathbf{I}_{n_\xi,n_\xi} & \Phi^{N_{end}-1}\Gamma & \Phi^{N_{end}-2}\Gamma & ... & \Gamma \end{bmatrix}. \tag{4.9}$$

As $\Omega^\dagger$ is computed offline, the desired steady-state control increment is obtained only by implementing of a direct matrix multiplication. A terminal equality constraint can be alternatively used as boundary condition, but the optimal solution is not so straightforward. It is assumed that the control increment target provides an admissible solution in steady-state which depends on the trajectory planning layer. This result allows the optimal implicit feedforward action in order to reduce the sensitivity of the tracking error with respect to the design parameters. It should be remarked that any minimal state space realization can be used in this approach as $\xi[k|k]$ is an artificial variable with no experimental meaning as the input-output relations are preserved with respect to the desired trajectory and only the receptance information is required. Notice that this approach was developed to be used in the receptance-based MPC, but the state-space MPC may be designed with the same type of cost function modification.

A periodic reference was assumed thus it is possible to recover $\Delta\nu[k]$ from $y_r[k]$ from the solution of (4.7a)-(4.7c), but such premise is not necessarily needed. Another alternative for systems with minimum or non-minimum phase with known future reference is simply invert the virtual model used and recover $\nu[k]$ from $y_r[k]$ and calculate $\Delta\nu[k] = \nu[k] - \nu[k-1]$.

4.3 Numerical case study

The simulation case study has been already investigated in relevant related work (ARAÚJO et al., 2021; BETTEGA; RICHIEDEI, 2023) due to its interesting control challenges. Considering the potential of its practical application, the multibody system used to evaluate the receptance-based MPC is a two-link arm with revolute joints where first one (A) is actuated by a DC motor and the other one (B) is a passive joint with a torsional spring. This represents an under-actuated, non-square control problem which can evaluate a challenging and applicable issue to robotics applications.

Two absolute rotation angles ($\theta_1(t)$ and $\theta_2(t)$) define the degree of freedom ($n = 2$) while the torque ($T_m(t)$) is the only control action ($m = 1$) as illustrated in Fig. 8.

The control objective is to perform a pick-and-place task in a repetitive cycle moving of the tip point defined by the coordinate $w_{1,tip}$ from $w_{1,\text{pick}} = (l_1 + l_2)sin(20^o\pi/180^o)$ to $w_{1,\text{place}} = (l_1 + l_2)sin(-20^o\pi/180^o)$. The variation in $w_2(t)$ is neglected in this problem (BETTEGA; RICHIEDEI, 2023). As $w_1(t) = l_1 sin(\theta_1(t)) + l_2 sin(\theta_2(t))$, then $y(t) = w_1(t)$ is defined as the measured variable. The time interval between the $w_{1,\text{pick}}$ and $w_{1,\text{place}}$ is defined by T_{task}. The torque is assumed to be bounded by $\|T_m(t)\|_\infty \leq 0.40\ Nm$ as proposed in (BETTEGA; RICHIEDEI, 2023) where $u(t) = T_m(t)$.

The control problem is nonlinear, constrained, underactuated with a non-minimum phase description. For simulation purposes, the nonlinear model is given by

$$\begin{bmatrix} J_\ell + J_d + J_1 + m_{2,eq} l_1^2 & \frac{1}{2} m_2 l_1 l_2 \cos(\theta_1(t) - \theta_2(t)) \\ \frac{1}{2} m_2 l_1 l_2 \cos(\theta_1(t) - \theta_2(t)) & J_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1(t) \\ \ddot{\theta}_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{2} m_2 l_1 l_2 \sin(\theta_1(t) - \theta_2(t)) \dot{\theta}_2(t)^2 \\ -\frac{1}{2} m_2 l_1 l_2 \sin(\theta_1(t) - \theta_2(t)) \dot{\theta}_1(t)^2 \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1(t) \\ \dot{\theta}_2(t) \end{bmatrix} + \begin{bmatrix} \left(\frac{1}{2} m_1 + m_{2,eq}\right) g l_1 \sin(\theta_1(t)) \\ \frac{1}{2} m_2 g l_2 \sin(\theta_2(t)) \end{bmatrix} + \begin{bmatrix} k_s & -k_s \\ -k_s & k_s \end{bmatrix} \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} T_m(t), \tag{4.10}$$

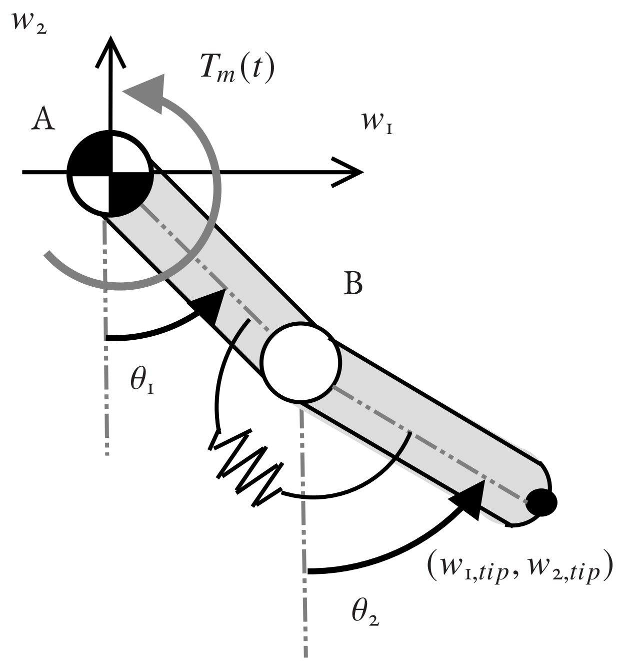

Figure 8 – Illustration of the two-link control problem.

where the parameters are presented in Table 2 from the similar problem seen in (BETTEGA; RICHIEDEI, 2023).

Table 2 – System parameters.

System Parameters	**Units**	**Values**
k_s	$\frac{\text{Nm}}{\text{rad}}$	0.1772
J_ℓ	kgm^2	2.70×10^{-5}
J_d	kgm^2	1.08×10^{-5}
m_1	kg	0.050
m_2	kg	0.021
l_1	m	0.170
l_2	m	0.155
J_1	kgm^2	4.82×10^{-4}
J_2	kgm^2	1.68×10^{-4}
m_{enc}	kg	0.100
m_b	kg	0.025
g	$\frac{\text{m}}{\text{s}^2}$	9.81
c_{11}	$\frac{\text{Nms}}{\text{rad}}$	1.20×10^{-2}
c_{12}	$\frac{\text{Nms}}{\text{rad}}$	4.00×10^{-4}
c_{21}	$\frac{\text{Nms}}{\text{rad}}$	4.00×10^{-4}
c_{22}	$\frac{\text{Nms}}{\text{rad}}$	3.00×10^{-4}

A linearized model around the equilibrium ($\theta_e = [0\ 0]^\top$) was used in (BETTEGA;

RICHIEDEI, 2023), but the knowledge of $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ is demanded. In this work, the receptance information can be directly applied such that $\mathbf{P}(s) = \mathbf{H}(s)\mathbf{B}$ is simply

$$\mathbf{P}(s) = \frac{1}{s^4 + 4.47s^3 + 1521s^2 + 3661s + 8.048 \cdot 10^4} \begin{bmatrix} 233.5s^2 + 416.9s + 2.684 \cdot 10^5 \\ \\ -384.5s^2 - 555.8s + 2.462 \cdot 10^5 \end{bmatrix}, \tag{4.11}$$

and $\mathbf{L} = [l_1 \; l_2]$.

A fifth-degree motion law is used to interpolate the pick and place points in order that the desired trajectory is define by the trajectory function $f(t)$.

$$f(t) = f_0 + f_3 t^3 + f_4 t^4 + f_5 t^5, \; 0 \leq t < T_{\text{task}}, \tag{4.12}$$

or simply for a periodic trajectory,

$$f(t) = f_0 + f_3(T_{\text{task}} - t)^3 + f_4(T_{\text{task}} - t)^4 + f_5(T_{\text{task}} - t)^5, \; T_{\text{task}} \leq t < 2T_{\text{task}}, \tag{4.13}$$

in which $f_0 = w_{1,\text{place}} - w_{1,\text{pick}}$, $f_3 = 10f_0/(T_{\text{task}}^3)$, $f_4 = -15f_0/(T_{\text{task}}^4)$, $f_5 = 6f_0/(T_{\text{task}}^5)$. The interpolation is a sampling procedure given by $r_k = f(kT_s)$, $0 \leq k < T_{\text{task}}/T_s$, $N_{end} = 2T_{\text{task}}/T_s$, and $r_{k+N_{end}} = r_k$.

Since the velocity measurement is not needed for this type of position control problem based on top of a receptance based MPC strategy, Euler's approximation for velocity estimation is not used such that the sampling period is defined as $T_s = 100 \; ms$ in this case instead of $T_s = 1 \; ms$ that was used in (BETTEGA; RICHIEDEI, 2023). This enlarged sampling interval alleviates the MPC computational time bounds and it also requires smaller prediction and control horizons as the effective horizon time interval is defined by $T_s N$. The MPC prediction and control horizons are $N = 15$ and $N_u = 5$ for the further analyses of this section, and $\tilde{N} = 100$ (DMC only). These values are typical prediction choices if the sampling period follows classical textbook rules (CAMACHO; ALBA, 2013). In order to illustrate the MPC challenges, two set of typical matrix weightings have been considered: design (i) - ($Q = 1, R = 1$) and design (ii) - ($Q = 1, R = 0.01$).

Moreover, two pick and place intervals are analyzed ($T_{\text{task}} = 1 \; s$ and $T_{\text{task}} = 3 \; s$) such that an enlarged sampling interval also reduces the number of samples for a trajectory cycle ($N_{end} = 20$ and $N_{end} = 60$). All simulations are performed using Matlab's Simulink with the nonlinear model and a quantization error due to an optical encoder is also considered for each angle $\theta_i(t)$ by assuming $N_{ppr} = 500$ (consider the quantization interval as $2\pi/N_{ppr} \; s$) (BETTEGA; RICHIEDEI, 2023). In order to clarify the novelties of the modified strategy, the simulations are organized in two subsections, the standard MPC cases using the GPC and DMC proposed in Annex A and the modified MPC cases proposed into this chapter.

4.3.1 Standard MPC

Firstly, $T_{\text{task}} = 1\ s$ is considered with the standard GPC and DMC. Both designs (i) and (ii) are investigated. Figure 9 shows the output responses and the tracking error of the nonlinear system controlled with the GPC. This result illustrates that the tracking error is very sensitive to the design parameters. If the pair $(Q = 1, R = 0.01)$ is used, a reduced steady state tracking error is verified despite the nonlinear effect and the encoder quantization error. However, the design defined by $(Q = 1, R = 1)$ generates a significant steady state error. Notice that $\sum_{j=1}^{N} \|\mathbf{r}[k+j] - \hat{y}[k+j|k]\|_Q^2$ demands a tracking error minimization while $\sum_{j=0}^{N_u-1} \|\Delta u[k+j|k] - \Delta\boldsymbol{\nu}[k+j|k]\|_{Q_u}^2$ imposes the minimization of the control increment (torque variation). When $Q = 1$ and $R = 0.01$, then the tracking error almost dominates the optimal solution as the control increment impact is significantly small. As expected, the receptance-based GPC can be used to perform the pick and place task, but the tracking error is really sensitive to the MPC design parameters. Similar results are observed with the DMC as indicates in Figure 10. Indeed, as the modeling error is not significant in this angle ranges, the DMC and the GPC present almost identical closed-loop responses. In all scenarios with the modified receptance-based MPC, the maximum absolute error is smaller than 2.5 mm and the computation time of the control law is smaller than 10 ms which indicates the feasibility of the proposed strategy.

4.3.2 Modified MPC

The sensitivity of the tracking error may be significantly reduced by avoiding the conflicting objectives with the new cost function. Therefore, simulated results for the modified GPC and DMC responses are depicted in Fig. 11. As expected, the proposed modified receptance-based GPC and DMC strategies were able to achieve a reduced steady-state tracking error despite a conservative design $(Q = 1, R = 1)$. A small transient error is observed just after the pick and place task initialization due to the conservative design, but the steady-state tracking error sensitivity is significantly reduced.

The performance was improved in steady-state because tracking error objective and control increment costs are aligned. The control signals are depicted in Figure 12. Notice that GPC/DMC with $(Q = 1, R = 0.01)$ and the modified GPC/DMC with $(Q = 1, R = 1)$ presents similar control action responses in steady-state. Furthermore, the reference of the torque increment and the torque increment responses are presented in Figure 13. The standard GPC/DMC penalizes the torque increment signal in terms of the prediction error based on the value weighting factors Q and R deals with a torque increment minimization while the proposed strategy penalizes the difference between the torque increment $(\Delta\mathbf{u}[k])$

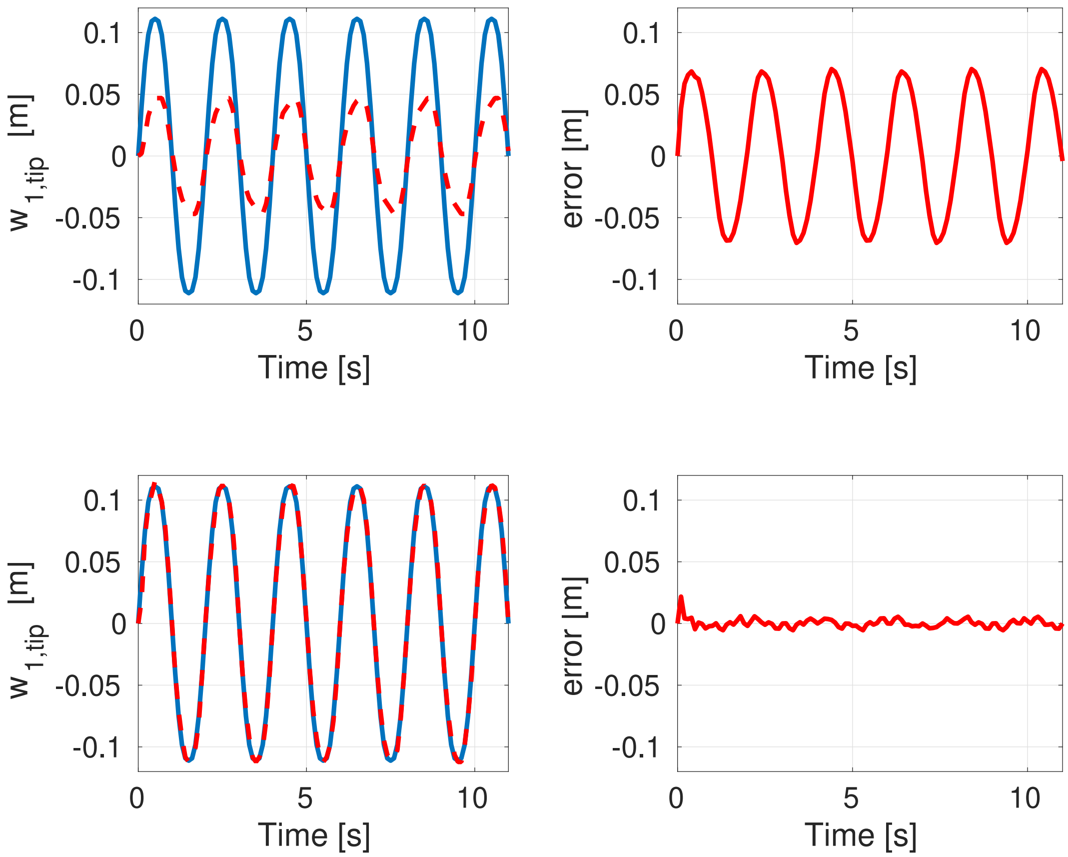

Figure 9 – Standard **GPC** comparison of the tracking error with alternative design parameters: top figures - $(Q = 1, R = 1)$ and bottom figures - $(Q = 1, R = 0.01)$. Output responses $(w_{1,\text{tip}})$: solid lines represent the trajectory target and dashed lines describe the output responses.

and its new reference $(\Delta\mathcal{V}[k])$, which is a non-conflicting objective, having its minimum at (or close to) zero. The main result of this analysis can be observed from the evolution of the cost function as shown in Fig. 14. Due to the absence of conflicts in steady-state $\lim_{k\to\infty} J_m(\Delta\mathbf{u}_k; \mathcal{W}_k, \Delta\mathcal{V}_k, \mathcal{Y}_k^{\text{free}}) \approx 0$, which does not happen in the standard GPC/DMC strategies. Hence, the proper choice of the cost function is a relevant aspect of tracking MPC for multibody systems with time-varying references.

To illustrate that the steady-state tracking error sensitivity is also reduced for other task period, the design defined by $(Q = 1, R = 1)$ was also used with $T_{\text{task}} = 3\ s$ as considered in (BETTEGA; RICHIEDEI, 2023). The results with $T_{\text{task}} = 3\ s$ are shown in Figure 15. Notice that the initial transient effect and the nonlinear modeling error in steady-state were reduced as the pick and place task is slower in this case. Anyway, the overall conclusions with respect to the steady-state responses are exactly the same as the previous case with $T_{\text{task}} = 1\ s$. Details of the steady-state tracking errors are presented in Figure 16. For the case with $T_{\text{task}} = 1\ s$ the Root-Means Square (RMS) errors are 1.18 mm (GPC) and 1.15 mm (DMC) while the case with $T_{\text{task}} = 3\ s$ the modified receptance-based

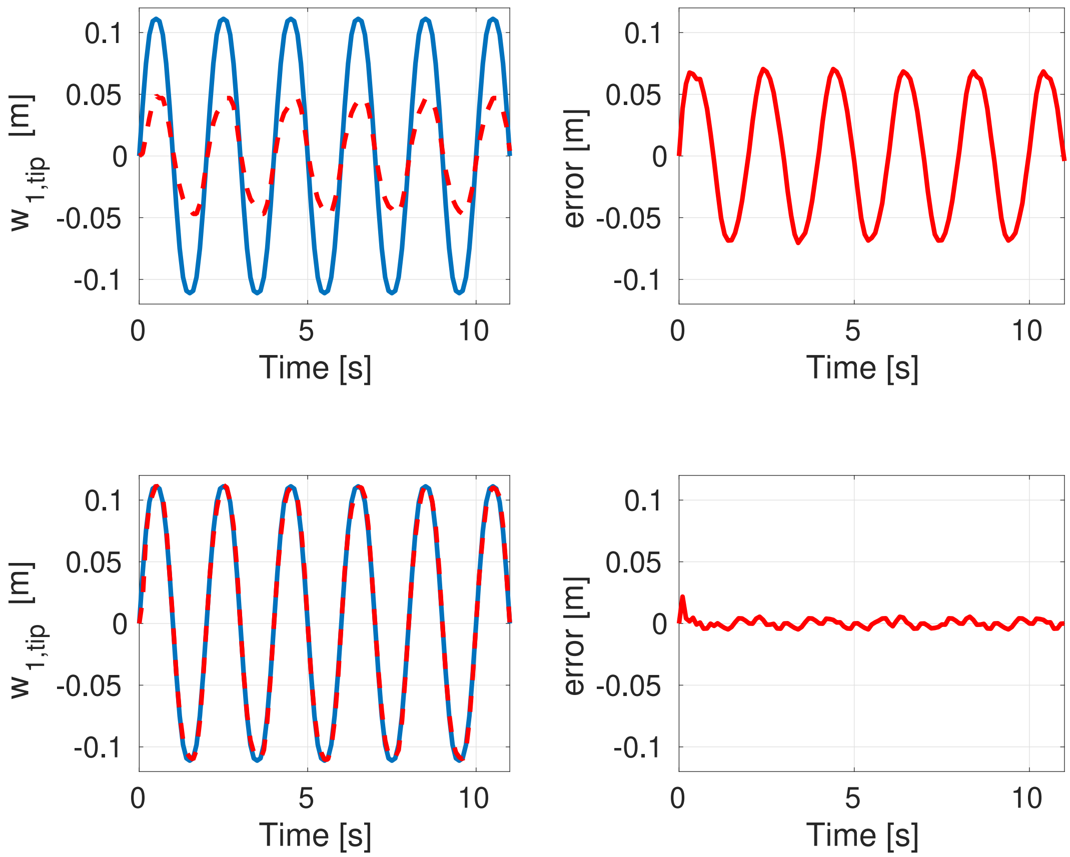

Figure 10 – Standard **DMC** comparison of the tracking error with alternative design parameters: top figures - $(Q = 1, R = 1)$ and bottom figures - $(Q = 1, R = 0.01)$. Output responses $(w_{1,\text{tip}})$: solid lines represent the trajectory target and dashed lines describe the output responses.

strategies provided 1.10 mm (GPC) and 1.10 mm (DMC) RMS errors.

4.3.3 Discussions

4.3.3.1 Disturbance Rejection

The disturbance rejection property of the proposed MPC is observed in Figure 17. The difference between GPC and DMC strategies is clear in such results, mainly because the distinct disturbance models of both strategies. Whilst DMC uses a constant output disturbance model, the GPC approach uses the CARIMA model, which generates different behaviors when the system is subject to a constant input step disturbance at time $t = 5s$, maintaining the reference tracking property. In this case, it is clearly seen the impact of the input disturbance in the control signal for both standard methods.

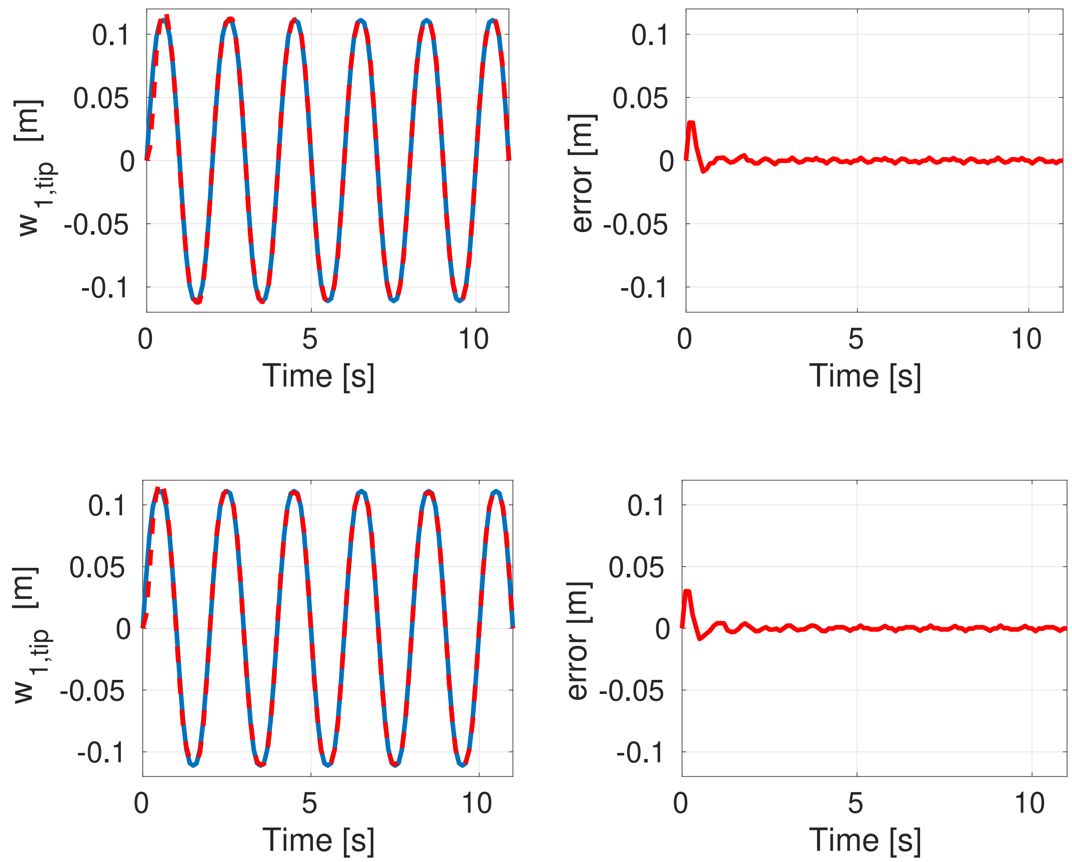

Figure 11 – Modified **GPC** and **DMC** comparison with $(Q = 1, R = 1)$. Top figure - **GPC**, bottom figure - **DMC**. Output responses $(w_{1,\text{tip}})$: solid lines represent the trajectory target and dashed lines describe the output responses.

4.3.3.2 Standard GPC with double integrator

A slight modification could be done in the GPC formulation in terms of the disturbance model originally proposed in Appendix A.3. For that, it should be considered the following input-output prediction model

$$\tilde{\tilde{\mathbf{A}}}(z^{-1})\mathbf{Y}(z) = (1 - z^{-1})\tilde{\mathbf{A}}(z^{-1})\mathbf{Y}(z) = \mathbf{B}(z^{-1})\Delta^2\mathbf{U}(z) + \mathbf{E}(z), \tag{4.14}$$

where $\Delta^2 = (1 - z^{-1} - z^{-2})$ is the double shift operator. As a matter of illustration, if the system is SISO, the variables of decision are based on future values of $\Delta^2 u[k] = u[k] - u[k-1] - u[k-2]$, which means that a double integrator is added to the GPC solution. Therefore the standard cost function with double integrator $J_\Delta(\Delta^2\mathbf{u}_k; \mathcal{W}_k, \mathcal{Y}_k^{\text{free}})$ to be optimized is simply

$$J_\Delta(\Delta^2\mathbf{u}_k; \mathcal{W}_k, \mathcal{Y}_k^{\text{free}}) = \sum_{j=1}^{N_{end}} \|y_r[k+j] - \hat{y}[k+j|k]\|_Q^2 + \sum_{j=0}^{N_u-1} \|\Delta^2 u[k+j|k]\|_R^2. \tag{4.15}$$

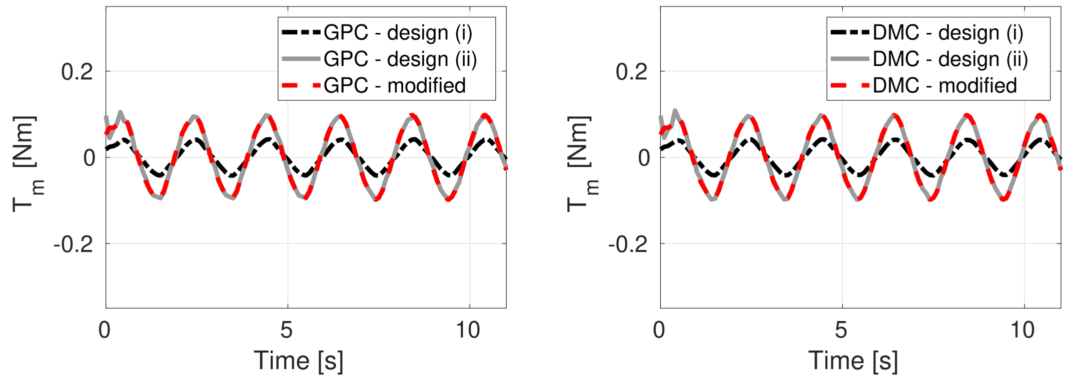

Figure 12 – Torque comparison in several combinations. Design (i) - ($Q = 1, R = 1$) and Design (ii) - ($Q = 1, R = 0.01$).

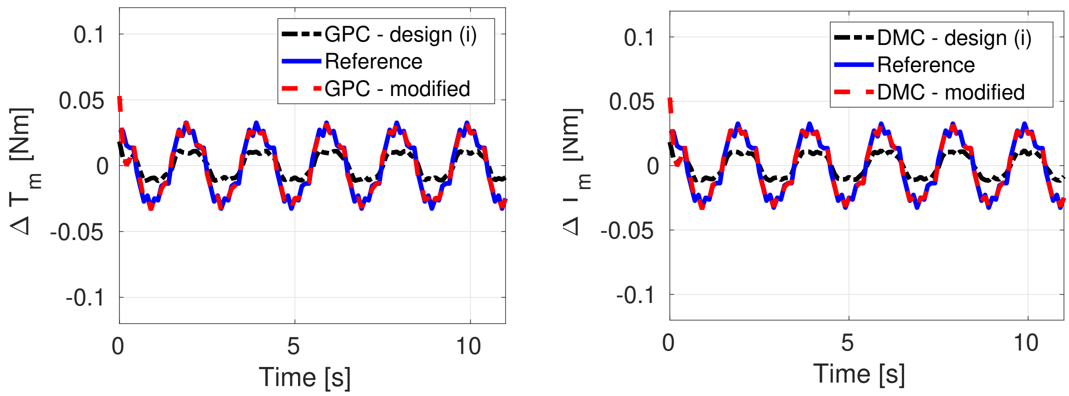

Figure 13 – Torque comparison in several combinations. Design (i) - ($Q = 1, R = 1$) and Design (ii) - ($Q = 1, R = 0.01$).

The results of the controlled system using the GPC with a ramp disturbance model is shown in Figure 18 without the addition of the virtual reference, considering the same conservative design, i.e $Q = 1$, and $R = 1$. Only the proposed modified GPC with a ramp signal as a model disturbance has been evaluated to verify the tracking properties of the solution.

As expected, the double integrator improves the tracking properties when compared to the standard approach with the same conservative tuning (see Figure 9), since it provides an increasing gain due to the additional integrator. However, error evolution shows that it not fully solves the tracking problem since the contradiction on equilibrium cost still holds.

4.3.3.3 Main results

The standard strategies such as GPC, DMC and State Space can be effectively used to control multibody systems if the design parameters are suitable. However, aggressive control tuning may impose practical problems such as reduced robustness margins. It

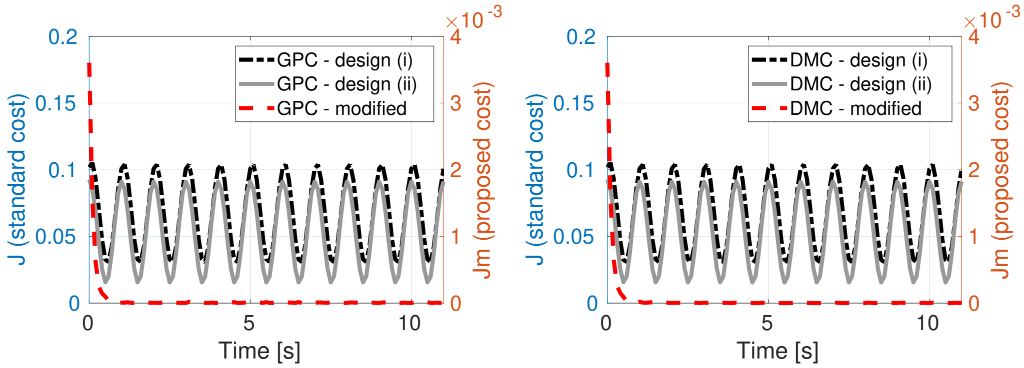

Figure 14 – Cost function comparison is several combinations. J stands for the standard GPC/DMC costs while J_m represent the costs of the proposed strategies (modified GPC/DMC).

should be highlighted that GPC and DMC are computed from input-output data with input-output models. Hence, a receptance-based model can be directly used without any additional state estimator.

The proposed modified receptance-based strategy has a remarkable property: only the receptance model is required for the prediction mechanism. As emphasized in Section 4.2, receptance models are quite simple to obtain, requiring only a few natural frequencies and suitable degrees of freedom. Successful examples of this approach can be found in (MOTTERSHEAD et al., 2012) and (DANTAS; DOREA; ARAUJO, 2021). In (MOTTERSHEAD et al., 2012), a 3-DoF AugustaWestland W30 helicopter receptance model is fully obtained through experimental procedures. In (DANTAS; DOREA; ARAUJO, 2021), experimental data is used to obtain the receptance model of the proposed system. In the latter solution, the obtained model is applied to design state feedback controllers for partial pole placement in a time delay scenario. The numerical results with a nonlinear simulation model illustrates that the proposed approach is an interesting alternative to the augmented state-space approach. The nonlinear simulation model with the encoder quantization effect was previously validated in a recent work (BETTEGA; RICHIEDEI, 2023). Furthermore, the proposed cost modification may also be used to the augmented state-space MPC due to the generality of the objective function modification. Hence, the main benefit of the proposed approach comes from the fact that the MPC design problem for multibody control is significantly simplified because the matrix weighting (Q and R) becomes easier to define due to the cost function correction in steady-state.

4.4 Conclusions

This chapter proposes a receptance based MPC for control multibody systems with time-varying references. The GPC and the DMC have been used with a receptance

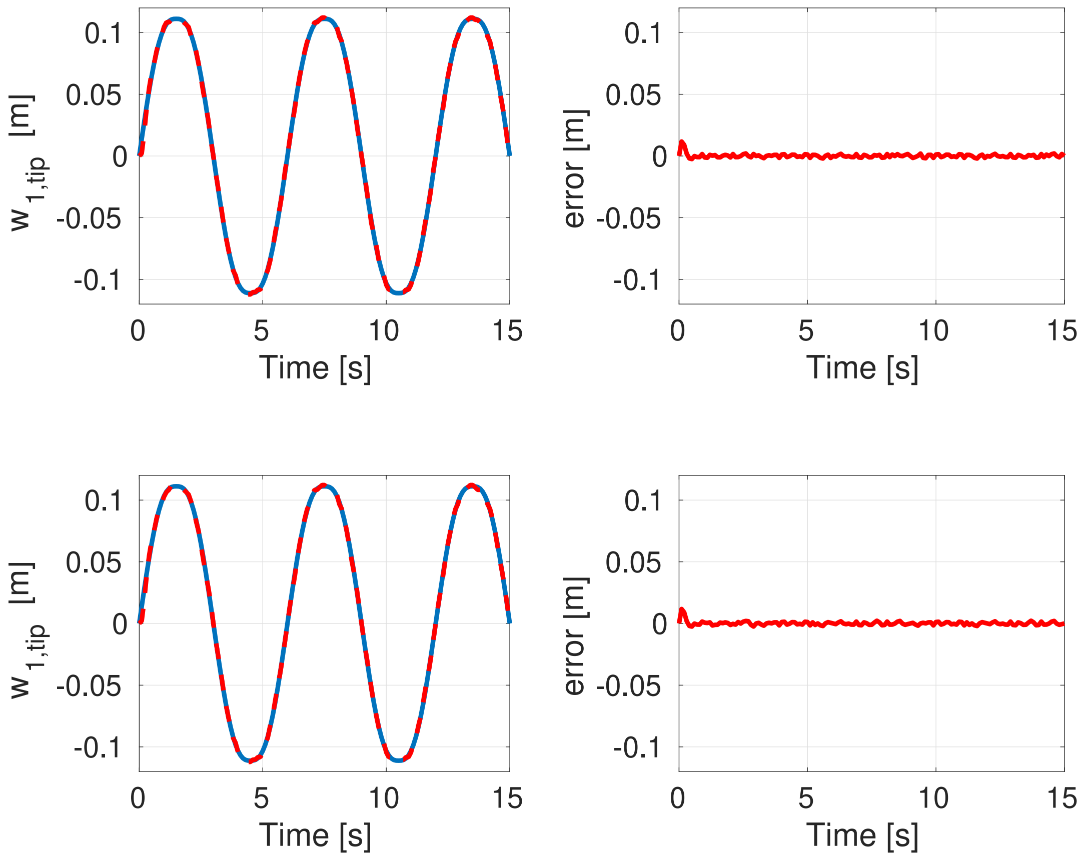

Figure 15 – Modified **GPC** and **DMC** comparison with ($Q = 1, R = 1$) - case with $T_{\text{task}} = 3$ s. Top figure - **GPC**, bottom figure - **DMC**. Output responses ($w_{1,\text{tip}}$): solid lines represent the trajectory target and dashed lines describe the output responses.

modelling framework for multibody systems in order to simplify the identification process and to alleviate the modelling requirements. The conditions to provide negligible error with almost zero delay have been presented in the proposed framework. A modified MPC for the receptance-based MPC with time-varying references is also proposed to significantly reduce the sensitivity of the tracking error with respect to the design parameters. It is shown that the new strategy implicitly imposes an optimal feedforward control in terms of the desired input increment set by the known sequence of future references. The linear MPC is simulated with a nonlinear model to illustrate the usefulness of the analysis and the benefits of the proposed modified algorithms.

The approach proposed in this chapter successfully avoids the conflict of the cost function used when the system is subject to non-constant references since it reduces the sensitivity of the tuning parameters. However, the proposed GPC and DMC do not assure recursive feasibility to robust constraint satisfaction. Therefore, a robust approach will be presented in the next chapter.

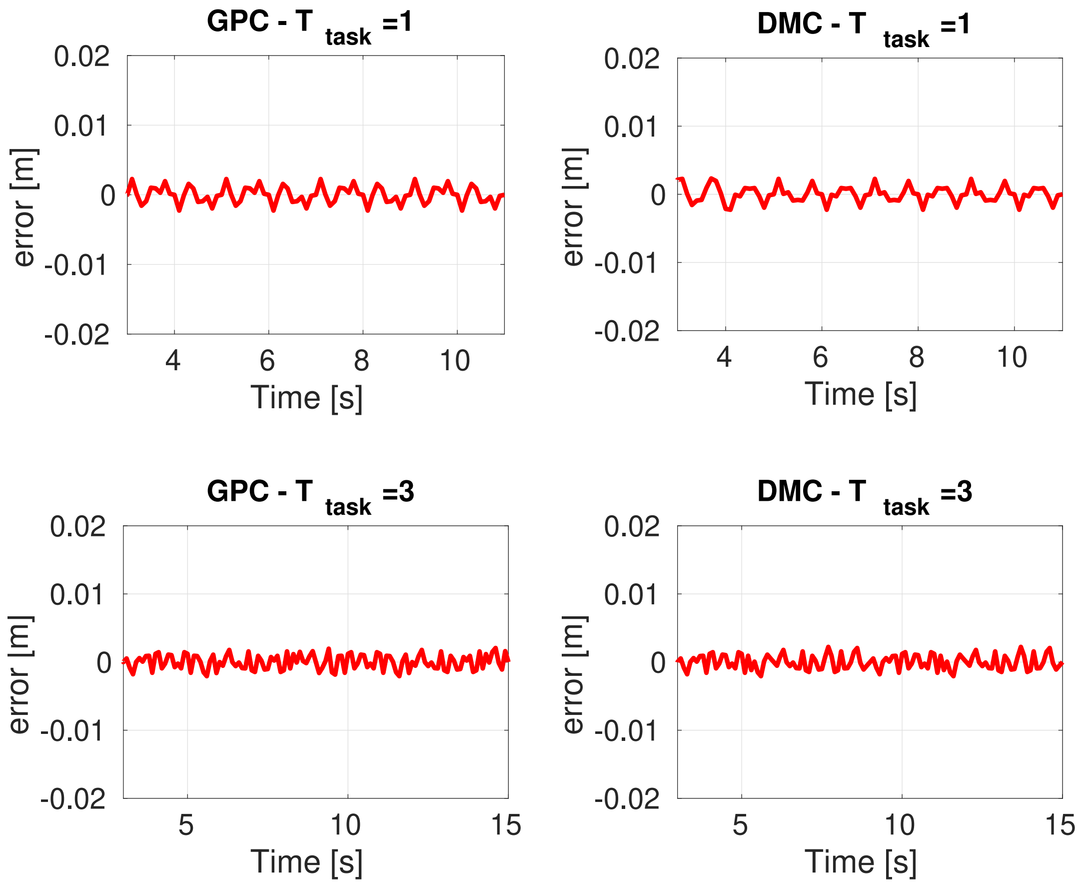

Figure 16 – Steady-state tracking error in steady-state (after 3 s) with different task intervals.

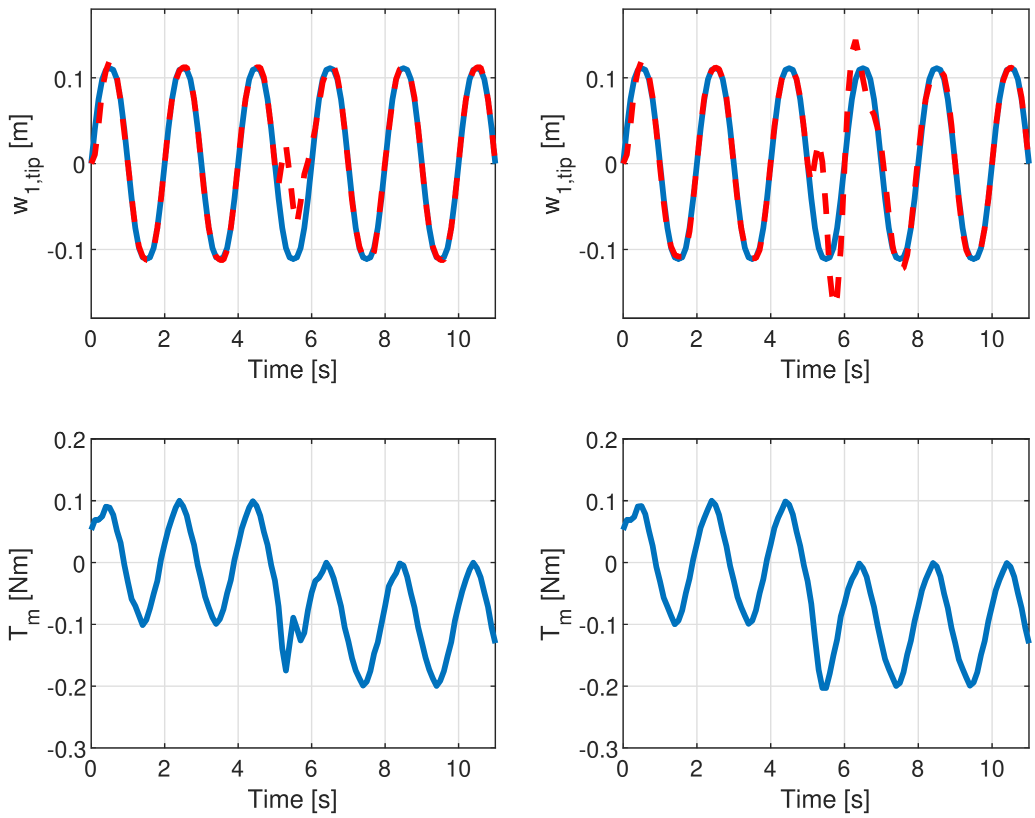

Figure 17 – Comparison between modified GPC (left) and DMC (right) output (top) and input (bottom) when the system is subject to a step disturbance in $t = 5s$.

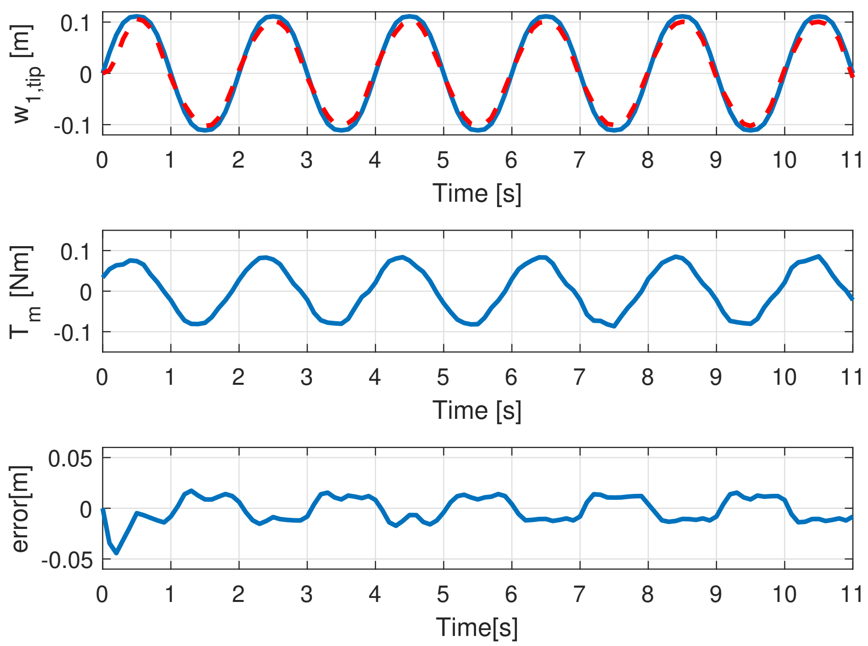

Figure 18 – Standard GPC with double integrator ($Q = 1$ and $R = 1$). Output evolution (top), control signal (middle), and error evolution (bottom).

5 Linear Robust MPC for tracking dynamic target signals

The concept of dynamic operation has been elaborated in (KÖHLER; MÜLLER; ALLGÖWER, 2024), in which could be determined as an operation that is non-stationary, references (or modes of operation) are unpredictable and its behavior could change online without prior knowledge of the controller, and the desired mode of operation is not directly determined based on a given sequence of setpoint of the system state.

This definition means that the state trajectory from a given cost function is non-trivial, thus the objective does not simply steers the state optimally from the initial state to a desired setpoint. Therefore, dynamic target signals can represent periodic targets, but the opposite is not true. The path following problem and the trajectory control of dynamic target needs to be faced as different concepts from control perspectives (AGUIAR; HESPANHA; KOKOTOVIC, 2005) since trajectory tracking needs to follow time and position jointly.

The issue of the MPC while dealing with time-varying references has been theoretically been explored recently, by studying applications on periodic targets, as in (LIMÓN et al., 2015; PEREIRA et al., 2016b), targets defined as dynamic signals imposed by a virtual model (KÖHLER; MÜLLER; ALLGÖWER, 2020; KÖHLER; MÜLLER; ALLGÖWER, 2019), and path following missions as seen in (FAULWASSER; FINDEISEN, 2015) and (SÁNCHEZ et al., 2023). Such theoretical studies are motivated by practical applications such as trajectory tracking control (FAULWASSER et al., 2016) applied to robotics, control of renewable energy systems (XU et al., 2019), and other system with periodic behaviors (PEREIRA et al., 2016a).

In general, setpoint tracking MPC are able to achieve null tracking error if the target is defined to be piecewise constant. Even though, such strategies could not succeed in the tracking of time-varying references. In such condition, a tracking error is expected from the Internal Model Principle since the optimization cost function is based on the assumption that the future reference is constant.

It has been seen along this work that various MPC strategies are able to deal with time-varying references, but a computational complexity may be added in both offline stage (KÖHLER; MÜLLER; ALLGÖWER, 2019) or during online execution (SÁNCHEZ et al., 2023), since stabilizing ingredients (See Section 2.2) and the optimization problem should deal with the nature of the target. To illustrate such affirmation, if an artificial target approach is used, as seen in (2.35a) and used in (LIMÓN et al., 2015; PEREIRA et al., 2016b; KÖHLER; MÜLLER; ALLGÖWER, 2020; SÁNCHEZ et al., 2023; PEREIRA et al., 2016a), the number of additional decision variables linearly depends on the prediction horizon chosen.

Nominal trajectory tracking control of periodic references have been considered in the state of the art, but always limited to particular optimal strategies. For instance, a tracking scheme for linear systems was applied in (LIMÓN et al., 2015) and a robust extension is found in (PEREIRA et al., 2016b). The nonlinear MPC for trajectory tracking was proposed in (KÖHLER; MÜLLER; ALLGÖWER, 2020) and a robust extension is discussed in the same work. The path following algorithms are explored in various works, as in (FAULWASSER; FINDEISEN, 2015; FAULWASSER et al., 2016; SÁNCHEZ et al., 2023), but these works does not consider a general approach, defining its solutions into the scope of the respective MPC algorithm.

This chapter explores the trajectory tracking control of linear systems subject to bounded disturbances, in which a robust approach needs to be considered. Apart from the cited works, an analytical target modification method is proposed in a way that any RMPC for tracking piecewise constant reference could use if recursive feasibility are preserved during (mainly) in conditions of reference changes. The proposed analytical modification can be interpreted as a modular reference computation layer that does not depend on the RMPC algorithm. In this solution, the trajectory is not required to be periodic as seen in related works (LIMÓN et al., 2015; PEREIRA et al., 2016b; PEREIRA et al., 2016a) and the stabilizing objectives are simplified if compared to (KÖHLER; MÜLLER; ALLGÖWER, 2020; KÖHLER; MÜLLER; ALLGÖWER, 2019) since only piecewise constant reference tracking property is required.

The main drawback comes from the fact that the transient performance of (KÖHLER; MÜLLER; ALLGÖWER, 2020; KÖHLER; MÜLLER; ALLGÖWER, 2019) cannot be achieved as the future reference is not considered in the optimization criterion. In summary, the proposed analytical reference compensation is an alternative to provide trajectory tracking properties to a class of RMPC strategies for set-point tracking with reduced number of decision variables and simplified stabilizing ingredients.

To illustrate the benefits and drawbacks of the proposal, a simulation study on an UGV will be presented, where the system will be driven in a leminiscate trajectory and compared to the robust MPC proposed in (SANTOS; CUNHA, 2023) without the target modification. Additionally, results on top of an experimental case are presented to illustrate practical applicability, steady state and transient properties of the analytical target modification.

5.1 Preliminary statements

The original robust optimal controller based on (SANTOS; CUNHA, 2023) is designed by considering the linear disturbed system shown in (2.1)-(2.2). Moreover, consider the following assumptions to be considered:

1. the pair (A,B) is controllable;
2. the pair (A,C) is observable;
3. the state $x[k]$ is measured, thus no observer is needed for the following proposal.
4. the states, inputs and disturbances are bounded by compact and convex sets, such as $x[k] \in \mathbb{X}$, $u[k] \in \mathbb{U}$, and $w[k] \in \mathbb{W}$

Then, a linear MPC for piecewise constant reference is defined by the control law

$$u[k] = \kappa(x[k], y_t[k]), \tag{5.1}$$

such as $y_t[k]$ is a piecewise constant output target to be tracked. The control law in (5.1) is defined on a basis of a strictly convex optimization problem with a quadratic cost function, as seen in many linear MPC strategies.

Several RMPC strategies provides offset free reference tracking in the presence of asymptotic constant disturbances. This means that the algorithm aims to reach $\lim_{k\to\infty} y_t[k] = y_{ss}$ and $\lim_{k\to\infty} w[k] = w_{ss}$, such as the set-point target is an admissible steady-state target considering the disturbances and constraints. Therefore, the controller achieves the condition $\lim_{k\to\infty} ||y[k] - y_{ss}|| = 0$ in steady state for any admissible $y_s s$. Such condition is succeeded to reach offset-free tracking property for the condition established, but potentially fail to reach null steady state tracking error when time-varying references occurs, even in the absence of disturbances.

Therefore, this work defines the time-varying reference from a dynamic target signal target taken from a linear nominal model (KÖHLER; MÜLLER; ALLGÖWER, 2020):

$$x_t[k+1] = Ax_t[k] + Bu_t[k], \tag{5.2}$$

$$y_t[k] = Cx_t[k], \tag{5.3}$$

where the pair (x_t, u_t) defines the state and input target, and defines an admissible target candidate, that also should consider system constraints in its definition.

The definition of the pair $(x_t[k], u_t[k])$ from a given output sequence $y_t[k], k \geq 0$ can be interpreted as a simple inversion problem or an unknown input observer if the system has minimum phase. An extra attention should be taken in the case of a non-minimum phase system in order to guarantee input convergence as seen in (GEORGE; VERHAEGEN; SCHERPEN, 1999; NADERI; KHORASANI, 2018). Moreover, such solution for non-minimum phase systems could be applied with no loss of generality. Since the system is observable, the observability matrix

$$\mathcal{O} = \begin{bmatrix} C^\top & (CA)^\top & \dots & (CA^j)^\top \end{bmatrix}^\top, \tag{5.4}$$

and the controllability matrix is defined as

$$\mathcal{B} = \begin{bmatrix} \mathbf{0}_{n,n} & (CB)^\top & \dots & (CA^{j-1}B)^\top \end{bmatrix}^\top. \tag{5.5}$$

The controllability is full rank for a suitable $j \leq n-1$. Therefore, from the vector of future output targets $\mathcal{Y}_t(k) = \begin{bmatrix} y_t[k]^\top & y_t[k+1]^\top & \dots & y_t[k+j]^\top \end{bmatrix}^\top$ and the vector of future input targets $\mathbf{u}_t(k) = \begin{bmatrix} u_t[k]^\top & u_t[k+1]^\top & \dots & u_t[k+j] \end{bmatrix}^\top$, a nominal observer can be achieved by the following:

$$x_t[k] = \mathcal{O}^\dagger(\mathcal{Y}_t(k) - \mathcal{B}\mathbf{u}_t(k)), \tag{5.6}$$

where the $\mathcal{O}^\dagger$ is the pseudo-inverse of $\mathcal{O}$. This reference observer can be used since the reference is not subject to uncertainties and noise effects, thus it can be applied to generate dynamic target signal from the output reference for a minimum phase system. In that case, the sequence defined from $u_t[k-d]$ (defining d the discrete delay that ensures a causal output-to-input relationship) can be recovered from $y_t[k]$, and $x_t[k]$ can be recovered from both $u_t[k]$ and $y_t[k]$ from the nominal observer shown in (5.6).

From the various robust strategies for dealing with offset free tracking objectives presented, the artificial target idea from (LIMÓN et al., 2010) brings interesting characteristics, such as: (i) recursive feasibility for any target value; (ii) enlarged domain of attraction when compared to standard MPC for regulation; (iii) the number of decision variables is similar to a standard MPC for regulation. Since recursive feasibility does not depend on the target at instant k, an RMPC strategy will be revisited so further discussions could be done regarding potential applications of the proposed target modification layer.

5.2 RMPC for tracking piece-wise constant references with artificial targets

The proposed robust MPC for tracking is described in this section based on (SANTOS; CUNHA, 2023) with no loss of generality from other RMPC strategies that follows the strategy that limits the constraints based on the worst-case scenario of disturbances.

The system is subject to the control law in (2.24), thus the strategy is defined on top of the following assumptions:

Assumption 3 *(1) The matrices $Q \in \mathbb{R}^{n\times n}$, $R \in \mathbb{R}^{m\times m}$ and $T \in \mathbb{R}^{n\times n}$ and $Q \geq 0$, $R > 0$, and $T > 0$.*
(2) The tuned system (pair $(Q^{1/2}, A)$) is detectable.
(3) The matrix $P \in \mathbb{R}^{n\times n}$ be a positive definite matrix such that $(A+BK)^\top P(A+BK)-P \leq -C^\top QC - K^\top RK$.
(4) $\mathbb{X}_{\infty,inner} \subset \mathbb{X}_\infty$ and $\mathbb{U}_{\infty,inner} \subset \mathbb{U}_\infty$.
(5) $\mathcal{Z}_f$ is an augmented robust invariant set for tracking such that

$$(x, \theta) \in \mathcal{Z}_f \Rightarrow ((A + BK)x + BL\theta + (A + BK)^N w, \theta) \in \mathcal{Z}_f, \forall w \in \mathbb{W},$$

where $x \in \mathbb{X}_N$, $Kx - L\theta \in \mathbb{U}_N$, $M_x\theta \in \lambda\mathbb{X}_{\infty,inner}$, and $M_u\theta \in \lambda\mathbb{U}_{\infty,inner}$, for $0 < \lambda < 1$.

Notice that from Assumption 3-(4) and 3-(5), the artificial state and input target never reaches the boundary of the constraint set by $\mathbb{X}_\infty$ and $\mathbb{U}_\infty$. Therefore, the tighter sets for robust constraint satisfaction and recursive feasibility are defined from the initial sets $\mathbb{X}_0 = \mathbb{X}$ and $\mathbb{U}_0 = \mathbb{U}$[1] as follows:

$$\mathbb{X}_j = \mathbb{X}_{j-1} \ominus (A + BK)^{j-1}\mathbb{W}, \tag{5.7}$$

$$\mathbb{U}_j = \mathbb{U}_{j-1} \ominus K(A + BK)^{j-1}\mathbb{W}, \tag{5.8}$$

which are used recursively as constraints of the RMPC problem, similar to the MPC for tracking proposed in (2.35f).

$$\begin{aligned}
\underset{\mathbf{u}(k),\theta(k)}{\text{minimize}} \quad & J_N(x(k), y_t(k), \mathbf{u}(k), \theta(k)), && \text{(5.9a)}\\
\text{subject to} \quad & x[0|k] = x[k], && \text{(5.9b)}\\
& (x_s[k], u_s[k]) = M_\theta\theta(k), && \text{(5.9c)}\\
& y_s[k] = M_y\theta(k), && \text{(5.9d)}\\
& x[i+1|k] = Ax[i] + Bu[i], \quad i \in \mathbb{N}_{[0,N-1]}, && \text{(5.9e)}\\
& x[i|k] \in \mathbb{X}_i, \quad i \in \mathbb{N}_{[0,N-1]}, && \text{(5.9f)}\\
& u[i|k] \in \mathbb{U}_i, \quad i \in \mathbb{N}_{[0,N-1]}, && \text{(5.9g)}\\
& (x[N], \theta) \in \mathcal{Z}_f, && \text{(5.9h)}
\end{aligned}$$

[1] Consider the Pontryagin set difference denoted by $\mathbb{S} \ominus \mathbb{T} = \{s \in \mathbb{S} | s + t \in \mathbb{S}, \forall t \in \mathbb{T}\}$.

where the cost function $J_N(x(k), y_t(k), \mathbf{u}(k), \theta(k))$ is similar to the formulation of the MPC for tracking proposed in (2.34).

$$J_N(x(k), y_t(k), \mathbf{u}(k), \theta(k)) = \sum_{i=0}^{N-1} ||x[i|k] - x_s[k]||_Q^2 + ||u[i|k] - u_s[k]||_R^2 + \dots$$
$$\dots + ||x[N|k] - x_s[k]||_P^2 + ||y_s[i] - y_t[k]||_T^2. \quad (5.10)$$

The offset cost function in (5.10) uses a norm-2 ($||y_s[i] - y_t[k]||_T^2$) as discussed in Section 2.3, but other norms can be used as disposed in (SANTOS; CUNHA, 2023) such that conditions of the offset cost function V_o are fulfilled. Also, the robust invariant set $\mathcal{Z}_f$ for tracking is taken based on the inner sets $\mathbb{X}_{\infty,inner}$ and $\mathbb{U}_{\infty,inner}$ to avoid the artificial reference to reach the constraint border.

By solving the optimization problem, the optimal sequence of inputs $\mathbf{u}^0[0|k]$ available and the receding horizon control law $\kappa(x[k], y_t[k])$ is defined by

$$u[k] = \kappa(x[k], y_t[k]) = u^0[0|k]. \quad (5.11)$$

The main objective of this chapter is to provide a simple analytical target modification to be combined with the robust MPC algorithms for piecewise constant references, which is taken from

$$\tilde{y}_t[k] = h(x_t[k], u_t[k]), \quad (5.12)$$
$$u[k] = \kappa(x[k], \tilde{y}_t[k]). \quad (5.13)$$

The target modification is done such that $(y[k] - y_t[k]) \in C\mathcal{X}_\infty$, for $k = k_0, k_0 + 1, \dots, \infty$, where k_0 is the initial instant and $\mathcal{X}_\infty$ is a Minimal Robust Positively Invariant Set (minRPI). Notice that if $x[k_0] - x_t[k_o] \in \mathcal{X}_\infty$ and the sequence $\{y_t[k_0], y_t[k_0+1], \dots, y_t[\infty]\}$ defines an admissible target, then the desired convergence is achieved.

5.3 Target modification for time-varying references

To propose the novel MPC strategy, some premises should be clarified to the scope of this problem. First, it is assumed that the values of $y_t[k]$, $x_t[k]$, and $u_t[k]$ are known at a given instant k. The values could be calculated from the output target $y_t[k]$ as discussed in Section 5.1.

Consider that an RMPC could be synthesized from a general optimization problem given by

$$\underset{v}{\text{minimize}} \quad J(v, z) \tag{5.14a}$$

$$\text{subject to} \quad Fv \leq Mz + l, \tag{5.14b}$$

where v is the vector of decision variables, z represents a vector of known parameters, and the cost function $J(v, z)$ is quadratic in v and is a strictly convex function, i.e. $\frac{\delta^2}{\delta v^2} J(v, z) > 0$. The constraints are represented by (5.14b) and represents a set $\mathbb{V}_{z_0} = Fz \leq h$ which is assumed to be non-empty and contains the origin in its interior. For instance, the RMPC shown in Section 5.2 could be represented by selecting $z = (x[k], y_t[k])$ and $v = (\mathbf{u}(k), \theta(k))$, which reduces the optimization problem (5.14a)-(5.14b) to the problem stated in (5.9a)-(5.9h).

If v^o returns the optimal solution of the stated problem with no active constraints, that is $Fv^o < Mz + l$, then the problem converges to the unconstrained optimal solution such as $\nabla_v J(v^o, z) = 0$. This conclusion allows to propose the target modification if recursive feasibility is ensured by the RMPC for any output target $y_t[k]$. Roughly speaking, a new modified target is proposed such that $u[k]$ converges to $u_t[k]$ if the tracking condition $(y[k] = y_t[k])$ is reached.

5.3.1 Target modification

The quadratic cost function $J(v, z)$ could be rewritten generically in the form of

$$J(v, z) = \frac{1}{2} v^\top \mathcal{H} v + z^\top \mathcal{S} v + f^\top v + h_z(z) + c, \tag{5.15}$$

with the matrices $\mathcal{H}$, $\mathcal{S}$ and $f^\top$ properly chosen depending on optimization problem. The optimal unconstrained solution is then defined by solving $\nabla_v J(v, z) = 0$

$$v^o = -\mathcal{H}^{-1}(\mathcal{S}^\top z + f). \tag{5.16}$$

Since $z = (x[k], y_t[k])$ and $v = (\mathbf{u}(k), \theta(k))$ for the RMPC, the receding horizon control law can be defined as follows:

$$u[k] = u^o[0|k] = \mathcal{K}x[k] + \mathcal{L}y_t[k] + r, \tag{5.17}$$

for $[\mathcal{K} \quad \mathcal{L}] = [\mathbf{I}_m \quad \mathbf{0}_{m,N.m}]\mathcal{H}^{-1}\mathcal{S}^T$ and $r = [\mathbf{I}_m \quad \mathbf{0}_{m,N.m}]f$, with the matrices $\mathcal{K} \in \mathbb{R}^{m,n}$ and $\mathcal{L} \in \mathbb{R}^{m,m}$ being simply the selection of the first m rows of $\mathcal{H}^{-1}\mathcal{S}^T$ and f respectively.

The optimal solution is time-invariant (MAYNE et al., 2000) in the unconstrained region of the state-space partition, then a reverse target computation can be used to find a new value of $\tilde{y}_t[k]$. This is only possible if $\det(\mathcal{L}) \neq 0$. The proof for such condition is taken from defining (5.17) as the unconstrained control law that ensures piecewise constant reference tracking in the absence of disturbance. Then, assuming that the constant steady state target $\bar{y}_t$ is

$$\bar{x}_t[k] = A\bar{x}_t[k] + B\bar{u}_t[k], \tag{5.18}$$

$$\bar{y}_t[k] = C\bar{x}_t[k], \tag{5.19}$$

and defining $r = 0$ and $\mathcal{L}\bar{y}_t[k] = \bar{u}_t[k] - \mathcal{K}\bar{u}_t[k]$ to guarantee offset free tracking from $\bar{u}_t[k] = \mathcal{K}(x[k] - \bar{x}_t[k]) + \bar{u}_t[k]$ in such condition. If the trivial parameterization of the artificial target in (2.26) and (2.27) is used for $\bar{x}_t[k] = M_x\theta[k]$, $\bar{u}_t[k] = M_u\theta$, and $\bar{y}_t[k] = CM_x\theta$, thus $\mathcal{L}\bar{y}_t[k] = M_u\theta - \mathcal{K}M_x\theta = \theta$ or $\mathcal{L}(CM_x)\theta = \theta$. By consequence, $\mathcal{L}CM_x = \mathbf{I}_m$ which inflicts that both $\mathcal{L}$ and CM_x are full rank matrices, which leads to $\det(\mathcal{L}) \neq 0$.

Therefore, the reverse computation of the new output target $\tilde{y}_t[k] = h(x_t[k], u_t[k])$ is

$$\tilde{y}_t[k] = \mathcal{L}^{-1}(u_t[k] - \mathcal{K}x_t[k] - r). \tag{5.20}$$

It is important to mention that the condition $r = 0$ is taken only to prove the inversibility of $\mathcal{L}$ and is considered for constant set-point tracking and is maintained in the final formulation for the sake of generality of the discussion.

The new target value is calculated such that

$$u[k] = \mathcal{K}x[k] + \mathcal{L}\tilde{y}_t[k] + r \tag{5.21}$$

$$= \mathcal{K}(x[k] - x_t[k]) + u_t[k]. \tag{5.22}$$

Therefore, a minRPI set $\mathcal{X}_\infty$ can be defined around the desired target from Equations (2.1), (5.2) and (5.22) based on the evolution of $(x[k] - x_t[k])$

$$(x[k+1] - x_t[k+1]) = (A + B\mathcal{K})(x[k] - x_t[k]) + w[k], \tag{5.23}$$

where the set $\mathcal{X}_\infty$ is defined as follows

$$\mathcal{X}_\infty = \sum_{j=0}^{\infty}(A + B\mathcal{K})^j\mathbb{W}. \tag{5.24}$$

Finally, if the controller parameters and the sequence of targets are defined such that $\{x_t[k]\} \oplus \mathcal{X}_\infty \times \{\tilde{y}_t[k]\}$ is a subset of the optimal unconstrained partition, then $\{x[k_0] - x_t[k_0]\} \oplus \mathcal{X}_\infty$ ensures that $\{x[k] - x_t[k]\} \oplus \mathcal{X}_\infty$ for any $k_0 \geq 0$. The controller gain $\mathcal{K}$ is not required to be the optimal infinite horizon solution of the unconstrained optimization problem, but if local optimality property is assured, then $\mathcal{K}$ is the solution of the LQR problem.

With the output target modification properly set, a concern on the characterization of the unconstrained region should be considered for computational and algorithm synthesis purposes. The partitions from the unconstrained control law (5.17) could be used to define the admissible region, but computational cost to determine all inequalities may be avoided. A simplified characterization can be directly derived from the unconstrained solution v^o for the generalized optimization problem presented in (5.14a)-(5.14b) and the cost function (5.15), resulting in the set $\mathbb{V}^o$ as follows

$$\mathbb{V}^o = \{z \in \mathbb{R}^{n_z} | -(M + F\mathcal{H}^{-1}\mathcal{S}^\top)z < F\mathcal{H}^{-1}f + l\}, \tag{5.25}$$

where n_z is the dimension of the vector z. From the proposed target modification, $z = (x, \tilde{y}_t)$. Defining the sequence $\mu[k] = (x_t[k], u_t[k], y_t[k])$, an augmented sequence $\mu^a[k]$ is obtained from (5.17), such as $\mu^a[k] = (x_t[k], u_t[k], y_t[k], \tilde{y}_t[k])$. From that, consider the following conditions of admissibility and robust admissibility of the target.

- A target sequence is admissible if $(x_t[k], \tilde{y}_t[k]) \in \mathbb{V}^o, \forall k \geq 0$.
- The target sequence is robustly admissible if $\{x_t[k]\} \oplus \mathcal{X}_\infty \times \{\tilde{y}_t[k]\} \subseteq \mathbb{V}^o, \forall k \geq 0$.

The synthesis of an outer approximation such as $\mathcal{X}_\infty \subset \mathcal{X}_{outer,\infty}$ may be computationally expensive, thus a tighter set can be defined, such as $\bar{\mathbb{V}}^o$ can be computed by using any particular set representation such as zonotopes and orthopes from the following

$$\bar{\mathbb{V}}^o = \mathbb{V}^o \ominus (\mathcal{X}_{outer,\infty} \times \{0\}). \tag{5.26}$$

Moreover, the proposed strategy follows a dynamic signal target tracking property, in which an RMPC control law for piecewise constant reference tracking with recursive feasibility such as (SANTOS; CUNHA, 2023) despite target changes is defined by the control law $u[k] = \kappa(x[k], y_t[k])$. Then, if the following conditions are met:

1. The modified target $\tilde{y}_t[k]$ is defined from the pair $(x_t[k], u_t[k])$ as in (5.20) with $r = 0$ to ensure offset free tracking;
2. The control law is a function of the current state and the modified target, i.e. $u[k] = \kappa(x[k], \tilde{y}_t[k]$;
3. The pair of targets $(x_t[k], \tilde{y}_t[k])$ respects the set of constraints considering the effect of the disturbance $\mathbb{W}$, that is, $(x_t[k], \tilde{y}_t[k]) \in \bar{\mathbb{V}}^o, \forall k \geq k_0$, and;

4. $(x[k_0] - x_t[k_0]) \in \mathcal{X}_\infty$,

then the deviation $(x[k] - x_t[k])$ remains in the minRPI set for any $k \geq k_0$.

For that, a general practical solution is to use a mode of operation to use a piecewise constant reference during an initialization time interval $0 \leq k < k_0$ such as the condition $(x[k_0] - x_t[k_0]) \in \mathcal{X}_\infty$ is achieved. After such condition is achieved, the proposed solution can be used to track the desired reference.

Also it is important to notice that the transient performance and the admissible region are naturally degraded when compared to other MPC with time-varying reference considered in the objective function as in (SANTOS; CUNHA, 2023). This remark is accepted since the main objective of the target modification is to preserve online implementation complexity at the same time that the algorithm provide a simplified trajectory tracking solution for RMPC strategies that are not able to avoid offset free tracking in the presence of dynamic signal targets. This characteristic of the controller will be exposed in simulation and experimental analysis in the next sessions.

Also, the simplicity and flexibility of the target modification proposed are useful properties because it can be applied in several other strategies and applications. For instance, avoidance features can be achieved by using the ideas of (SANTOS; FERRAMOSCA; RAFFO, 2024), and chance constraints can be considered (D'JORGE et al., 2020), as other RMPC strategies.

5.4 Case study - Motion control of a skid steered UGV

To illustrate the main characteristics of the RMPC combined with the general target modification, a case study on the motion control of a skid steered UGV will be studied in the the following section. Such class of mobile robots are widely used in various applications such as autonomous outdoor navigation (YI et al., 2009) and indoor service robotics (HASHEMI; HE; JOHANSSON, 2022) for transportation, logistic missions, and others. For the scope of this work, a Clearpath Husky A200 unit will be used as object of study, where a motion control interface is provided by using ROS framework. Figure 19 presents the UGV available at the Robotics Laboratory of the Electrical and Computer Engineering Department of the UFBA.

5.4.1 Output tracking for skid steered UGV

Low-level controllers are available for the wheels in order to provide the desired commands of linear velocity ($v_c(t)$) and angular velocity ($\omega_c(t)$) of the robot. Figure 20 illustrates position and orientation of the UGV $\mathbf{p}(t) = \begin{bmatrix} x(t) & y(t) & \theta(t) \end{bmatrix}$ and a virtual robot is considered following a desired trajectory with $\mathbf{p_r}(t) = \begin{bmatrix} x_r(t) & y_r(t) & \theta_r(t) \end{bmatrix}$.

Figure 19 – Cleapath Husky A200 in Robotics Laboratory - UFBA.

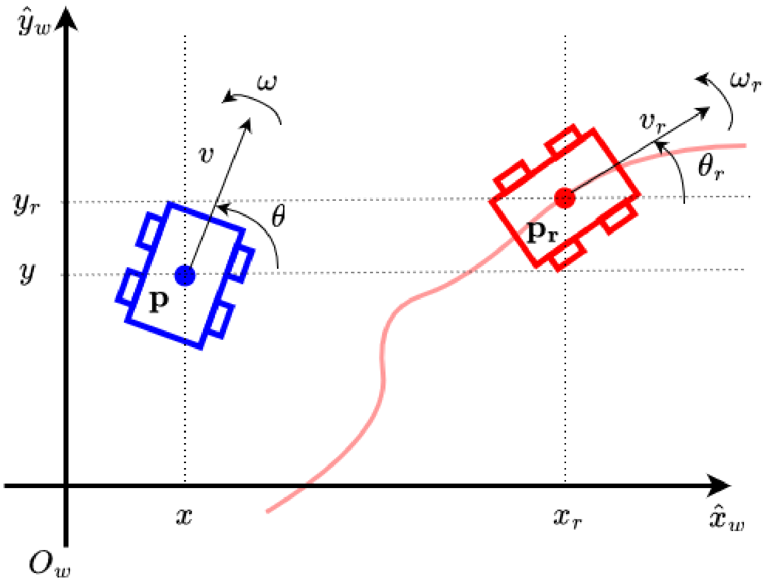

Figure 20 – Coordinate system representation for controlled and virtual robot.

The kinematic model of the mobile robot with respect to the world frame, is described as follows, considering the linear and angular velocity of the vehicle represented by $v(t)$ and $\omega(t)$ respectively. The same applies to the virtual robot that will be used for reference computation.

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos(\theta(t)) & 0 \\ \sin(\theta(t)) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix}. \tag{5.27}$$

Therefore the following kinematic model considering the linear and velocity commands of the robot will be used in the scope of this strategy:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos(\theta(t)) & 0 \\ \sin(\theta(t)) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_c(t) \\ w_c(t) \end{bmatrix} + \begin{bmatrix} \epsilon_x(t) \\ \epsilon_y(t) \\ \epsilon_\theta(t) \end{bmatrix}, \tag{5.28}$$

where $\epsilon_x(t)$, $\epsilon_y(t)$ and $\epsilon_\theta(t)$ are bounded disturbance signals that complies multiple sources of modeling mismatches, external disturbances and measurement noises.

5.4.2 Feedback Linearization

In order to apply the robust linear strategies presented in this chapter, a feedback linearization based on the strategy proposed in (SCIAVICCO; SICILIANO, 2012, Chapter 11) is used for modeling simplicity. The linear feedback design is defined from the new virtual states:

$$z_1(t) = x(t) + b\cos(\theta(t)), \tag{5.29}$$

$$z_2(t) = y(t) + b\sin(\theta(t)), \tag{5.30}$$

where $b > 0$ is a parameter freely chosen that defines a future virtual point displaced from the center of the vehicle. The new kinematic model is defined by simply acquiring $\dot{z}_1$ and $\dot{z}_2$, resulting in:

$$\begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos(\theta(t))v_c(t) - b\sin(\theta(t))w_c(t) \\ \sin(\theta(t))v_c(t) + b\cos(\theta(t))w_c(t) \\ w_c(t) \end{bmatrix} + \begin{bmatrix} \epsilon_{z,1}(t) \\ \epsilon_{z,2}(t) \\ \epsilon_\theta(t) \end{bmatrix}, \tag{5.31}$$

and the new disturbance signals is

$$\epsilon_{z,1}(t) = \epsilon_x(t) - b\sin(\theta(t))\epsilon_\theta(t),$$
$$\epsilon_{z,2}(t) = \epsilon_y(t) + b\cos(\theta(t))\epsilon_\theta(t).$$

The relationship between the virtual signals $u_1(t)$ and $u_2(t)$ and the robot's input command signals are:

$$\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} \cos(\theta(t)) & -b\sin(\theta(t)) \\ \sin(\theta(t)) & b\cos(\theta(t)) \end{bmatrix} \begin{bmatrix} v_c(t) \\ w_c(t) \end{bmatrix}, \tag{5.32}$$

such as the inverse transformation is simply

$$\begin{bmatrix} v_c(t) \\ w_c(t) \end{bmatrix} = \begin{bmatrix} \cos(\theta(t)) & \sin(\theta(t)) \\ -\sin(\theta(t))/b & \cos(\theta(t))/b \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}. \tag{5.33}$$

Thus, the partial input output linear model results in the model:

$$\begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} + \begin{bmatrix} \epsilon_{z,1}(t) \\ \epsilon_{z,2}(t) \end{bmatrix}, \tag{5.34}$$

such as $\dot{\theta} = (-\sin(\theta(t))u_1(t) + \cos(\theta(t))u_2(t))/b + \epsilon_\theta(t)$ defines the zero dynamics. Therefore, the control law is applied based on the discretized model, such as the control signal are applied with a zero-order holder with the sampling interval T_s. This way, $v_c(t) = v_c(kT_s)$, $w_c(t) = w(kT_s)$, for $kT_s \leq t < (k+1)T_s$. Appliying Euler's discretization for sampled data approximation was used, such as the discrete signals are $x[k] = x(kT_s)$, $y[k] = y(kT_s)$, $\theta[k] = \theta(kT_s)$, $z_1[k] = z_1(kT_s)$, $z_2[k] = z_2(kT_s)$. Finally, the discrete model for the RMPC design is given by

$$\begin{bmatrix} z_1[k+1] \\ z_2[k+1] \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_1[k] \\ z_2[k] \end{bmatrix} + \begin{bmatrix} T_s & 0 \\ 0 & T_s \end{bmatrix} \begin{bmatrix} u_1[k] \\ u_2[k] \end{bmatrix} + \begin{bmatrix} \eta_1[k] \\ \eta_2[k] \end{bmatrix}. \tag{5.35}$$

The new disturbances $\eta_1[k]$ and $\eta_2[k]$ also includes discretization errors and the the discrete evolution of θ can be either

$$\theta[k+1] = \theta[k] + T_s w_c[k] + \eta_\theta[k], \tag{5.36}$$

or

$$\theta[k+1] = \theta[k] + T_s[-\sin(\theta[k])u_1[k] + \cos(\theta(t))u_2[k]]/b + \eta_\theta[k] \tag{5.37}$$

5.4.3 Reference computation from feedback linearization

The transformed reference is defined from the position reference $x_r[k] = x_t(kT_s)$ and $y_r[k] = y_t(kT_s)$. It is assumed the information of $x_r[k]$, $x_r[k+1]$, $x_r[k+2]$, $y_r[k]$, $y_r[k+1]$ and $y_r[k+2]$ are known at a given instant k. This way, $\theta_r[k] = atan2(y_r[k+1] - y_r[k], x_r[k+1] - x_r[k])$ and $\theta_r[k+1] = atan2(y_r[k+2] - y_r[k+1], x_r[k+2] - x_r[k+1])$. Finally, the transformed references are given by:

$$z_{r,1}[k] = x_r[k] + b\cos(\theta_r[k]), \tag{5.38}$$

$$z_{r,2}[k] = y_r[k] + b\sin(\theta_r[k]), \tag{5.39}$$

$$u_{r,1}[k] = \frac{z_{r,1}[k+1] - z_{r,1}[k]}{T_s}, \tag{5.40}$$

$$u_{r,2}[k] = \frac{z_{r,2}[k+1] - z_{r,2}[k]}{T_s}. \tag{5.41}$$

From the new references, a state-feedback control law for nominal trajectory tracking could be defined as follows

$$u_1[k] = K(z_1[k] - z_{r,1}) + u_{r,1}[k], \tag{5.42}$$

$$u_2[k] = K(z_2[k] - z_{r,2}) + u_{r,2}[k]. \tag{5.43}$$

Hence, the evolution of the error is calculated from $e_1[k] = z_1[k] - z_{r,1}$ and $e_2[k] = z_2[k] - z_{r,2}$:

$$\begin{bmatrix} e_1[k+1] \\ e_2[k+1] \end{bmatrix} = \begin{bmatrix} 1+KT_s & 0 \\ 0 & 1+KT_s \end{bmatrix} \begin{bmatrix} e_1[k] \\ e_2[k] \end{bmatrix} + \begin{bmatrix} \eta_1[k] \\ \eta_2[k] \end{bmatrix}. \tag{5.44}$$

The proposed linearizing state-feedback strategy does not deal naturally with restrictions, which motivates the application of a robust approach. The next sections presents a simulation analysis and experimental demonstration of the Robust MPC proposed by (SÁNCHEZ et al., 2023) and the RMPC with the target modification proposed in (5.20).

5.4.4 Simulation results

The trajectory of a skid-steering UGV is analyzed using the kinematic model in (5.27) through numerical simulation to evaluate the nominal performance of the proposed controllers.

The feedback linearization (5.29)-(5.30) is used such that the virtual model (5.35) is used for the standard RMPC (5.9a)-(5.9h) and the proposed RMPC with the target modification (5.20) are compared to evaluate the tracking properties with time-varying trajectory when the system is subject to a lemniscate reference defined by:

$$x_r[k] = x_d - 2\beta\frac{\sin(kT_s\omega_n)}{1 + (\alpha/\beta)^2\cos(kT_s\omega_n)^2}, \tag{5.45}$$

$$y_r[k] = y_d - 2\alpha\frac{\sin(kT_s\omega_n)\cos(kT_s\omega_n)}{1 + (\alpha/\beta)^2\cos(kT_s\omega_n)^2}, \tag{5.46}$$

with natural frequency $\omega_n = 2\pi/40$ rad/s, $\alpha = 1.3$, $\beta = 1.0$, and $x_d = y_d = 0$ m. The initial position and orientation vector is $\mathbf{p}_0 = \begin{bmatrix} 0.0 & 0.0 & -\pi/2 \end{bmatrix}$ and the workspace is limited to hard state constraints $2.0 \leq x[k] \leq 4.0$ m and $2.0 \leq y[k] \leq 4.0$ m. Input constraint in both linear velocity $0 \leq v_c[k] < 0.7$ (m/s) and angular velocity $|\omega_c| < 1$ (rad/s) are set and a sampling interval $T_s = 0.2$ s is set.

By choosing the feedback linearization parameter $b = 0.3$, the constraints of the workspace should be adjusted for the linearized model, considering the worst-case scenario in (5.29) and (5.30). Therefore the set could be constricted in a conservative way such that $z_1[k]$ and $z_2[k]$ never leads the position of the robot to outside of the defined workspace, that is $|x[k]| \leq 1.7$ (m), and $|y[k]| \leq 1.7$ (m).

The MPC was designed with the tuning gains $Q = R = diag(1, 1)$, with P and K being the LQR solution, and $T = 1000||P||$. The disturbances w were assumed to be bounded such that $|w| \leq \begin{bmatrix} 0.03 & 0.03 \end{bmatrix}^\top$.

In Figure 21, it can be seen the evolution of the UGV's position ($x(t)$ and $y(t)$) and orientation $\theta[k]$. A difference between the the reference and the position/orientation output is easily seen on the original RMPC method, while the proposed method reduces the tracking error in steady state condition.

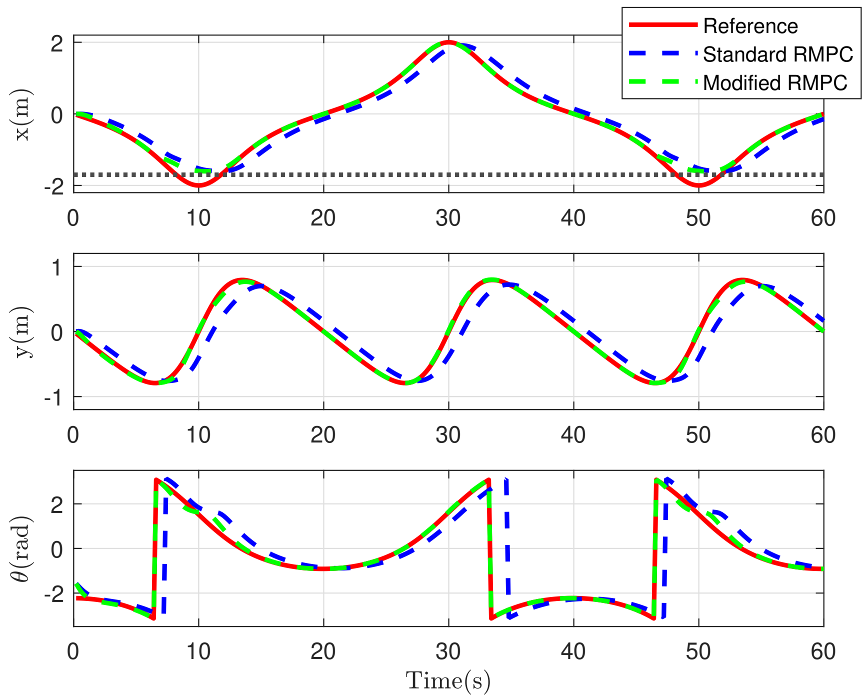

Figure 21 – Comparison of the position and orientation evolution of the UGV in simulation analysis.

Between 15 and 25 seconds, it is visible that the target converges to the closest nominal admissible reference even if the reference chosen is not admissible. In such condition, the target for both strategies presents a conservative optimal value caused by the robust constraint set adjustment such that $|w[k]| < 0.03$ in a way that the operation is safe considering the worst-case scenario of disturbance during the entire operation, even when the target is not valid from the workspace perspective. Additionally, Figure 21 presents the lemniscate trajectory following of both strategies in the workspace plane, where it is possible to observe that the modified strategy is able to track with null steady state error while the target is admissible. When the target is not admissible, a necessary non-zero tracking error is achieved to avoid the vehicle to crash on the limit of the workspace, ensuring safe operation condition for the navigation mission.

Therefore, the enhancement in steady-state performance is significant, as anticipated from the theoretical analysis presented in this chapter. However, it should be noted that the transient response is slightly degraded compared to the original controller. This degradation can be attributed to the suboptimal nature of the proposed solution, which does not account for future reference signals in its design.

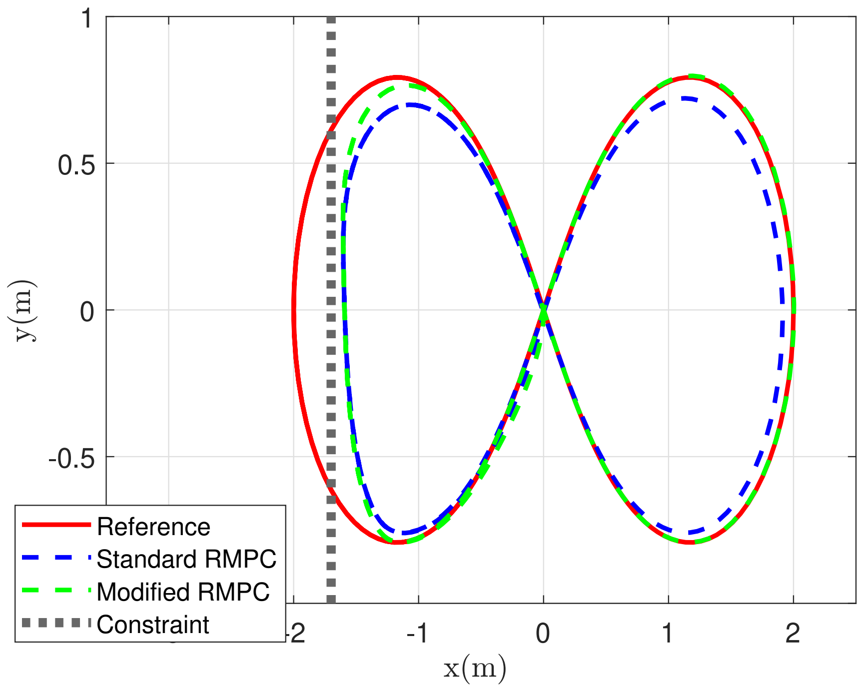

Figure 22 – UGV's lemniscate trajectory in simulation analysis.

Control inputs are depicted in Figure 23, which shows that constraints conditions are respected during entire analysis. Moreover, the modified RMPC presents a more aggressive control signal during initialization since the target modification proposal causes an indirect feedforward signal based on the target reference chosen.

Figure 24 presents a comparison of the time needed to solve the optimization problema in each simulation step. The Matlab's *quadprog* function was used for this analysis in a computational platform with the following configuration: intel Core i7 11800H, 16GB RAM, Windows 11. From this comparison, it is possible to conclude that,

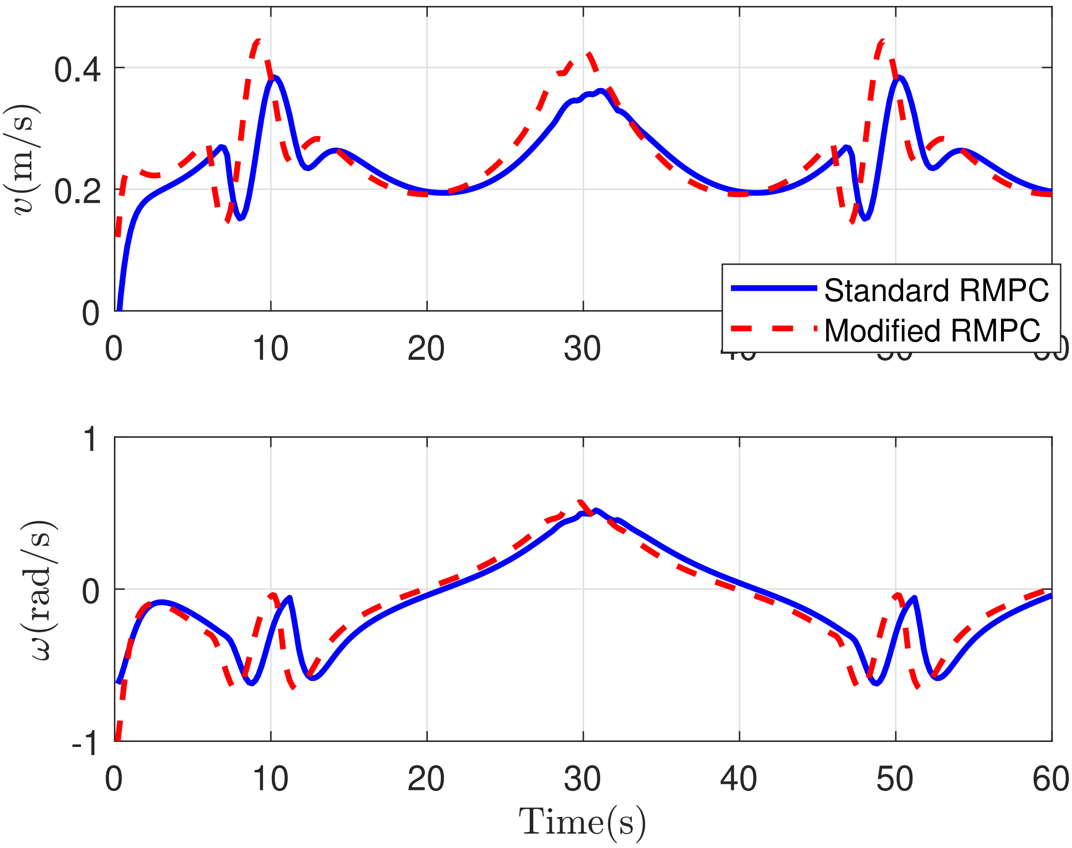

Figure 23 – Comparison of the control signals of linear and angular velocity commands in simulation analysis.

despite the degraded transient performance, the proposed solution significantly improves trajectory tracking under time-varying references and maintaining the computation time when comparing to the original solution.

The performance between the two methods could be analyzed in Figure 25 that shows the tracking error evolution. It is evident that the RMPC with analytical target modification improves the tracking properties. Different from the standard RMPC for tracking piecewise constant reference, the modified RMPC presents a null tracking error when the target is admissible.

5.4.5 Experimental results

The experimental results of the trajectory tracking of the Cleapath Husky UGV vehicle were execute on an epoxy industrial floor, in which the control law for both strategies were calculated on a computational platform AMD® Ryzen® 7 5700u, 32GB RAM, Ubuntu 20.04 LTS. To gather the global position and orientation of the robot, the optical motion tracking system OptiTrack (NaturalPoint, 2024) was used. This system estimates the pose $\mathbf{p} = \begin{bmatrix} x(t) & y(t) & \theta(t) \end{bmatrix}^\top$ related to the global reference frame $O_w \hat{x}_w \hat{y}_w$,

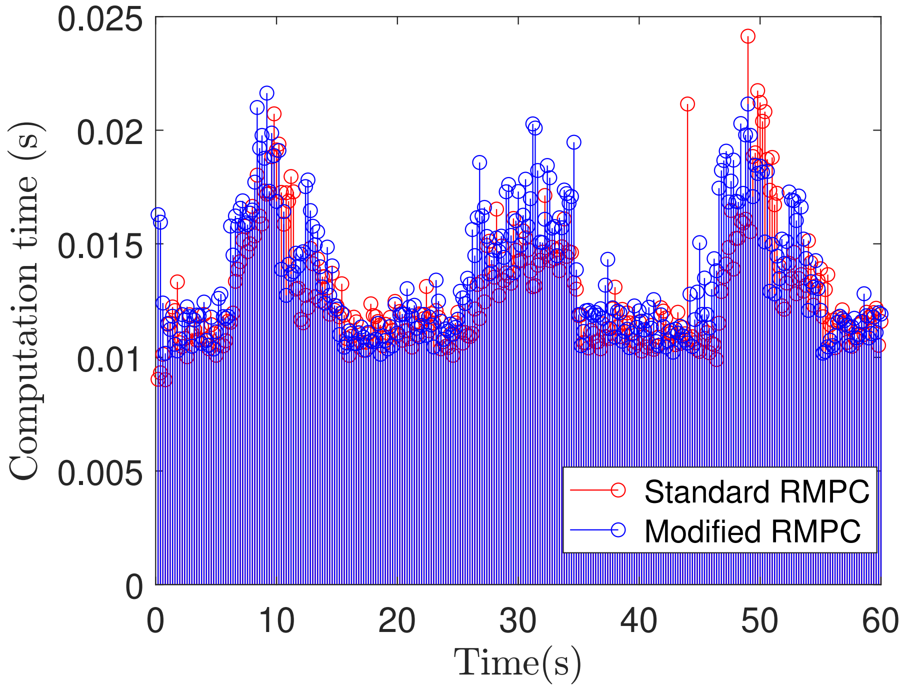

Figure 24 – Comparison of the computation time of the optmization solution during the entire simulation.

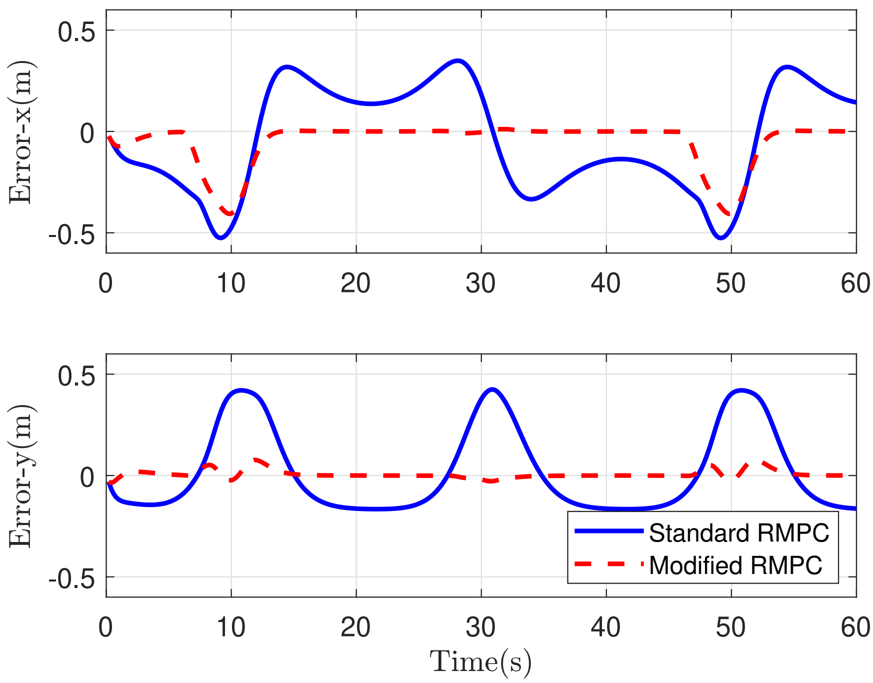

Figure 25 – Tracking error evolution from simulation analysis.

as shown in Figure 20. The optmization step was solved using the Python library *qpsolvers* (CARON et al., 2024).

The same sampling interval is used ($T_S = 0.2$ s and the linear and angular velocity commands are bounded such as $0 \leq v_c[k] \leq 0.7$ (m/s) and $|\omega_c[k]| < 1$ (rad/s). Similar to the simulated case, the workspace is limited by $|x[k]| \leq 2$ (m) and $|y[k]| \leq 2$ (m). The parameters chosen for the lemniscate are $\alpha = 1.3$, $\beta = 1.0$, $x_d = 0.0$ m, $y_d = 0.0$ m, and the natural frequency of the trajectory is $\omega_n = 2\pi/40$ rad/s.

The MPC design parameters are similar to the one presented in the simulated results. $Q = R = diag(1, 1)$, P and K is the LQR solution, and $T = 1000||P||$. The disturbances w were assumed to be bounded such that $|w| \leq \begin{bmatrix} 0.03 & 0.03 \end{bmatrix}^\top$. Also, the same feedback linearization parameter was chosen ($b = 0.3$) and the hard constraints are reorganized as follows ($z_1[k]| \leq 1.7$ (m) and $|z_2[k]| \leq 1.7$ (m)).

With the start point set to the center of the lemniscate trajectory, i.e. $x[0] = 0$, $y[0] = 0$, Figure 26 and 27 show that the effectiveness of the modified RMPC in a practical application, in which presents an offset free behavior when facing a time-varying (and periodic) reference.

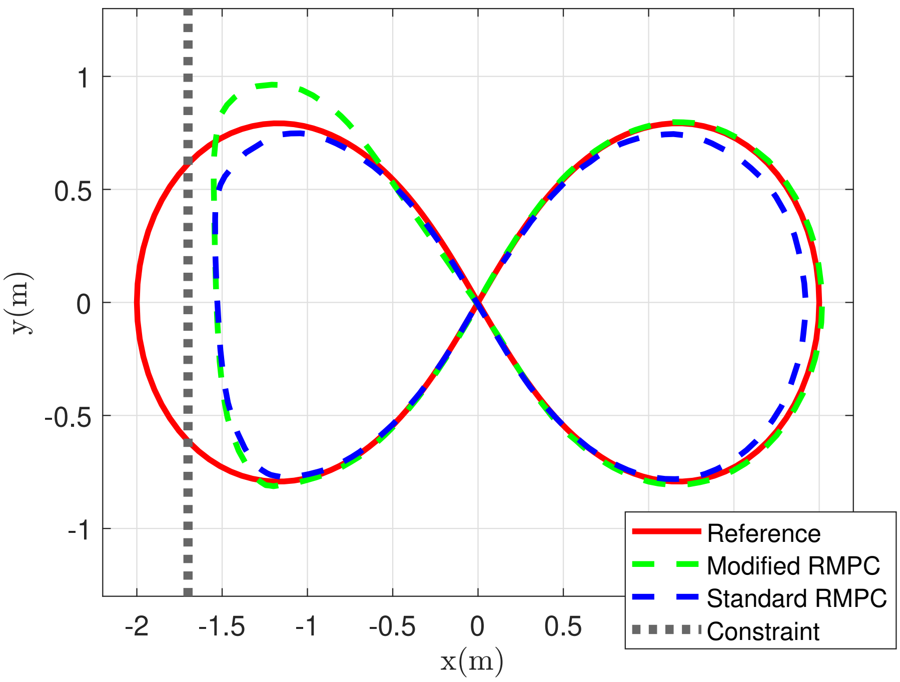

Figure 26 – Experimental Lemniscate trajectory comparison of the UGV. Demonstration of the experiment is available at <https://youtu.be/CfayXUjJWuw>.

The degraded transient performance is also observed in Figure 27 while recovering from the unfeasible reference. The trajectory taken from the controller in such condition is feasible even if the reference surpasses the hard constraint, which is done by the optimization of the offset cost function chosen, selecting best possible target from the optimization criteria.

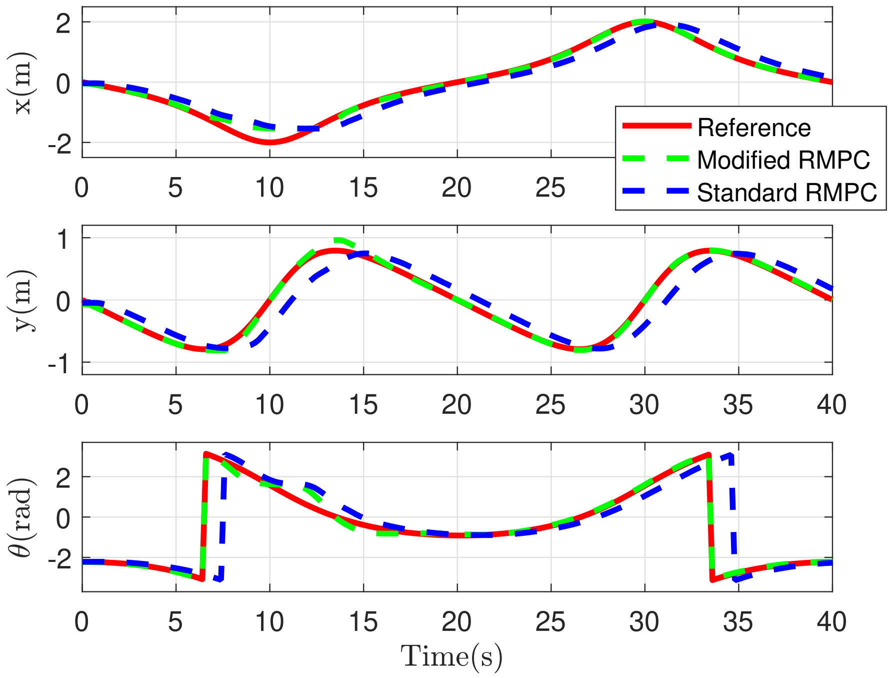

Figure 27 – Comparison of the position and orientation of the UGV.

Figure 28 presents the control inputs calculated from the optimal controller of the modified RMPC (v_c and ω_c). Additionally, the result shows also the estimated linear and angular velocity gathered by the OptiTrack, showing that some high-frequency signals are imposed to the system as additive disturbances, in which the system responds as expected, having an null-error offset tracking property even in such condition which is visible on the tracking error evolution shown in Figure 29.

5.5 Conclusions

This chapter presents a synthesis of the RMPC that was proposed in (SANTOS; CUNHA, 2023) to track piecewise constant reference in which the constraint tightening is executed based on the propagation of the worst case of disturbance within the prediction horizon. Such feature combined with the additional artificial target as in (LIMÓN et al., 2008) provides a safe operation of the controlled system even if the desired target is not admissible, converging to the closest admissible equilibrium given an offset optimization criteria. The main contribution of this chapter is done by a new method for trajectory tracking by means of a reverse target computation strategy.

The main benefits of the novel strategy is verified from a simulation analysis and an experimental evaluation. An offset-free tracking error is evaluated from the new approach, and is observed that steady-state performance is significantly improved for

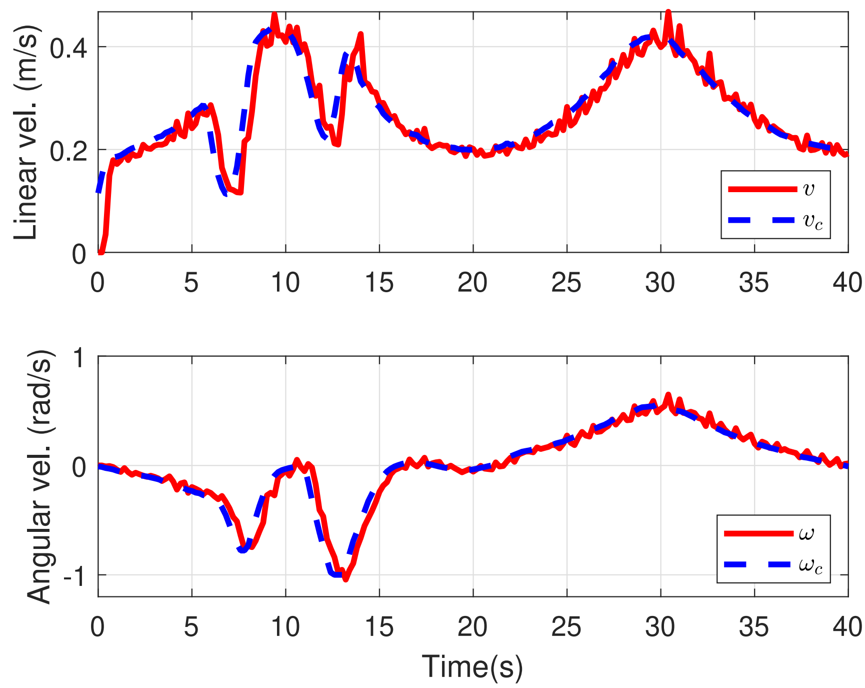

Figure 28 – Linear (v_c) and angular (ω_c) velocity.

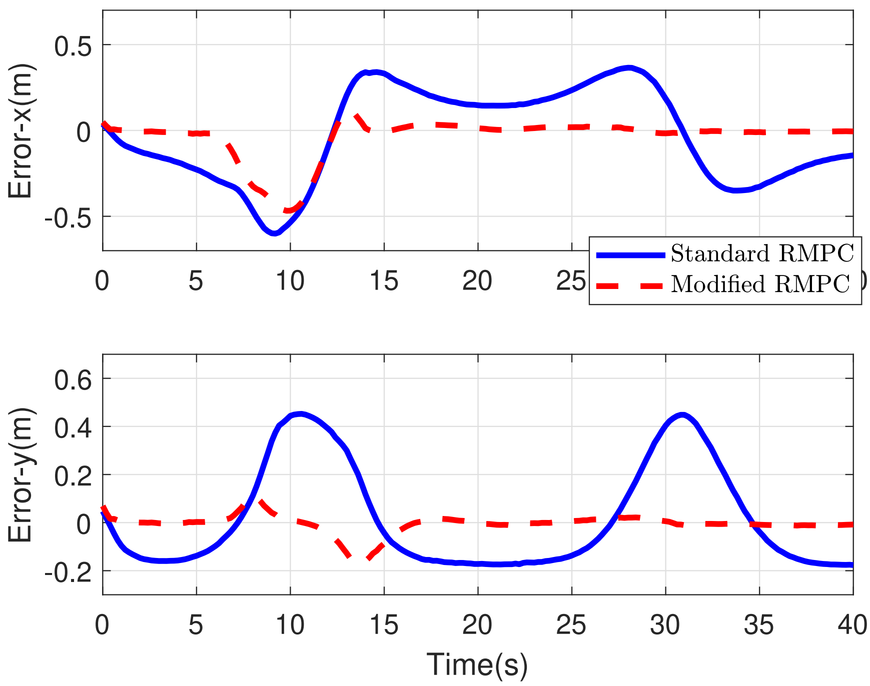

Figure 29 – Tracking error evolution from experimental analysis.

admissible targets when the system is subject to a periodic lemniscate trajectory. Recursive feasibility is ensured and average time to compute the control law is preserved from the original formulation by comparing the time for solving the optimization on both strategies. Sufficient conditions for target admissibility have been shown by forcing the reference to activate hard constraints of the system and its feasibility on experimental cases have been demonstrated by using a skid steered UGV.

Also, there is a difference between the simulated and experimental results since the model used does not fully covers the skid steering model in a way that the lateral slips seen in (HASHEMI; HE; JOHANSSON, 2022) are not considered. Despite that, the robot is still able to track the reference with null offset tracking error when the target is admissible, and following safely the closest admissible target set when the reference activates the workspace restriction.

6 Conclusions

This thesis presented contributions about simplified approaches for the application cases of the MPC for tracking time-varying references. Initially, it was presented two simplified strategies that were able to handle setpoints set by dynamic signals.

A DMC evaluation under the tracking problem of periodic references has been done, in which the compromise between tracking index and robustness was analytically demonstrated, such as tracking properties should be achieved with a direct loss of robustness as consequence of an aggressive tuning. The filtered DMC is then analyzed for the tracking properties. The prediction error filter then played a fundamental role on establish robustness properties as an additional degree of freedom of the MPC, allowing the system to follow references properly by reducing the effects of internal and measurement noises that an aggressive standard DMC is not able to deal with without alleviating tuning configurations. The theoretical analysis have been demonstrated through simulated and experimental results of a nonlinear model of a temperature control of a thermorresistive sensor, which was compared against the standard DMC. It has been seen that the effect of the reduced bandwidth of the filter results in degraded regulatory performance, although the effect of voltage (input) variation is significantly reduced in such scenarios. Experimental results reinforced the simulated analysis and it was opportune due to the natural occurrence of noise effects, modelling errors, and external disturbances in both step response identification and test execution.

A receptance based MPC for tracking multibody system has been proposed as a data-driven approach that is able to track time-varying targets successfully without the full knowledge of the internal modelling parameters of the multi-body system and a modified cost function was proposed in order to provide null tracking error for periodic references. The premise stated for periodic references could be extended for some non-periodic time-varying target under simple statement that the system is minimum phase. It is also shown that the proposed strategy implicitly imposes an optimal feed-forward control in terms of the desired input increments, which is defined by the known sequence of future references. To evaluate the properties of the new proposal, a linear MPC is simulated on a underactuated two-link system and its results was compared by standard GPC and DMC approaches. The results illustrate the main advantages and benefits of the strategy, in which the tracking properties are significantly improved under time-varying references, while it is possible to alleviate sensitivity of the tracking error with respect to the design parameters.

Finally, a new RMPC for tracking targets defined by dynamic signals has been proposed based on a simple target modification based on a reverse target computation. Despite this work presents the main benefits of the modified approach on a specific RMPC based on constraint tightening, such idea can be extended on any similar strategy following the same concepts for tracking piecewise constant reference as in (SÁNCHEZ et al., 2023; LIMÓN et al., 2010). The controller has been evaluated in both simulation and experimental scenario, by testing its effectiveness on a *Clearpath Husky A200* UGV vehicle. The comparison from a standard robust optimal controller showed improvement of steady-state trajectory tracking under admissible targets, and respecting safe operation when the trajectory is not admissible, by converging naturally the target to an admissible target considering a robust set of constraints. Despite the simplicity of the target modification, computational complexity is maintained, showing potential extensions in other applications such as obstacle avoidance features, and consideration of chance constraints. A preliminary result regarding a stochastic approach has been presented in the *Congresso Brasileiro de Automática*, but has not been discussed in this thesis, since the discussion is out of the deterministic scope of this work, but it is an interesting topic for further research.

6.1 Main contributions

This thesis provided the following main contributions to the subject of MPC for tracking time-varying references:

- The evaluation of the filtered DMC applied to output tracking under varying references, whose prediction filter guarantees aggressive tunings in the same time that robustness to deal with internal and measurement noise is assured;
- The role of high-noise disturbance effects was analyzed in the analytical solution of the FDMC. Thus, a proper filter design allows to deal with such uncertainties without interfere with the DMC control law, demonstrating the addition of the degree of freedom in this controller;
- The conflicting objectives of the standard MPC cost function is discussed, showing that it is not possible to achieve tracking properties with a null cost function value under condition of time-varying references;
- A simple modification of the cost function is proposed to ensure null values of the cost function while tracking non-constant references, such as the decision variables (input increments) are penalized in terms of the deviation between the values chosen and a virtual input target. The virtual target added can be defined by means of an unconstrained optimization problem based on the premise that the reference is periodic;

- A receptance modelling framework for prediction of a DMC and/or GPC strategy. The proposal have been applied to control a multibody mechanical system which simplifies significantly the modelling process;

- A simple target modification has been proposed for a set of robust MPC for tracking piecewise constant references, which improves the property of trajectory tracking of the system when it is subject to periodic references, although it can be extended to other dynamic operations;

- The experimental evaluation of MPC for tracking time-varying references have been demonstrated, showing the effectiveness of a filtered DMC and a modified RMPC on different types of application, showing promising and computationally efficient solutions for other practical applications.

6.2 List of publications

The following lists presents the published papers developed within the scope of this thesis.

6.2.1 Paper published in journal

- **(Pereira; Santos; Araújo)** Receptance-based model predictive control of multibody systems with time-varying references. *Meccanica*. v. 59, n. 1, p. 33-48, 2024.

6.2.2 Accepted works in congress proceedings

- **(Pereira; Santos)** DMC filtrado aplicado ao controle de sistemas sujeitos a referências variantes no tempo. In: *Anais do Simpósio Brasileiro de Automação Inteligente*. Brazil - Manaus, 2023.

- **(Santos; Pereira)** Analytical reference compensation for tracking dynamic target signals with linear robust MPC strategies. In *Proceedings of the American Control Conference*. Canada - Toronto, 2024.

- **(Pereira; Santos)** Indirect-feedback Stochastic MPC for tracking piecewise constant references. In: *Anais do Congresso Brasileiro de Automática*. Brazil - Rio de Janeiro, 2024.

6.3 Perspective for future investigation

- Evaluate an extension of the filtered DMC approach for tracking time-varying references to unstable systems;
- Evaluate the applicability of the receptance-based MPC with a modified cost function on practical applications;
- Investigate transient performance, the error evolution outside the admissible region, and extensions to the nonlinear case of the modified RMPC;
- Extend the experimental analysis of the filtered DMC, the receptance-based MPC for tracking, and the modified RMPC under influence of time-varying and non-periodic references.
- Consider stochastic MPC approaches for tracking non-constant references.

Bibliography

AGUIAR, A. P.; HESPANHA, J. P.; KOKOTOVIC, P. V. Path-following for nonminimum phase systems removes performance limitations. *IEEE Transactions on Automatic Control*, IEEE, v. 50, n. 2, p. 234–239, 2005. Cited in page 75.

ALVARADO, I. Model predictive control for tracking constrained linear systems. *Doctor of Philosophy, Department of Systems Engineering and Automation, University of Sevilla Sevilla*, 2007. Cited in page 44.

ANDERSSON, J. A. E. et al. CasADi – A software framework for nonlinear optimization and optimal control. *Mathematical Programming Computation*, Springer, v. 11, n. 1, p. 1–36, 2019. Cited in page 43.

ARAÚJO, J. M. et al. Vibration control of a two-link flexible robot arm with time delay through the robust receptance method. *Applied Sciences*, MDPI AG, v. 11, n. 21, p. 9907, 2021. Cited in page 62.

ARAÚJO, J. M.; SANTOS, T. L. Control of second-order asymmetric systems with time delay: Smith predictor approach. *Mechanical Systems and Signal Processing*, v. 137, p. 106355, 2020. Special issue on control of second-order vibrating systems with time delay. Cited in page 58.

ARAÚJO, J. M.; SANTOS, T. L. M. Control of a class of second-order linear vibrating systems with time-delay: Smith predictor approach. *Mechanical Systems and Signal Processing*, v. 108, p. 173–187, 2018. Cited in page 58.

ÅSTRÖM, K. J.; WITTENMARK, B. *Computer-controlled systems: theory and design*. [S.l.]: Courier Corporation, 2013. Cited 2 times in pages 59 and 121.

BELLMAN, R. Dynamic programming and the numerical solution of variational problems. *Operations Research*, JSTOR, p. 277–288, 1957. Cited in page 38.

BEMPORAD, A.; CASAVOLA, A.; MOSCA, E. Nonlinear control of constrained linear systems via predictive reference management. *IEEE transactions on Automatic Control*, IEEE, v. 42, n. 3, p. 340–349, 1997. Cited in page 27.

BETTEGA, J.; RICHIEDEI, D. Trajectory tracking in an underactuated, non-minimum phase two-link multibody system through model predictive control with embedded reference dynamics. *Mechanism and Machine Theory*, Elsevier, v. 180, p. 105165, 2023. Cited 7 times in pages 57, 58, 62, 63, 64, 66, and 70.

BLANCHINI, F.; MIANI, S. Any domain of attraction for a linear constrained system is a tracking domain of attraction. *SIAM Journal on Control and Optimization*, SIAM, v. 38, n. 3, p. 971–994, 2000. Cited in page 27.

CAMACHO, E. F.; ALBA, C. B. *Model predictive control*. [S.l.]: Springer science & business media, 2013. Cited 8 times in pages 23, 25, 43, 64, 111, 114, 117, and 122.

CARON, S. et al. *qpsolvers: Quadratic Programming Solvers in Python*. 2024. Disponível em: <https://github.com/qpsolvers/qpsolvers>. Cited in page 93.

CHISCI, L.; ZAPPA, G. Dual mode predictive tracking of piecewise constant references for constrained linear systems. *International Journal of Control*, Taylor & Francis, v. 76, n. 1, p. 61–72, 2003. Cited in page 27.

CLARKE, D. W.; MOHTADI, C.; TUFFS, P. S. Generalized predictive control—part i. the basic algorithm. *Automatica*, Elsevier, v. 23, n. 2, p. 137–148, 1987. Cited in page 24.

CUTLER, C. R.; RAMAKER, B. L. Dynamic matrix control?? a computer control algorithm. In: *joint automatic control conference*. [S.l.: s.n.], 1980. p. 72. Cited in page 24.

DAI, Y. et al. A fast tube model predictive control scheme based on sliding mode control for underwater vehicle-manipulator system. *Ocean Engineering*, v. 254, p. 111259, 2022. ISSN 0029-8018. Disponível em: <https://www.sciencedirect.com/science/article/pii/S0029801822006539>. Cited in page 28.

DANTAS, N. J.; DOREA, C. E.; ARAUJO, J. M. Partial pole assignment using rank-one control and receptance in second-order systems with time delay. *Meccanica*, Springer, v. 56, n. 2, p. 287–302, 2021. Cited in page 70.

de J. Souza, E. L.; SANTOS, T. L. Transient compensation for thermistor-based sensors in constant temperature configuration. *Sensors and Actuators A: Physical*, v. 305, p. 111920, 2020. ISSN 0924-4247. Disponível em: <https://www.sciencedirect.com/science/article/pii/S0924424719317121>. Cited in page 50.

D'JORGE, A. et al. Stochastic model predictive control for tracking linear systems. *Optimal Control Applications and Methods*, Wiley Online Library, v. 41, n. 1, p. 65–83, 2020. Cited in page 84.

FARONI, M. et al. Predictive inverse kinematics for redundant manipulators with task scaling and kinematic constraints. *IEEE Transactions on Robotics*, IEEE, v. 35, p. 278–285, 2 2019. ISSN 1552-3098. Disponível em: <https://ieeexplore.ieee.org/document/8477138/>. Cited in page 31.

FAULWASSER, T.; FINDEISEN, R. Nonlinear model predictive control for constrained output path following. *IEEE Transactions on Automatic Control*, IEEE, v. 61, n. 4, p. 1026–1039, 2015. Cited 2 times in pages 75 and 76.

FAULWASSER, T. et al. Implementation of nonlinear model predictive path-following control for an industrial robot. *IEEE Transactions on Control Systems Technology*, IEEE, v. 25, n. 4, p. 1505–1511, 2016. Cited 2 times in pages 75 and 76.

FERNANDES, D. et al. Dmc controller design for an integrated allam cycle and air separation plant. *Computers & Chemical Engineering*, Elsevier, v. 141, p. 107019, 2020. Cited in page 24.

FERNANDES, D. et al. Dmc controller design for an integrated allam cycle and air separation plant. *Computers & Chemical Engineering*, v. 141, p. 107019, 2020. ISSN 0098-1354. Cited in page 45.

FERRAMOSCA, A. et al. Mpc for tracking with optimal closed-loop performance. In: IEEE. *2008 47th IEEE Conference on Decision and Control*. [S.l.], 2008. p. 4055–4060. Cited 4 times in pages 27, 35, 42, and 44.

FNADI, M. et al. Constrained model predictive control for dynamic path tracking of a bi-steerable rover on slippery grounds. *Control Engineering Practice*, v. 107, p. 104693, 2021. ISSN 0967-0661. Disponível em: <https://www.sciencedirect.com/science/article/pii/S096706612030263X>. Cited in page 28.

FRANKLIN, T. S.; ARAÚJO, J. M.; SANTOS, T. L. Receptance-based robust stability criteria for second-order linear systems with time-varying delay and unstructured uncertainties. *Mechanical Systems and Signal Processing*, v. 149, p. 107191, 2021. ISSN 0888-3270. Cited in page 58.

GARCIA, C. E.; MORSHEDI, A. Quadratic programming solution of dynamic matrix control (qdmc). *Chemical Engineering Communications*, Taylor & Francis, v. 46, n. 1-3, p. 73–87, 1986. Cited in page 24.

GE, L. et al. Towards longitudinal and lateral coupling control of autonomous vehicles using offset free mpc. *Control Engineering Practice*, v. 121, p. 105074, 2022. ISSN 0967-0661. Disponível em: <https://www.sciencedirect.com/science/article/pii/S0967066122000065>. Cited in page 28.

GEORGE, K.; VERHAEGEN, M.; SCHERPEN, J. M. Stable inversion of mimo linear discrete time non-minimum phase systems. In: UNIVERSITY OF GRONINGEN, RESEARCH INSTITUTE OF TECHNOLOGY AND MANAGEMENT. *7th Mediterranean Conference on Control and Automation (MED99) Haifa, Israel*. [S.l.], 1999. p. 267–281. Cited in page 78.

GILBERT, E. G.; KOLMANOVSKY, I.; TAN, K. T. Nonlinear control of discrete-time linear systems with state and control constraints: A reference governor with global convergence properties. In: IEEE. *Proceedings of 1994 33rd IEEE Conference on Decision and Control*. [S.l.], 1994. v. 1, p. 144–149. Cited in page 27.

HASHEMI, E.; HE, X.; JOHANSSON, K. H. Robust slip-aware fusion for mobile robots state estimation. *IEEE Robotics and Automation Letters*, IEEE, v. 7, n. 3, p. 7896–7903, 2022. Cited 2 times in pages 84 and 96.

JIANG, Q. et al. Multimodel generalized predictive control of a heat-pipe reactor coupled with an open-air brayton cycle. *Energy*, Elsevier, v. 279, p. 128032, 2023. Cited in page 25.

JIN, Z. et al. Gaussian process-based nonlinear predictive control for visual servoing of constrained mobile robots with unknown dynamics. *Robotics and Autonomous Systems*, v. 136, p. 103712, 2021. ISSN 0921-8890. Disponível em: <https://www.sciencedirect.com/science/article/pii/S0921889020305522>. Cited in page 29.

JUNIOR, J. R. C. et al. Deteccao de falhas incipientes de sensor em um processo térmico com controle preditivo tipo dynamic matrix control. In: *Anais do Simpósio Brasileiro de Automática 2013*. [S.l.: s.n.], 2013. p. 1–6. Cited in page 53.

KANG, E. et al. Neural network-based model predictive tracking control of an uncertain robotic manipulator with input constraints. *ISA Transactions*, v. 109, p. 89–101, 3 2021. ISSN 00190578. Disponível em: <https://linkinghub.elsevier.com/retrieve/pii/S0019057820304043>. Cited in page 31.

KARG, B.; LUCIA, S. Efficient representation and approximation of model predictive control laws via deep learning. *IEEE Transactions on Cybernetics*, IEEE, v. 50, n. 9, p. 3866–3878, 2020. Cited in page 31.

KERRIGAN, E. C.; MACIEJOWSKI, J. M. Soft constraints and exact penalty functions in model predictive control. In: *Control 2000 Conference, Cambridge*. [S.l.: s.n.], 2000. p. 2319–2327. Cited in page 26.

KLOPOT, T. et al. Tuning strategy for dynamic matrix control with reduced horizons. *ISA transactions*, Elsevier, v. 76, p. 145–154, 2018. Cited in page 24.

KLOPOT, T. et al. Tuning strategy for dynamic matrix control with reduced horizons. *ISA Transactions*, v. 76, p. 145–154, 2018. ISSN 0019-0578. Cited in page 45.

KÖHLER, J.; MÜLLER, M. A.; ALLGÖWER, F. A nonlinear model predictive control framework using reference generic terminal ingredients. *IEEE Transactions on Automatic Control*, IEEE, v. 65, n. 8, p. 3576–3583, 2019. Cited 2 times in pages 75 and 76.

KÖHLER, J.; MÜLLER, M. A.; ALLGÖWER, F. A nonlinear tracking model predictive control scheme for dynamic target signals. *Automatica*, Elsevier, v. 118, p. 109030, 2020. Cited 4 times in pages 27, 75, 76, and 77.

KÖHLER, J.; MÜLLER, M. A.; ALLGÖWER, F. Analysis and design of model predictive control frameworks for dynamic operation—an overview. *Annual Reviews in Control*, Elsevier, v. 57, p. 100929, 2024. Cited 2 times in pages 32 and 75.

KÖHLER, M. et al. Transient performance of mpc for tracking. *IEEE Control Systems Letters*, IEEE, 2023. Cited 2 times in pages 27 and 44.

KÖHLER, M.; MÜLLER, M. A.; ALLGÖWER, F. Distributed mpc for self-organized cooperation of multi-agent systems. *IEEE Transactions on Automatic Control*, IEEE, 2024. Cited in page 27.

KOUVARITAKIS, B.; ROSSITER, J.; CHANG, A. Stable generalised predictive control: an algorithm with guaranteed stability. In: IET. *IEE Proceedings D (Control Theory and Applications)*. [S.l.], 1992. v. 139, n. 4, p. 349–362. Cited in page 26.

LIMA, D. M.; NORMEY-RICO, J. E.; SANTOS, T. L. M. Temperature control in a solar collector field using filtered dynamic matrix control. *ISA transactions*, Elsevier, v. 62, p. 39–49, 2016. Cited 4 times in pages 24, 46, 48, and 50.

LIMÓN, D. et al. Mpc for tracking of piece-wise constant references for constrained linear systems. *IFAC Proceedings Volumes*, Elsevier, v. 38, n. 1, p. 135–140, 2005. Cited 2 times in pages 27 and 36.

LIMÓN, D. et al. Mpc for tracking piecewise constant references for constrained linear systems. *Automatica*, Elsevier, v. 44, n. 9, p. 2382–2387, 2008. Cited 6 times in pages 31, 35, 42, 43, 44, and 94.

LIMÓN, D. et al. Robust tube-based mpc for tracking of constrained linear systems with additive disturbances. *Journal of Process Control*, Elsevier, v. 20, n. 3, p. 248–260, 2010. Cited 2 times in pages 78 and 98.

LIMÓN, D. et al. Nonlinear mpc for tracking piece-wise constant reference signals. *IEEE Transactions on Automatic Control*, IEEE, v. 63, n. 11, p. 3735–3750, 2018. Cited 2 times in pages 27 and 31.

LIMÓN, D. et al. Mpc for tracking periodic references. *IEEE Transactions on Automatic Control*, IEEE, v. 61, n. 4, p. 1123–1128, 2015. Cited 3 times in pages 27, 75, and 76.

LIU, X. et al. Mpc-based high-speed trajectory tracking for 4wis robot. *ISA Transactions*, v. 123, p. 413–424, 2022. ISSN 0019-0578. Disponível em: <https://www.sciencedirect.com/science/article/pii/S0019057821002731>. Cited in page 28.

MAGNI, L.; SEPULCHRE, R. Stability margins of nonlinear receding-horizon control via inverse optimality. *Systems & Control Letters*, Elsevier, v. 32, n. 4, p. 241–245, 1997. Cited in page 26.

MARKOVSKY, I.; DÖRFLER, F. Behavioral systems theory in data-driven analysis, signal processing, and control. *Annual Reviews in Control*, Elsevier, v. 52, p. 42–64, 2021. Cited 2 times in pages 24 and 45.

MARQUIS, P.; BROUSTAIL, J. Smoc, a bridge between state space and model predictive controllers: application to the automation of a hydrotreating unit. *IFAC Proceedings Volumes*, Elsevier, v. 21, n. 4, p. 37–45, 1988. Cited in page 116.

MAYNE, D. Nonlinear model predictive control: Challenges and opportunities. *Nonlinear model predictive control*, Springer, p. 23–44, 2000. Cited 4 times in pages 23, 26, 37, and 116.

MAYNE, D. Q. et al. Constrained model predictive control: Stability and optimality. *Automatica*, Elsevier, v. 36, n. 6, p. 789–814, 2000. Cited 7 times in pages 25, 26, 35, 38, 43, 44, and 81.

MINNITI, M. V. et al. Whole-body mpc for a dynamically stable mobile manipulator. *IEEE Robotics and Automation Letters*, IEEE, v. 4, p. 3687–3694, 2 2019. ISSN 2377-3766. Disponível em: <https://ieeexplore.ieee.org/document/8758922/http://arxiv.org/abs/1902.10415http://dx.doi.org/10.1109/LRA.2019.2927955>. Cited in page 29.

MOTTERSHEAD, J. E.; RAM, Y. M. Inverse eigenvalue problems in vibration absorption: passive modification and active control. *Mechanical systems and signal processing*, Elsevier, v. 20, n. 1, p. 5–44, 2006. Cited in page 57.

MOTTERSHEAD, J. E. et al. Active vibration control experiments on an agustawestland w30 helicopter airframe. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, v. 226, n. 6, p. 1504–1516, 2012. Cited in page 70.

MUSKE, K. R. Steady-state target optimization in linear model predictive control. In: IEEE. *Proceedings of the 1997 American control conference (Cat. No. 97CH36041)*. [S.l.], 1997. v. 6, p. 3597–3601. Cited in page 26.

MUSKE, K. R.; RAWLINGS, J. B. Model predictive control with linear models. *AIChE Journal*, Wiley Online Library, v. 39, n. 2, p. 262–287, 1993. Cited in page 26.

NADERI, E.; KHORASANI, K. Inversion-based output tracking and unknown input reconstruction of square discrete-time linear systems. *Automatica*, Elsevier, v. 95, p. 44–53, 2018. Cited in page 78.

NaturalPoint. *Motion Capture Systems - OptiTrack Webpage*. 2024. [Accessed: July-2024]. Disponível em: <https://optitrack.com>. Cited in page 91.

NICOLAO, G. D.; MAGNI, L.; SCATTOLINI, R. On the robustness of receding-horizon control with terminal constraints. *IEEE Transactions on Automatic Control*, IEEE, v. 41, n. 3, p. 451–453, 1996. Cited in page 26.

NICOLIS, D.; ALLEVI, F.; ROCCO, P. Operational space model predictive sliding mode control for redundant manipulators. *IEEE Transactions on Robotics*, IEEE, v. 36, p. 1348–1355, 8 2020. ISSN 1552-3098. Disponível em: <https://ieeexplore.ieee.org/document/9026917/>. Cited in page 30.

NORMEY-RICO, J. Controle preditivo baseado em modelo. *Enciclopédia de Automática*, Blucher São Paulo, v. 2, 2007. Cited in page 114.

NUBERT, J. et al. Safe and fast tracking on a robot manipulator: Robust mpc and neural network control. *IEEE Robotics and Automation Letters*, Institute of Electrical and Electronics Engineers Inc., v. 5, p. 3050–3057, 4 2020. ISSN 23773766. This paper proposes. Cited in page 31.

PECCIN, V. B. et al. Fast constrained dynamic matrix control algorithm with online optimization. *Revista Iberoamericana de Automática e Informática industrial*, 2022. Cited in page 45.

PEREIRA, B. S.; SANTOS, T. L.; ARAÚJO, J. M. Receptance-based model predictive control of multibody systems with time-varying references. *Meccanica*, Springer, v. 59, n. 1, p. 33–48, 2024. Cited in page 57.

PEREIRA, B. S. S.; SANTOS, T. L. M. Dmc filtrado aplicado ao controle de sistemas sujeitos a referências variantes no tempo. In: *Anais do Simpósio Brasileiro de Automação Inteligente 2023*. [S.l.: s.n.], 2023. Cited in page 45.

PEREIRA, M. et al. Application to a drinking water network of robust periodic mpc. *Control Engineering Practice*, Elsevier, v. 57, p. 50–60, 2016. Cited 2 times in pages 75 and 76.

PEREIRA, M. et al. Robust model predictive controller for tracking changing periodic signals. *IEEE Transactions on Automatic Control*, IEEE, v. 62, n. 10, p. 5343–5350, 2016. Cited 2 times in pages 75 and 76.

PICCINELLI, N.; MURADORE, R. A bilateral teleoperation with interaction force constraint in unknown environment using non linear model predictive control. *European Journal of Control*, v. 62, p. 185–191, 2021. ISSN 0947-3580. Disponível em: <https://www.sciencedirect.com/science/article/pii/S0947358021000881>. Cited in page 30.

POPESCU, R. I. et al. Design and development of a novel type of table tennis aerial robot player with tilting propellers. *Mechatronics*, v. 74, p. 102483, 2021. ISSN 0957-4158. Disponível em: <https://www.sciencedirect.com/science/article/pii/S0957415821000015>. Cited in page 28.

QIN, S. J.; BADGWELL, T. A. A survey of industrial model predictive control technology. *Control engineering practice*, Elsevier, v. 11, n. 7, p. 733–764, 2003. Cited 2 times in pages 24 and 57.

RAM, Y. M.; MOTTERSHEAD, J. E. Receptance method in active vibration control. *AIAA journal*, v. 45, n. 3, p. 562–567, 2007. Cited in page 57.

RICHALET, J. et al. Model predictive heuristic control. *Automatica (journal of IFAC)*, Pergamon Press, Inc. Elmsford, NY, USA, v. 14, n. 5, p. 413–428, 1978. Cited in page 23.

ROSSITER, J.; KOUVARITAKIS, B.; GOSSNER, J. R. Guaranteeing feasibility in constrained stable generalised predictive control. *IEE Proceedings-Control Theory and Applications*, IET, v. 143, n. 5, p. 463–469, 1996. Cited in page 27.

ROSSITER, J. A.; KOUVARITAKIS, B.; RICE, M. A numerically robust state-space approach to stable-predictive control strategies. *Automatica*, Elsevier, v. 34, n. 1, p. 65–73, 1998. Cited in page 26.

SÁNCHEZ, I. J. et al. Tracking periodic parametric references using model predictive control. *International Journal of Robust and Nonlinear Control*, Wiley Online Library, v. 33, n. 13, p. 7452–7470, 2023. Cited 4 times in pages 75, 76, 88, and 98.

SANDEEPKUMAR, R. et al. A unified ship manoeuvring model with a nonlinear model predictive controller for path following in regular waves. *Ocean Engineering*, v. 243, p. 110165, 2022. ISSN 0029-8018. Disponível em: <https://www.sciencedirect.com/science/article/pii/S0029801821014864>. Cited in page 28.

SANTOS, M. A. dos; FERRAMOSCA, A.; RAFFO, G. V. Set-point tracking mpc with avoidance features. *Automatica*, Elsevier, v. 159, p. 111390, 2024. Cited in page 84.

SANTOS, T. L. Modified reference tracking mpc for constrained linear systems. *International Journal of Systems Science*, Taylor & Francis, v. 49, n. 8, p. 1674–1684, 2018. Cited in page 41.

SANTOS, T. L.; CUNHA, V. M. Tractable robust mpc for tracking piecewise constant references based on nominal predictions. *Journal of Process Control*, Elsevier, v. 129, p. 103030, 2023. Cited 7 times in pages 76, 77, 78, 80, 83, 84, and 94.

SANTOS, T. L.; NORMEY-RICO, J. E. A generalised dynamic matrix control for unstable processes based on filtered predictions. *ISA transactions*, Elsevier, v. 136, p. 297–307, 2023. Cited 6 times in pages 24, 45, 48, 117, 118, and 119.

SANTOS, T. L. M. et al. Contribuições para o controle preditivo com compensação de atraso robusta. Florianópolis, SC, 2011. Cited 2 times in pages 26 and 113.

SANTOS, V. C. V.; SANTOS, T. L. M. On the variance propagation of thermistor-based measurements with a digital temperature controller. *Measurement*, v. 192, p. 110863, 2022. ISSN 0263-2241. Cited in page 51.

SATOH, T. et al. Predictive functional control of an axis positioning system with an estimator-based internal model. *Control Engineering Practice*, v. 86, p. 1–10, 2019. ISSN 0967-0661. Disponível em: <https://www.sciencedirect.com/science/article/pii/S0967066118304441>. Cited 2 times in pages 24 and 45.

SCIAVICCO, L.; SICILIANO, B. *Modelling and control of robot manipulators.* [S.l.]: Springer Science & Business Media, 2012. Cited 3 times in pages 27, 58, and 86.

SHEAD, L.; ROSSITER, J. A. Feasibility for non-square linear mpc. In: IEEE. *2007 American Control Conference*. [S.l.], 2007. p. 4464–4469. Cited in page 26.

SHI, Y. et al. Dual-mode fast dmc algorithm for the control of orc based waste heat recovery system. *Energy*, Elsevier, v. 244, p. 122664, 2022. Cited in page 45.

SLEIMAN, J. P. et al. A unified mpc framework for whole-body dynamic locomotion and manipulation. *IEEE Robotics and Automation Letters*, v. 6, p. 4688–4695, 2021. ISSN 23773766. Cited in page 29.

SOLOPERTO, R.; KÖHLER, J.; ALLGÖWER, F. A nonlinear mpc scheme for output tracking without terminal ingredients. *IEEE Transactions on Automatic Control*, IEEE, v. 68, n. 4, p. 2368–2375, 2022. Cited in page 27.

TEHRANI, M. G.; MOTTERSHEAD, J. An overview of the receptance method in active vibration control. *IFAC Proceedings Volumes*, Elsevier, v. 45, n. 2, p. 1174–1178, 2012. Cited in page 57.

WANG, L. *Model predictive control system design and implementation using MATLAB®*. [S.l.]: Springer Science & Business Media, 2009. Cited 2 times in pages 111 and 116.

WILLEMS, J. C. et al. A note on persistency of excitation. *Systems & Control Letters*, Elsevier, v. 54, n. 4, p. 325–329, 2005. Cited 2 times in pages 24 and 45.

WU, J. et al. Vision-based neural predictive tracking control for multi-manipulator systems with parametric uncertainty. *ISA Transactions*, v. 110, p. 247–257, 2021. ISSN 0019-0578. Disponível em: <https://www.sciencedirect.com/science/article/pii/S0019057820304602>. Cited in page 30.

XIANG, J.; ZHEN, C.; LI, D. Partial pole assignment with time delay by the receptance method using multi-input control from measurement output feedback. *Mechanical Systems and Signal Processing*, Elsevier, v. 66, p. 743–755, 2016. Cited in page 57.

XU, Q. et al. An offset-free composite model predictive control strategy for dc/dc buck converter feeding constant power loads. *IEEE Transactions on Power Electronics*, IEEE, v. 35, n. 5, p. 5331–5342, 2019. Cited in page 75.

XU, X. et al. Data-driven plant-model mismatch estimation for dynamic matrix control systems. *International Journal of Robust and Nonlinear Control*, Wiley Online Library, v. 30, n. 17, p. 7103–7129, 2020. Cited 2 times in pages 24 and 45.

YAN, Z. et al. Robust nonlinear model predictive control of a bionic underwater robot with external disturbances. *Ocean Engineering*, v. 253, p. 111310, 2022. ISSN 0029-8018. Disponível em: <https://www.sciencedirect.com/science/article/pii/S0029801822007028>. Cited in page 28.

YI, J. et al. Kinematic modeling and analysis of skid-steered mobile robots with applications to low-cost inertial-measurement-unit-based motion estimation. *IEEE transactions on robotics*, IEEE, v. 25, n. 5, p. 1087–1097, 2009. Cited in page 84.

ZHANG, C. et al. A variable self-tuning horizon mechanism for generalized dynamic predictive control on dc/dc boost converters feeding cpls. *IEEE Journal of Emerging and Selected Topics in Power Electronics*, IEEE, v. 11, n. 2, p. 1650–1660, 2022. Cited in page 25.

ZHANG, D.; HU, Y.; GAO, Y. Optimization control of a 330 mw drum boiler unit based on dmc algorithm and deb strategy. *ISA transactions*, Elsevier, v. 128, p. 435–449, 2022. Cited 2 times in pages 24 and 45.

ANNEX A – Implementation Aspects of the MPC

A.1 State Space MPC based on incremental control action

For the linear and time-invariant state space representation of a dynamic system written in form of (2.1) and (2.2) such as $f(.) = Ax[k] + Bu[k]$ and $h(.) = Cx[k] + Du[k]$, the prediction model is manipulated to consider $\Delta u[k] = u[k] - u[k-1]$ as the new system input,

$$\xi[k+1] = A_a\xi[k] + B_a\Delta u[k], \tag{A.1}$$

$$y[k] = C_a\xi[k] + D_a\Delta u[k], \tag{A.2}$$

and $\xi \in \mathbb{R}^{(n+p)}$ is the augment space vector such as $\xi[k] = \begin{bmatrix}\Delta x[k]^T & y[k]^T\end{bmatrix}^T$. This modification is widely used since it includes an integral action to the control law, which allows constant (or asymptotic constant) reference tracking and guarantees disturbance rejections (CAMACHO; ALBA, 2013).

Such predictor is presented in (WANG, 2009), and similar augmented representations are shown in (CAMACHO; ALBA, 2013). Consider that $\Delta x[k] = x[k] - x[k-1]$ the unitary difference of the system state. Despite the predictor modelling is slightly different from the from the usual state space equation, such formulation filters white noise coloured errors by inserting an integrator to the MPC formulation. Moreover regulation performance of the proposed controller is improved. Since the proposed system is time-invariant, the state evolution can be predicted from the iteration of (A.1) within a prediction horizon N_p.

$$\begin{aligned}
\xi[k+1|k] =& A_a\xi[k] + B_a\Delta u[k|k]\\
\xi[k+2|k] =& A_a\xi[k+1|k] + B_a\Delta u[k+1|k]\\
=& A_a^2\xi[k] + A_aB_a\Delta u[k|k] + B_a\Delta u[k+1|k]\\
\xi[k+3|k] =& A_a\xi[k+2|k] + B_a\Delta u[k+2|k]\\
=& A_a^3\xi[k] + A_a^2B_a\Delta u[k|k] + A_aB_a\Delta u[k+1|k] + B_a\Delta u[k+2|k]\\
&\vdots\\
\xi[k+N_p|k] =& A_a^N\xi[k] + A_a^{N_p-1}B_a\Delta u[k] + A_a^{N_p-2}B_a\Delta u[k+1] + \dots\\
&\dots + A_a^{N_p}B_a\Delta u[k+N_p-1].
\end{aligned}$$

Summarizing, the output prediction in any future instant j, such as $j < N_p$ is taken as follows

$$\hat{y}[k+j|k] = C_a A_a^j \xi[k] + \sum_{i=0}^{j-1} C_a A_a^{(j-i-1)} B_a \Delta u[k+i|k]. \tag{A.3}$$

Considering the sequence future input increments $\Delta\mathbf{u}$

$$\Delta\mathbf{u} = \begin{bmatrix} \Delta u[k|k] & \Delta u[k+1|k] & \dots & \Delta u[k+N_p-1|k] \end{bmatrix}^\top, \tag{A.4}$$

the predicted output evolution is determined as

$$\mathcal{Y} = \begin{bmatrix} \hat{y}[k+1|k] & \hat{y}[k+2|k] & \dots & \hat{y}[k+N_p|k] \end{bmatrix}^\top. \tag{A.5}$$

The output prediction $y[k+i|N]$ depends only by the current state $\xi[k]$ and the future control inputs to be properly chosen within an objective directive. For the sake of computational efficiency, a control horizon N_c is inserted in the proposed predictor in a way that $\Delta u[k+d-1|k] = 0$ for every $d > N_c$ and $N_c \leq N_p$. Therefore, the control horizon indicates how many future control inputs are available to steer the states and outputs within a prediction horizon N_p. Moreover, the sequence of future outputs in (A.5) is rewritten

$$\mathcal{Y}(\xi[k], \Delta\mathbf{u}) = \mathbf{F}\xi[k] + \Phi\Delta\mathbf{u}, \tag{A.6}$$

where the terms $\mathbf{F}\xi[k]$ and $\mathbf{\Phi}\Delta\mathbf{u}$ are the predicted free and forced responses respectively. The naming of these terms is straightforward since the first is simply the evolution of the system output without considering any input and taking account only its initial states at instant k, while the second is the evolution of the output considering only the input signals without considering any initial state. By simple inspection of (A.3) and considering the control horizon, the prediction matrices are defined as

$$\mathbf{F} = \begin{bmatrix} C_a A_a \\ C_a A_a^2 \\ C_a A_a^3 \\ \vdots \\ C_a A_a^{N_p} \end{bmatrix}, \tag{A.7}$$

$$\Phi = \begin{bmatrix} C_aB_a & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ C_aA_aB_a & C_aB_a & \mathbf{0} & \dots & \mathbf{0} \\ C_aA_a^2B_a & C_aA_aB_a & C_aB_a & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_aA_a^{N_p-1}B_a & C_aA_a^{N_p-2}B_a & C_aA_a^{N_p-3}B_a & \dots & C_aA_a^{N_p-N_c}B_a \end{bmatrix}, \tag{A.8}$$

with the matrix $\mathbf{0}$ being a matrix with zeros and appropriated dimensions. The proposed strategy considers that a objective function should be defined to have an optimal sequence of control inputs in which could consider regulation properties or economical problems. Independently from the choice of the objective function, the MPC controller usually consider the evolution of the predicted output (as the one shown in (A.6)) in its formulation. Inherent input delay should be considered in the predictor dynamics of the forced response matrix Φ by properly changing the general output prediction formulation (A.3) as stated in (SANTOS et al., 2011).

A.1.1 Objective function

For now, consider that the aim of the controller is that the future output y is steered to a reference signal $y_r[k]$ and the control effort Δu is simultaneously penalized within the prediction and control horizon. Thus, a common used objective function is quadratic as follows

$$J(N_p, N_c, y_r, \Delta u) = \sum_{j=1}^{N_p} ||\hat{y}[k+j|k] - y_r[k+j]||_Q^2 + \sum_{j=1}^{N_c} ||\Delta u[k+j-1|k]||_R^2, \tag{A.9}$$

where $Q \geq 0$ is the penalizing matrix that balances the quadratic error between future outputs and references, and $R > 0$ penalizes the control effort Δu. Without loss of generality for multivariable systems, consider that $Q \in \mathbb{R}^{p\times p}$ and $R \in \mathbb{R}^{m\times m}$. Defining the sequence of future references $\mathcal{W}$

$$\mathcal{W} = \begin{bmatrix} y_r[k+1] & y_r[k+2] & \dots & y_r[k+N_p] \end{bmatrix}^T, \tag{A.10}$$

the cost function is reinterpreted in a vectorized form

$$J = (\mathcal{Y} - \mathcal{W})^T \mathcal{Q} (\mathcal{Y} - \mathcal{W}) + \Delta\mathbf{u}^T \mathcal{R} \Delta\mathbf{u}, \tag{A.11}$$

such as $\mathcal{Q}$ and $\mathcal{R}$ are augmented forms[1] of Q and R respectively, as follows:

[1] Matrices Q and R are diagonally repeated N_p and N_c respectively.

$$\mathcal{Q} = \begin{bmatrix} Q & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & Q & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & Q & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & Q \end{bmatrix}, \tag{A.12}$$

$$\mathcal{R} = \begin{bmatrix} R & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & R & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & R & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & R \end{bmatrix}. \tag{A.13}$$

Therefore, the objective function proposed is defined as a scalar, positive definite function that consider the square of the prediction errors, and the squared value of increment of control inputs, both penalized by weighting matrices Q and R respectively. For now, the terminal cost is set to zero (i.e. $F(.) = 0$).

A.1.2 SSMPC Receding Horizon Control Law

The MPC strategy is done by finding the minimum of the cost function (A.11), subject to the constraints exposed in (2.8), resulting in an optimal sequence of input increments. The solution for the constrained problem is usually achieved by solving a general constrained QP problem stated as

$$\begin{aligned} \underset{\Delta u}{\text{minimize}} \quad & \Delta \mathbf{u}^\top H \Delta \mathbf{u} + b \Delta \mathbf{u} + f_0, \\ \text{subject to} \quad & \hat{y}[k+j|k] \in \mathbb{Y}, \quad j \in \mathbb{N}_{[1,N]}, \\ & u[k+j|k] \in \mathbb{U}, \quad j \in \mathbb{N}_{[0,N_u-1]}, \\ & \Delta u[k+j|k] \in \Delta\mathbb{U}, \quad j \in \mathbb{N}_{[0,N_u-1]}, \end{aligned} \tag{A.14}$$

where the decision variable Δu in the statement (A.14) results in the sequence of optimal inputs $\Delta \mathbf{u}^0$. Various solutions of the constrained QP problem is available at (CAMACHO; ALBA, 2013), hence this is not the focus of this work. Also, constructive aspects of the restrictions in (A.14) are depicted in (NORMEY-RICO, 2007).

The predicted outputs within a time horizon N are function of the initial state and the sequence of future input increments to be chosen as in (A.6), the cost function is then redefined in terms of $\Delta \mathbf{u}$ from (A.11):

$$J = \Delta \mathbf{u}^T (\mathcal{R} + \Phi^T \mathcal{Q} \Phi) \Delta \mathbf{u} + 2(\mathbf{F}\xi[k] - \mathcal{W})^T \mathcal{Q} \Phi \Delta \mathbf{u} + f_0, \tag{A.15}$$

such as b and the hessian matrix H is determined by inspection. Since the matrix $(\mathcal{R} + \Phi^T \mathcal{Q}\Phi)$ is quadratic and positive definite, there is a minimum of $J(.)$ and the solution for optimal sequence of control inputs exists under an admissible set of constraints $\Delta\tilde{\mathbf{u}} \in \mathcal{U}$

Albeit the minimization of the cost function provides the control sequencing along the control horizon at a given instant k, only the first element of the optimal sequence is applied as the control input. A new optimal sequence is then calculated again at instant $k+1$ time and the new first element applied, repeating this process at each sampling time. Such procedure defines the receding horizon control law of the MPC. Defining the optimal sequence $\Delta\mathbf{u}^0$ that provides a solution for the optimization problem

$$\Delta\mathbf{u}^0 = \begin{bmatrix} \Delta u^0[k|k] & \Delta u^0[k+1|k] & \dots & \Delta u^0[k+N_c-1|k] \end{bmatrix}, \tag{A.16}$$

the implicit state space MPC control law is the accumulation of the input increments $\Delta u^0[k|k]$ over time, i.e.

$$u[k] = u[k-1] + \Delta u^0[k|k], \tag{A.17}$$

considering the horizons N_p and N_c, and the penalizing gains Q and R, as offline tuning parameters of the proposed controller.

A.1.3 Explicit control law

As mentioned before, the solution of the implicit control law is done by means of QP solving. Still, an explicit control law may arise in a condition that no restrictions are imposed to the optimization problem, which is interesting to evaluate the closed-loop characteristics of the MPC.

Under the premise of no restrictions (i.e. $\mathbb{X} \in \mathbb{R}^n$ and $\mathbb{U} \in \mathbb{R}^m$), if $Q > 0$ and $R \geq 0$, the inequality $(\mathcal{R} + \Phi^T \mathcal{Q}\Phi) > 0$ is also true and the minimum of V exists and can be analytically given by solving

$$\frac{dV(.)}{d\Delta\tilde{\mathbf{u}}} = 2(\mathcal{R} + \Phi^T \mathcal{Q}\Phi)\Delta\tilde{\mathbf{u}} - 2\Phi^T Q(\mathcal{W} - \mathbf{F}\xi[k_i]) = 0, \tag{A.18}$$

therefore the optimal control increment sequence is

$$\Delta\mathbf{u}^0 = (\mathcal{R} + \Phi^T \mathcal{Q}\Phi)^{-1}\Phi^T \mathcal{Q}(\mathcal{W} - \mathbf{F}\xi[k_i]). \tag{A.19}$$

Still following the receding horizon principle, the control law is done by selecting the first m elements of $\Delta\mathbf{u}^0$. Therefore an explicit expression for $\Delta u^0[k_i|k_i]$ is achieved by taking the first m rows of (A.19).

$$\Delta u^0[k|k] = \overbrace{\begin{bmatrix}\mathbf{I}_m & \mathbf{0}_m & \mathbf{0}_m & \dots & \mathbf{0}_m\end{bmatrix}}^{N_c} \Delta \mathbf{u}^0, \tag{A.20}$$

and is reduced to the form

$$\Delta u^0[k|k] = K_r \mathcal{W} - K_{MPC}\xi[k]. \tag{A.21}$$

for $\mathbf{I}_m$ being an identity matrix with and $\mathbf{0}_m$ a square matrix of zeros with both having dimension $m \times m$.

Moreover, from (A.21), the unconstrained state space MPC should be interpreted as composition of linear state feedback control law and a feed-forward gain in terms of the future references, relying to be equivalent to a Internal Model Controller (MARQUIS; BROUSTAIL, 1988). The control law hereby stated is proposed on a premise of a measurable state, but a proper observer could be designed if the states are observable (WANG, 2009). Following such premise, if the state information is not directly available, a proper observer should be designed to feed the control loop.

The closed loop dynamics can be analyzed by substituting the control law (A.21) in the augmented state dynamics (A.1), resulting in

$$\xi[k+1] = (A - BK_{MPC})\xi[k] - BK_y\mathcal{W}, \tag{A.22}$$

having the closed-loop dynamics determined by the eigenvalues[2] of the matrix $(A - BK_{MPC})$, the unconstrained MPC operates akin to a state feedback control law. This allows for the placement of closed-loop poles in terms of K_{MPC}, enabling the state space MPC to effectively handle unstable and non-minimum phase systems. This capability relies on having a sufficiently accurate state space model that represent the plant dynamics under the necessary conditions of controlability and observability.

Despite the implication that MPC ensures stability even in unstable systems, this assertion is untrue in constrained scenarios, as optimality does not guarantee stability, particularly in nonlinear and constrained cases as stated in (MAYNE, 2000). However, it holds true that the optimal constrained solutions coincide with the unconstrained ones when any restrictions are activated within the MPC horizon.

The MPC exhibits an intriguing property, particularly evident in the initial controller and consistently observed in subsequent ones. The explicit control law clearly demonstrates the consideration of all future reference values within the prediction horizon during input calculation, achieved through the use of the reference gain K_y applied to the vector of future setpoints in (A.21). This feature offers a significant advantage, especially

[2] the eigenvalues λ are obtained by solving $det(\lambda I - (A - BK_{MPC})) = 0$.

in systems with anticipated references, such as robotics and industrial processes. Moreover, if only the current reference is available to the controller, the vector $\mathcal{W}$ is simply the repetition of this value over the prediction horizon.

A.2 Dynamic Matrix Control

Consider the SISO system with input $u[k]$ as input signal and output $y[k]$ or the sake of simplicity for initial understanding of the DMC algorithm, The main property of the DMC is the data-driven model for prediction that can be entirely defined from the response of a known input signal. Thereby the convolutional model output without disturbance is given by:

$$\bar{y}[k] = \sum_{i=1}^{\infty} g_i \Delta u[k-i], \tag{A.23}$$

defining $\Delta u[k]$ the input increment at instant k, $\bar{y}[k]$ the expected nominal output of $y[k]$ without disturbances, and g_j is the effect of the step signal on the output of the system measured in the instant $j \geq 0$. Therefore the convolutional model presented depends only by the step response coefficients g_i which could be achieved by using process data and is non-parametric, being able to represent any process behavior (SANTOS; NORMEY-RICO, 2023).

A.2.1 DMC Prediction Model

For generalization, the superposition principle is applied to consider the MIMO case. The multivariable step response from the input $r \in [1, \ldots, m]$ is defined as $g_k^r = [g_k^{1,r} \ldots g_k^{q,r}]$, in which $g_j^{l,r}$ represent the step response coefficients from a single input r to the output $l \in [1, \ldots, p]$, measured at instant $j \geq 0$. The MIMO convolutional model can be defined by simple experiments by applying isolated steps from each input at a time and gathering the effect of each output, resulting in

$$\bar{y}_l[k+j|k] = \sum_{r=1}^{m} \sum_{i=1}^{\infty} g_i^{l,r} \Delta u_r[k+j-i] \tag{A.24}$$

A disturbance model is typically defined in the prediction formulation of the MPC in order to achieve offset-free tracking responses in steady-state considering constant disturbances (CAMACHO; ALBA, 2013). Assume the prediction error $\eta_l[k] = y_l[k] - \bar{y}_l[k]$ at instant k. A candidate predictor is defined considering a constant prediction error:

$$y_l^c[k+j|k] = \sum_{r=1}^{m} \sum_{i=1}^{\infty} g_i^{l,r} \Delta u_r[k+j-i] + \eta_l[k], \tag{A.25}$$

and rewriting substituting the prediction error:

$$y_l^c[k+j|k] = \sum_{r=1}^{m}\sum_{i=1}^{j} g_i^{l,r}\Delta u_r[k+j-i] + \dots$$
$$\dots + \sum_{r=1}^{m}\sum_{i=j+1}^{\infty} g_i^{l,r}\Delta u_r[k+j-i] - \sum_{r=1}^{m}\sum_{i=1}^{\infty} g_i^{l,r}\Delta u_r[k-i] + y_l[k] \quad (A.26)$$
$$= \sum_{r=1}^{m}\sum_{i=1}^{j} g_i^{l,r}\Delta u_r[k+j-i] + \sum_{r=1}^{m}\sum_{i=1}^{\infty}(g_{j+i}^{l,r} - g_i^{l,r})\Delta u_r[k-i] + y_l[k]. \quad (A.27)$$

Moreover, the candidate predictor can be easily split into the forced and free response, in a way that the first part depends only by the future input increments, while the latter depends on the initial output measurement and past inputs. From the premise that the open-loop system is stable, the approximation $(g_{j+i}^{l,r} - g_i^{l,r}) \approx 0$ is true for $j \geq \tilde{N}_{l,r}$, considering $\tilde{N}_{l,r}$ a MPC parameter of the truncation time long enough to oversee the system dynamic behavior. Then the DMC predictor is given by:

$$\hat{y}_l[k+j|k] = \underbrace{\sum_{r=1}^{m}\sum_{i=1}^{j} g_i^{l,r}\Delta u_r[k+j-i]}_{\text{Forced Response}} + \underbrace{\sum_{r=1}^{m}\sum_{i=1}^{\tilde{N}_{l,r}}(g_{j+i}^{l,r} - g_i^{l,r})\Delta u_r[k-i] + y_l[k]}_{\text{Free Response}} \quad (A.28)$$

The following predictor shown in (A.28) is only valid for asymptotically stable systems. Otherwise, the value $(g_{j+i}^{l,r} - g_i^{l,r})$ does not converges to zero as time goes to $\tilde{N}_{l,r}$ and the predictor can't withhold the system dynamics. Thus the DMC will not solve the regulate problem. An extension of the DMC is applied for integrative systems, considering the approximation $(g_{j+i}^{l,r} - g_i^{l,r}) \approx k_l$ as true after the transient time, in which k_l is the velocity gain of the model (SANTOS; NORMEY-RICO, 2023).

For a given response of the output l occurred from the input r within a time horizon N, the expression is reinterpreted for a matrix form of (A.28). First, consider the vectors

$$\mathcal{Y}_l[k] = \begin{bmatrix}\hat{y}_l[k+1|k] & \dots & \hat{y}_l[k+N|k]\end{bmatrix}^\top, \quad (A.29)$$
$$\mathbf{\Delta u}_r[k] = \begin{bmatrix}\Delta u_r[k|k] & \dots & \Delta u_r[k+N_u-1|k]\end{bmatrix}^\top, \quad (A.30)$$
$$\Delta\mathcal{U}_r[k] = \begin{bmatrix}\Delta u_r[k-1] & \dots & \Delta u_r[k-\tilde{N}_{\max}]\end{bmatrix}^\top, \quad (A.31)$$

as the predicted values of output l, the future values of the input increment r, and past values of the same input respectively. For a generalization of the MIMO system, first consider the the response matrices of the output l from the input r, G^{lr} and G_f^{lr}:

$$G^{l,r} = \begin{bmatrix} g_1^{l,r} & 0 & \dots & 0 \\ g_2^{l,r} & g_1^{l,r} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_N^{l,r} & g_{N-1}^{l,r} & \dots & g_{N-N_u+1}^{l,r} \end{bmatrix},$$

$$G_f^{l,r} = \begin{bmatrix} g_2^{l,r} - g_1^{l,r} & g_3^{lr} - g_2^{l,r} & \dots & g_{\tilde{N}_{\max}+1}^{l,r} - g_{\tilde{N}_{\max}}^{lr} \\ g_3^{l,r} - g_1^{l,r} & g_4^{l,r} - g_2^{l,r} & \dots & g_{\tilde{N}_{\max}+2}^{l,r} - g_{\tilde{N}_{\max}}^{l,r} \\ \vdots & \vdots & \ddots & \vdots \\ g_{N+1}^{l,r} - g_1^{l,r} & g_{N+2}^{l,r} - g_2^{l,r} & \dots & g_{\tilde{N}_{\max}+N}^{l,r} - g_{\tilde{N}_{\max}}^{l,r} \end{bmatrix},$$

Thus, the MIMO DMC predictor is

$$\begin{bmatrix} \mathcal{Y}_1[k] \\ \mathcal{Y}_2[k] \\ \vdots \\ \mathcal{Y}_p[k] \end{bmatrix} = \begin{bmatrix} G^{1,1} & G^{1,2} & \dots & G^{1,p} \\ G^{2,1} & G^{2,2} & \dots & G^{2,p} \\ \vdots & \vdots & \ddots & \vdots \\ G^{p,1} & G^{p,2} & \dots & G^{p,m} \end{bmatrix} \begin{bmatrix} \mathbf{\Delta u}_1[k] \\ \mathbf{\Delta u}_2[k] \\ \vdots \\ \mathbf{\Delta u}_m[k] \end{bmatrix} + \dots$$
$$\dots + \begin{bmatrix} G_f^{1,1} & G_f^{1,2} & \dots & G_f^{1,p} \\ G_f^{2,1} & G_f^{2,2} & \dots & G_f^{2,p} \\ \vdots & \vdots & \ddots & \vdots \\ G_f^{p,1} & G_f^{p,2} & \dots & G_f^{p,m} \end{bmatrix} \begin{bmatrix} \Delta\mathcal{U}_1[k] \\ \Delta\mathcal{U}_2[k] \\ \vdots \\ \Delta\mathcal{U}_m[k] \end{bmatrix} + \begin{bmatrix} \mathbb{1}y_1[k] \\ \mathbb{1}y_2[k] \\ \vdots \\ \mathbb{1}y_p[k] \end{bmatrix}, \tag{A.32}$$

or in a reduced form:

$$\vec{\mathcal{Y}}[k] = \mathbf{G\Delta u}[k] + \mathcal{G}\Delta\mathcal{U}[k] + \vec{Y}[k] \tag{A.33}$$

The DMC predictor algorithm is then expressed in terms of the forced response matrix $\mathbf{G}$ and free response matrix $\mathcal{G}$. Additionally, consider that $\mathbb{1}y_l[k]$ is the measured output repeated N times in a column vector. As mentioned before, the prediction is only valid for stable system or integrative processes (the last one if an approximation is done by considering the velocity gain of the model. To handle with this issue, (SANTOS; NORMEY-RICO, 2023) proposes a filtered predictor that allows to consider unstable systems for DMC formulation.

A.2.2 Objective Function

The DMC strategy is defined from the optimal control sequence resulted from the minimization of the optimization problem that considers the future references, the weighting matrices ($R > 0$ and $Q \geq 0$) and its convex sets ($\mathbb{Y}, \mathbb{U}$ and $\Delta\mathbb{U}$) that describes the system restrictions, as follows:

$$\begin{aligned}
\underset{\Delta u}{\text{minimize}} \quad & \sum_{j=1}^{N} ||\hat{y}[k+j|k] - y_r[k+j]||_Q^2 + \sum_{j=0}^{N_u-1} ||\Delta u[k+j|k]||_R^2, && \text{(A.34a)}\\
\text{subject to} \quad & \hat{y}[k+j|k] \in \mathbb{Y}, \quad j \in \mathbb{N}_{[1,N]}, && \text{(A.34b)}\\
& u[k+j|k] \in \mathbb{U}, \quad j \in \mathbb{N}_{[0,N_u-1]}, && \text{(A.34c)}\\
& \Delta u[k+j|k] \in \Delta\mathbb{U}, \quad j \in \mathbb{N}_{[0,N_u-1]}, && \text{(A.34d)}
\end{aligned}$$

defining the vector of predicted outputs $\hat{y}[k] = [\hat{y}_1[k] \ \dots \ \hat{y}_p[k]]^\top$ and $y_r[k] = [y_{r,1}[k] \ \dots \ y_{r,p}[k]]^\top$ the output references. Alternatively, considering the vector of future references $\vec{\mathcal{W}} = [\mathcal{W}_1 \ \mathcal{W}_2 \ \dots \ \mathcal{W}_p]^\top$ for the DMC strategy, such as $\mathcal{W}_l = [y_{r,l}[k+1] \dots y_{r,l}[k+N]]^\top$ and $y_{r,l}[k]$ the reference of the output l at instant k. This, way, the minimization problem (A.34a)-(A.34d) can be rewritten, as follows

$$\begin{aligned}
\underset{\mathbf{\Delta u}}{\text{minimize}} \quad & (\vec{\mathcal{Y}} - \vec{\mathcal{W}})^\top \bar{\mathcal{Q}} (\vec{\mathcal{Y}} - \vec{\mathcal{W}}) + \mathbf{\Delta u}^\top \bar{\mathcal{R}} \mathbf{\Delta u}, && \text{(A.35a)}\\
\text{subject to} \quad & \vec{\mathcal{Y}} \in \mathbb{Y}, && \text{(A.35b)}\\
& \mathbf{u} \in \mathbb{U}, && \text{(A.35c)}\\
& \mathbf{\Delta u} \in \Delta\mathbb{U}. && \text{(A.35d)}
\end{aligned}$$

The optimal control sequence $\mathbf{\Delta u}^0 = [\Delta u[k|k]^\top \dots \Delta u[k+N_u-1|k]^\top$ is the solution of the problem stated in (A.35a)-(A.35d) which is weighted by the future errors and the control effort, considering the tuning parameters Q and R as quadratic and positive definite matrices, which are inserted in the augmented matrices

$$\bar{\mathcal{Q}} = \begin{bmatrix} Q_1 & \mathbf{0}_N & \dots & \mathbf{0}_N \\ \mathbf{0}_N & Q_2 & \dots & \mathbf{0}_N \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_N & \mathbf{0}_N & \dots & Q_p \end{bmatrix}, \tag{A.36}$$

$$\bar{\mathcal{R}} = \begin{bmatrix} R_1 & \mathbf{0}_{N_u} & \dots & \mathbf{0}_{N_u} \\ \mathbf{0}_{N_u} & R_2 & \dots & \mathbf{0}_{N_u} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{N_u} & \mathbf{0}_{N_u} & \dots & R_m \end{bmatrix}. \tag{A.37}$$

Differently from the state space formulation, caution must be taken during the DMC design, considering the matrices Q_i and R_i quadratic matrices with the ponder value for the selected output and input respectively, having the appropriated size for each case.

Defining the sequence of future inputs as $\mathbf{u} = [\mathbf{u}_1[k] \dots \mathbf{u}_m[k]]$ such as $u_r[k] = u_r[k-1] + \Delta u_r[k|k]$ for any input of the system, the selected input to the system is defined as the first element of each optimal input sequence after solving the optimization problem stated in (A.34a)-(A.34d) or (A.35a)-(A.35d) (which are equivalent), and the optimization process occurs again in the next control loop for a new measured output and past inputs following the receding horizon principle.

A.3 Generalized Predictive Control

Defining the discrete dynamic system model from the continuous transfer function $P(s)$ of the process to be controlled, where T_s defines the sampling period, $u[k] = \mathbf{u}(kT_s)$, $x[k] = x(kT_s)$, $\mathbf{u}(t) = \mathbf{u}(kT_s)$ for $kT_s \leq t < (k+1)T_s$, the nominal zero-order hold discretized model is defined by

$$\mathbf{X}(z) = \mathbf{P}(z)\mathbf{U}(z) + \mathbf{H_d}(z)\mathbf{E}(z), \tag{A.38}$$

where z denotes the unilateral $\mathcal{Z}$-transform, $\mathbf{P}(z) = \mathcal{Z}\{\mathcal{L}^{-1}\{\mathbf{P}(s)/s\}|_{t=kT_s}\}$ is the zero-order hold discretization of $\mathbf{P}(s) = \mathbf{H}(s)\mathbf{B}$, and $\mathbf{H_d}(z)$ is a disturbance model that can be directly defined in discrete-time as shown in (ÅSTRÖM; WITTENMARK, 2013, Chapter 10).

The Controlled Auto Regressive and Integrated Moving Average (CARIMA) model is an established representation used in GPC applications. Initially consider a Matrix Fraction Description given by $\mathbf{P}(z) = \mathbf{A}^{-1}(z^{-1})\mathbf{B}(z^{-1})$, then the Controller Auto-Regressive Moving-Average (CARMA) model is given by

$$\mathbf{A}(z^{-1})\mathbf{X}(z) = \mathbf{B}(z^{-1})\mathbf{U}(z) + \mathbf{A}(z^{-1})\mathbf{H_d}(z)\mathbf{E}(z), \tag{A.39}$$

excluding any input delay of the system. If a dead-time is considered, the model is simply modified by adding a discrete shift operator z^{-d} at each input fraction described in $\mathbf{B}(z^{-1})$ where d is the dead-time of the system.

The CARIMA model is obtained from the disturbance model defined by $\mathbf{H_d}(z) = \mathbf{A}^{-1}(z^{-1})\mathbf{T}(z^{-1})/(1 - z^{-1})$, where $\mathbf{T}(z^{-1})$ is freely specified. Notice that the process open-loop poles are preserved in the disturbance model and an implicit integrative action was included to represent constant disturbance effect. In general, $\mathbf{e}(kt_s) = \mathcal{Z}^{-1}\{\mathbf{E}(z)\}$ is assumed to be a white noise with null mean and $\mathbf{T}(z^{-1})$ may be used to modified the white noise properties.

Moreover,

$$\mathbf{A}(z^{-1})(1 - z^{-1})\mathbf{X}(z) = \mathbf{B}(z^{-1})(1 - z^{-1})\mathbf{U}(z) + \mathbf{T}(z^{-1})\mathbf{E}(z), \tag{A.40}$$

or simply

$$\tilde{\mathbf{A}}(z^{-1})\mathbf{X}(z) = \mathbf{B}(z^{-1})\Delta\mathbf{U}(z) + \mathbf{T}(z^{-1})\mathbf{E}(z), \tag{A.41}$$

where $\tilde{\mathbf{A}}(z^{-1}) = \mathbf{A}(z^{-1})(1-z^{-1})$, and $\Delta\mathbf{U}(z) = (1-z^{-1})\mathbf{U}(z)$ as discussed in (CAMACHO; ALBA, 2013, Chapter 4). If $\mathbf{T}(z^{-1}) \neq \mathbf{I}$, then the GPC is commonly identified as GPC-T (CAMACHO; ALBA, 2013, Chapter 4).

Now assume that $\tilde{\mathbf{A}}(z^{-1})$, $\mathbf{B}(z^{-1})$, and $\mathbf{T}(z^{-1})$ are given by

$$\tilde{\mathbf{A}}(z^{-1}) = \mathbf{I}_{n\times n} + \tilde{\mathbf{A}}_1 z^{-1} + ... + \tilde{\mathbf{A}}_{n_a+1} z^{-n_a-1}, \tag{A.42}$$

$$\mathbf{B}(z^{-1}) = z^{-1}(\mathbf{B}_0 + \mathbf{B}_1 z^{-1} + ... + \mathbf{B}_{n_b} z^{-n_b}), \tag{A.43}$$

$$\mathbf{T}(z^{-1}) = \mathbf{T}_0 + \mathbf{T}_1 z^{-1} + ... + \mathbf{T}_{n_t} z^{-n_t}. \tag{A.44}$$

In this representation, n_a+1, n_b, and n_t defines the polynomial order of the CARIMA model.

A classic solution of the GPC (or GPC-T) prediction is defined by the recursive solution of the Diophantine equation

$$\mathbf{I}_{n\times n} = \mathbf{E}_j(z^{-1})\tilde{\mathbf{A}}(z^{-1}) + z^{-j}\mathbf{F}_j(z^{-1}), \tag{A.45}$$

and the prediction of $x[k+j|k]$ is taken based on the elements of $\mathbf{E}_j(z^{-1})$ and $\mathbf{F}_j(z^{-1})$, leading to :

$$\hat{x}[k+j|k] = \mathbf{F}_j(z^{-1})x[k] + \mathbf{E}_j(z^{-1})\mathbf{B}(z^{-1})\Delta u[k+j-1] + \mathbf{E}_j(z^{-1})e[k+j], \tag{A.46}$$

such as the term $\mathbf{E}_j(z^{-1})\mathbf{B}(z^{-1})\Delta u[k+j-1]$ can be separated in future inputs to be chosen by the optimal solution and past inputs. In other words, since the Diophantine equation does not depend on the current states of the plant, the GPC prediction is simply separated from the measured state (or output if a proper linear transformation is applied, i.e. $y[k] = Cx[k]$) and past inputs - defining the free response evolution of the system, and future input increments which summarizes the forced response in (A.46).

Alternatively, The GPC-T predictions could be given recursively based on CARIMA polynomials

$$\hat{\mathbf{x}}[k+j|k] = -\sum_{\ell=1}^{n_a+1} \tilde{\mathbf{A}}_\ell \mathbf{x}[k+j-\ell|k] + \sum_{\ell=0}^{n_b} \mathbf{B}_\ell \Delta\mathbf{u}[k-1+j-\ell|k] + \sum_{\ell=0}^{n_t} \mathbf{T}_\ell \mathbf{e}[k+j-\ell|k], \tag{A.47}$$

where:

1. $\hat{\mathbf{x}}[k+j|k]$ represents a prediction for $\hat{\mathbf{x}}[k+j] = \mathbf{x}((k+j)T_s)$ given the information available at k;

2. $\hat{\mathbf{x}}[k+j|k] = \mathbf{x}[k]$ if $j \leq 0$;

3. $\Delta\mathbf{u}[k+j|k] = \mathbf{u}[k+j|k] - \mathbf{u}[k+j-1|k]$ is the expected control increments;

4. $\Delta\mathbf{u}[k+j|k] = \Delta\mathbf{u}[k+j] = \mathbf{u}[k+j] - \mathbf{u}[k+j-1]$ if $j < 0$;

5. $\mathbf{e}[k+j-\ell|k] = \mathbf{e}[k+j-\ell]$ if $j \leq \ell$, and;

6. $\mathbf{e}[k+j-\ell|k] = \mathbf{0}$ if $j > \ell$.

The vector $[\hat{\mathbf{x}}[k+1|k], \hat{\mathbf{x}}[k+2|k], ..., \hat{\mathbf{x}}[k+N|k]]$ defines a finite horizon prediction window and N is the prediction horizon.

Due to the linearity of the prediction model, the prediction is separated in terms of its free and forced response, that is $\hat{x}[k+j|k] = x^{\text{free}}[k+j|k] + x^{\text{forced}}[k+j|k]$, where $x^{\text{free}}[k+j|k]$ is given by the prediction evolution without future control variation and $x^{\text{forced}}[k+j|k]$ is the isolated effect of the future control variation. To distinguish both terms, firstly define the discrete time step function given by $\mathbb{1}_j = 0,\ j < 0$, and $\mathbb{1}_j = 1,\ j \geq 0$. Therefore, from $x^{\text{free}}[k+i|k] = x[k+i]$, $i \leq 0$ and $x^{\text{forced}}[k|k] = \mathbf{0}_{n\times n}$, then the prediction model may be expressed as follows

$$\begin{aligned} x^{\text{free}}[k+j|k] &= -\sum_{\ell=1}^{n_a+1} \tilde{\mathbf{A}}_\ell \mathbf{x}^{\text{free}}[k+j-\ell|k] + \sum_{\ell=0}^{n_b} \mathbb{1}_{\ell-j}\mathbf{B}_\ell \Delta\mathbf{u}[k-1+j-\ell] + \dots \\ &\quad \dots + \sum_{\ell=0}^{n_t} \mathbb{1}_{\ell-j}\mathbf{T}_\ell \mathbf{e}[k+j-\ell], \qquad \text{(A.48)} \\ x^{\text{forced}}[k+j|k] &= \sum_{\ell=0}^{n_b} \mathbb{1}_{j-\ell-1}\mathbf{B}_\ell \Delta\mathbf{u}[k-1+j-\ell|k]. \qquad \text{(A.49)} \end{aligned}$$

Actually, only $\mathbf{u}[k|k]$, $\mathbf{u}[k+1|k]$, ... $\mathbf{u}[k+N-1|k]$ are considered in the forced response.

The forced responses from the control increment to the output prediction are equivalent (GPC, DMC, State Space) as the nominal input-output relationships do not depend on the disturbance model. On the other hand, the free response is not unique due to the disturbance effect. In any case, the prediction vector ($\mathcal{X}_k = [\hat{x}^\top[k+1|k] \ ... \ \hat{x}^\top[k+N|k]]^\top$) may be decomposed as follows

$$\mathcal{X}_k = \mathcal{X}_k^{\text{free}} + \mathbb{G}\boldsymbol{\Delta}\mathbf{u}_k, \qquad \text{(A.50)}$$

where $\mathbb{G}$ is a suitable matrix which can be defined offline either experimentally or analytically, $\boldsymbol{\Delta}\mathbf{u}_k = [\Delta\mathbf{u}^\top[k|k] \ ... \ \Delta\mathbf{u}^\top[k+N-1|k]]^\top$, and $\mathcal{X}_k^{\text{free}} = [\mathbf{x}^{\text{free}\top}[k+1|k] \ ... \ \mathbf{x}^{\text{free}\top}[k+N|k]]^\top$. In the general case, a reduced control horizon is commonly used ($N_u < N$) with $\Delta\mathbf{u}[k+N_u|k] = 0$, $\Delta\mathbf{u}[k+N_u+1|k] = 0$, ..., $\Delta\mathbf{u}[k+N-1|k] = 0$ without losing the generality of (A.50).

An output for the linearized model may be given by $\mathbf{y}[k] = C\mathbf{x}[k]$, where $C \in \mathbb{R}^{p\times n}$ defines a linear combination of the displacement vector and p is the output dimension. This kind of approach is particularly useful for underactuated systems ($n < m$) as the tracking objective may be expressed with respect to the output. It is commonly assumed

$p = m$ in order that the output ($\mathbf{y}[k] \in \mathbb{R}^p$) can effectively track the desired output target ($\mathbf{r}[k] \in \mathbb{R}^p$). Then, the state prediction can be converted to output prediction as follows

$$\mathcal{Y}_k = \mathcal{C}\mathcal{X}_k^{\text{free}} + \mathbb{G}_{\mathbb{C}}\Delta\mathcal{U}_k, \tag{A.51}$$

where $\mathbb{G}_{\mathbb{C}} = \mathcal{C}\mathbb{G}$ and $\mathcal{C}$ is given by

$$\mathcal{C} = \begin{bmatrix} C & \mathbf{0}_{p,n} & \dots & \mathbf{0}_{p,n} \\ \mathbf{0}_{p,n} & C & \dots & \mathbf{0}_{p,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{p,n} & \mathbf{0}_{p,n} & \dots & C \end{bmatrix}. \tag{A.52}$$

In some applications, only $\mathbf{y}[k]$ is measured while $\mathbf{x}[k]$ is not available, which defines an output feedback control strategy. Then, $\mathcal{Y}_k^{\text{free}} = \mathcal{C}\mathcal{X}_k^{\text{free}}$ can be directly computed from the alternative factorization defined by $\mathcal{C}\mathbf{P}(z) = \mathbf{A}_{\mathcal{C}}^{-1}(z^{-1})\mathbf{B}_{\mathcal{C}}(z^{-1})$ where

$$\begin{aligned} \mathbf{y}^{\text{free}}[k+j|k] = -\sum_{\ell=1}^{n_a+1} \tilde{\mathbf{A}}_{\mathcal{C},\ell}\mathbf{y}^{\text{free}}[k+j-\ell|k] + \sum_{\ell=0}^{n_b} \mathbb{1}_{\ell-j}\mathbf{B}_{\mathcal{C},\ell}\Delta\mathbf{u}[k-1+j-\ell] + \dots \\ \dots + \sum_{\ell=0}^{n_t} \mathbb{1}_{\ell-j}\mathbf{T}_{\mathcal{C},\ell}\mathbf{e}[k+j-\ell], \end{aligned} \tag{A.53}$$

where the modified CARIMA polynomials are described as follows

$$\mathbf{A}_{\mathcal{C}}(z^{-1})(1-z^{-1}) = \mathbf{I}_{n\times n} + \tilde{\mathbf{A}}_{\mathcal{C},1}z^{-1} + \dots + \tilde{\mathbf{A}}_{\mathcal{C},n_a+1}z^{-n_a-1}, \tag{A.54}$$

$$\mathbf{B}_{\mathcal{C}}(z^{-1}) = z^{-1}(\mathbf{B}_{\mathcal{C},0} + \mathbf{B}_{\mathcal{C},1}z^{-1} + \dots + \mathbf{B}_{\mathcal{C},nb}z^{-n_b}), \tag{A.55}$$

also $\mathbf{y}^{\text{free}}[k+j|k] = \mathbf{y}[k+j],\ j<0$, and $\mathbf{T}_{\mathcal{C},\ell}(z^{-1})$ is a disturbance model that defined the GPC-T. The choice $\mathbf{T}_{\mathcal{C},\ell}(z^{-1}) = \mathbf{I}_p$ is commonly used in the GPC. Moreover, $\mathbb{G}_{\mathbb{C}} = \mathcal{C}\mathbb{G}$ can be directly obtained from the step response of $\mathbf{P}_{\mathcal{C}}(z) = \mathcal{C}\mathbf{P}(z)$.

A.3.1 Computation of the forced response

Considering the prediction vector $\mathcal{X}_k$ defined in (A.50), and the vector of future inputs $\mathbf{\Delta u}_k = [\Delta\mathbf{u}^\top[k|k] \ \dots \ \Delta\mathbf{u}^\top[k+N-1|k]]^\top$. Assume that $\mathbf{g}_k^r$ represents the step-response vector at instant k from the input r to the state vector with $\mathbf{g}_k = [\mathbf{g}_k^1 \ \dots \ \mathbf{g}_k^m]$ where $k=0$ defined the initial instant. Such vector could be either obtained analytically or measured from a experimental framework. As the model is linear and time-invariant, then

$$\mathbb{G} = \begin{bmatrix} \mathbf{g}_1 & \mathbf{0}_{n,m} & \dots & \mathbf{0}_{n,m} \\ \mathbf{g}_2 & \mathbf{g}_1 & \dots & \mathbf{0}_{n,m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{g}_N & \mathbf{g}_{N-1} & \dots & \mathbf{g}_1 \end{bmatrix}, \tag{A.56}$$

where $\mathbb{G} \in \mathbb{R}^{N \cdot n \times N \cdot m}$.

For the sake of generality, if the control horizon (N_u) is smaller than the prediction horizon (N), then $\boldsymbol{\Delta}\mathbf{u}_k = [\Delta \mathbf{u}^\top[k|k] \; ... \; \Delta \mathbf{u}^\top[k+N_u-1|k]]^\top$ and $\mathbb{G} \in \mathbb{R}^{N \cdot n \times N_u \cdot m}$ is given by

$$\mathbb{G} = \begin{bmatrix} \mathbf{g}_1 & \mathbf{0}_{n,m} & \cdots & \mathbf{0}_{n,m} \\ \mathbf{g}_2 & \mathbf{g}_1 & \cdots & \mathbf{0}_{n,m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{g}_N & \mathbf{g}_{N-1} & \cdots & \mathbf{g}_{N-N_u+1} \end{bmatrix}. \tag{A.57}$$

Notice that $\mathbb{G}$ may be either obtained from an experimentally or computed from a step-test simulation with the zero-order hold discretization of $\mathbf{P}(s) = \mathbf{H}(s)\mathbf{B}$, namely $\mathbf{P}(z)$. Furthermore, if $N_u = N$, then (A.57) becomes equivalent to (A.56).

If only the output measurement ($\mathbf{y}[k]$) is available, then the step response of $\mathbf{P}_{\mathcal{C}}(z) = \mathcal{C}\mathbf{P}(z)$ can be used to obtain $\mathbb{G}_{\mathbb{C}} = \mathcal{C}\mathbb{G}$. The output step response is represented by $\mathbf{g}_{out,k} = [\mathbf{g}^1_{out,k} \; \cdots \; \mathbf{g}^m_{out,k}]$, then

$$\mathbb{G}_{\mathbb{C}} = \begin{bmatrix} \mathbf{g}_{out,1} & \mathbf{0}_{n,m} & \cdots & \mathbf{0}_{n,m} \\ \mathbf{g}_{out,2} & \mathbf{g}_{out,1} & \cdots & \mathbf{0}_{n,m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{g}_{out,N} & \mathbf{g}_{out,N-1} & \cdots & \mathbf{g}_{out,N-N_u+1} \end{bmatrix}. \tag{A.58}$$

Similarly from other strategies (state space and DMC), the control law is defined by minimizing a cost function that considers the quadratic future errors and the control effort. Through the optimal sequence of control inputs, the control law is set by selecting the first sequence element, respecting the receding horizon principle and a new optimal vector is defined once again in every loop.

A.3.2 Objective function

The GPC control law is not different from the others presented in this work. The objective function here is simmilar from the presented in the optimization problem (A.34d) with the difference of the organization of the sequence of future inputs and estimation of future states. For that, consider the cost function that penalizes the control effort by means of the variance of the input increment signal and the quadratic error between predicted outputs and future references.

$$J(N, y[k], \mathcal{W}, \Delta\mathbf{u}, \Delta\mathcal{U}_k) = \sum_{j=1}^{N} ||\hat{y}[k+j|k] - y_r[k+j]||^2_Q + \sum_{j=0}^{N-1} ||\Delta u[k+j|k]||^2_R, \tag{A.59}$$

or simply in a vector form

$$J(y[k], \mathcal{W}, \Delta\mathbf{u}, \Delta\mathcal{U}_k) = (\mathcal{Y}_k - \mathcal{W})^\top \mathcal{Q}(\mathcal{Y}_k - \mathcal{W}) + \Delta\mathbf{u}^\top \mathcal{R}\Delta\mathbf{u}, \tag{A.60}$$

such as the predicted outputs is given by the GPC prediction model (A.51). Similar with the DMC algorithm, the cost function depends on the values of the past inputs $\Delta\mathcal{U}_k$ since the free response depends on such values and composes the estimated future values of the output.

A.3.3 GPC Receding Horizon Control Law

Finally, the GPC strategy is reduced to the same optimization problem as the other strategies shown until now,

$$\begin{aligned}
\underset{\boldsymbol{\Delta}\mathbf{u}}{\text{minimize}} \quad & (\mathcal{Y}_k - \mathcal{W})^\top \mathcal{Q}(\mathcal{Y}_k - \mathcal{W}) + \boldsymbol{\Delta}\mathbf{u}^\top \mathcal{R}\boldsymbol{\Delta}\mathbf{u}, && \text{(A.61a)}\\
\text{subject to} \quad & \mathcal{Y}_k \in \mathbb{Y}, && \text{(A.61b)}\\
& \mathbf{u} \in \mathbb{U}, && \text{(A.61c)}\\
& \boldsymbol{\Delta}\mathbf{u} \in \Delta\mathbb{U}, && \text{(A.61d)}
\end{aligned}$$

which gives the sequence of optimal inputs $\Delta\mathbf{u}^0 = [\Delta u^0[k|k]^\top \ldots \Delta u^0[k{+}N{-}1|k]^\top]^\top$ that minimize the cost function under the restrictions imposed. The receding horizon principle is applied by selecting only the first element of each input increment. The control law at a given time k_i is then defined as

$$u[k_i] = \kappa_{GPC}(y[k_i], \mathcal{W}) = u[k_i - 1] + \Delta u^0[k_i|k_i], \tag{A.62}$$

and the whole process of prediction and minimization is restarted in each control loop.