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A GENERALIZED VLASOV EQUATION

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A two-spacial dimension electronic system characterized by a plasma parameter $\Gamma \leqslant 1$ is analyzed; then, by using a rigorous non-equilibrium statistical mechanical theory, the evolution of distribution function is considered. A generalized Vlasov equation (GVE) is derived. Compared to the usual Vlasov equation, GVE presents an additional velocity-dependent correlation term. Taking as a starting point the GVE, the phenomenological approximation to two-particles function, $f_2(r_1 r_2 p_1 p_2; t) = f_1(r_1 p_1; t) f_1(r_2 p_2; t) g(r_1 - r_2)$, proposed by Singwi, Tosi, Landi and Sjolander is analyzed.

1. Introduction

Properties of low-dimensional electronic systems have been extensively studied in the literature [1–5]. Particularly important in this area is the system formed by electrons on a helium surface. Such a system we will call in this paper TDES (two-dimensional electronic system).

TDES is a system with correlational properties; it is characterized by an electron density c in the experimental range $10^5 \leqslant c \leqslant 10^9$ [4, 5]. In this situation the range of electrostatic energy per particle (2×10^{-16} ergs to 7×10^{-15} ergs) is comparable to a typical thermal energy ($\sim 1.4 \times 10^{-16}$ ergs). Then, the quantum effects are not important and TDES exhibits a classical behaviour, i.e. TDES can be treated as a classical plasma. Indeed, the electron system behaves like a non-degenerate plasma described by a two-dimensional Boltzmann distribution as a limit of the Fermi distribution at experimental densities and temperatures.

From a theoretical standpoint, the TDES has been studied by several authors [6–11]. In particular, Studart and Hipólito [12, 13] have considered the correlation effects by using the self-consistent field approximation proposed by Singwi, Tosi, Land and Sjolander (STLS) [14]. In the STLS approximation, the

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two-particle distribution function $f_2(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{v}_1, \boldsymbol{v}_2; t)$ is written as

$$f_2(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{v}_1, \boldsymbol{v}_2; t) = f_1(\boldsymbol{r}_1, \boldsymbol{v}_1; t)\, f_1(\boldsymbol{r}_2, \boldsymbol{v}_2; t)\, g(|\boldsymbol{r}_1 - \boldsymbol{r}_2|)\,, \qquad (1)$$

where $f_1(\boldsymbol{r}, \boldsymbol{v}; t)$ is the single-particle distribution function and $g(|\boldsymbol{r}_1 - \boldsymbol{r}_2|)$ is the static pair correlation function.

The STLS method is strongly based on the relation (1) which is a phenomenological hypothesis. Indeed, in this scheme the equation of motion of the one-particle distribution function is truncated under the assumption that the coupling of the particle to the medium is given by the function $g(r)$.

It is known that the main object of a partly intuitive or semi-phenomenological model is the calculation of the physical quantities from its fundamental equations, which are accepted more or less a priori. In consequence, one has not an analytical method to modify the equations of such models in order to obtain the best results for a given problem. The STLS model as it is presented in the literature can be included in this category of phenomenological models.

In the present paper, by using a rigorous non-equilibrium statistical mechanical theory, we will analyze relation (1). Our starting point is the dynamics of correlations (DC) developed mainly by Prigogine and Balescu [15, 16]. The DC has, at least, two aspects which are interesting to the study of the STLS approximation: (i) the DC has been used with success in the study of plasma [16, 17] (ii) by using DC theory one obtains the same diagrammatical representation for both classical and quantum plasmas [16]. This aspect (ii) can be useful in the analysis of the classical limit of a quantum system.

We will consider here the TDES characterized by a plasma parameter $\Gamma \approx 1$. The parameter Γ is defined as $\Gamma = \langle V \rangle / \langle K \rangle$ where $\langle V \rangle$ and $\langle K \rangle$ denote the averages of the potential and kinetic energies respectively. We will show that the correlation effects will be related to diagrams we will call 2nd order diagrams. Our analysis concerns to derivation of an equation which describes the behavior of the plasma over short periods of time, i.e., for $t = \mathcal{O}(t_p)$, where t_p is the plasma oscillation period, but since we will consider the case in which Γ is no longer very small we will need to include terms of order $e^2(e^2c)^n$. As a result, we obtain a generalized Vlasov equation (GVE). Compared to the usual Vlasov equation our equation presents an additional velocity-dependent correlation term. Taking as a starting point the GVE we will obtain, as a particular case, the equation related to hypothesis (1). Hence, we can show the principles guiding the choice of the diagrams in the STLS model; this feature allows to us to indicate as the STLS model can be improved.

In section 2 we present the notation we have used in the paper, the characteristic properties of TDES and the diagrams necessary to describe TDES by using DC theory. In section 3 we will derive the GVE. Section 4 contains an analysis of the STLS approximation seen as a particular case of our GVE. In section 5 we present final remarks and conclusions.

2. Notation and preliminaries

The TDES will be considered here as a two-dimensional N-particle system with interaction potential given by

$$V_{ij} = e/|\boldsymbol{r}_i - \boldsymbol{r}_j| , \qquad \boldsymbol{r} = \hat{\imath}x + \hat{\jmath}y, \quad i, j = 1, \ldots, N , \tag{2}$$

where $e = e_0[2/(1+\varepsilon)]^{1/2}$ with e_0 the electronic charge and ε the dielectric function. The system occupies a volume Ω. The perturbative solution of the Liouville equation in Fourier representation is, according to DC theory [16], given by

$$\rho_{\{k\}}(v; t) = -\frac{1}{2\pi i} \sum_{\{k'\}} \sum_{n=0}^{\infty} (-e^n) \langle \{k\} | R_0(\omega) [L' R_0(\omega)]^n | \{k'\} \rangle \, \rho_{\{k'\}}(v; 0) . \tag{3}$$

The coefficients $\rho_{\{k\}}$ are factored as [16]

$$\rho_{k_1, k_2, \ldots, k_s}(1, \ldots, s) = \prod_{j=1}^{s} \rho_{k_j}(j) + \rho_{[k_1 \ldots k_s]}(1, \ldots, s) . \tag{4}$$

The first term in eq. (4) is a product of s non-homogeneous factors. The second is the sum of all possible correlation patterns of s particles.

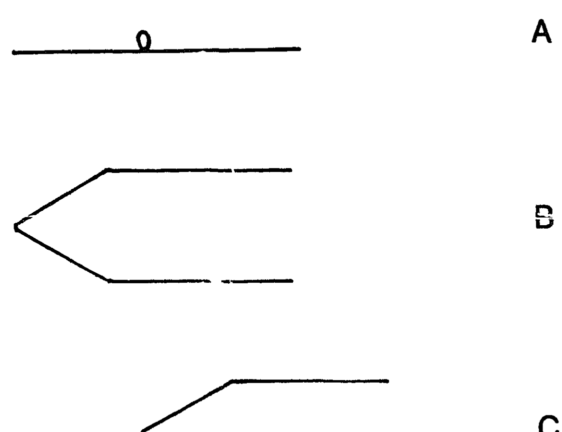

Fig. 1. Vertices which compose the relevant diagrams for calculation of ρ_0 and ρ_k.

Associated with eq. (3) there is a set of diagrams [15, 16]. The choice of relevant diagrams must be consistent with specifications of the studied system. In our case, the TDES is characterized by $\Gamma \leq 1$, and we will be interested here in a short-time process. Thus, the main contributions at eq. (3) will be diagrams corresponding to order $(e^2c)^n$, which we will call first order diagrams, and $e^2(e^2c)^n$, which we will call second order diagrams, being $c = N/\Omega$. The vertices of these diagrams are presented in fig. 1. Once the relevant diagrams have been chosen, we can make their summation in order to determine their contribution to $\rho_0(t)$ and $\rho_k(t)$.

3. A generalized Vlasov equation

3.1. First and second order contributions to ρ_0

The contributions to ρ_0 are derived from diagrms that have not external lines at left, i.e. the relevant diagrams are composed by A, B and C vertices shown in fig. 1. The vertices A and C are connected to B as is indicated in fig. 2.

Considering the summation of all the diagrams of fig. 2 the B vertices at the left can be factorized and we obtain for $\rho_0(t)$ the result

$$\begin{aligned}\rho_0(| \ldots ; t) = &-\frac{1}{2\pi\mathrm{i}}(-e^2)\int \mathrm{d}\omega\, \mathrm{e}^{\mathrm{i}\omega t} \sum_{k'_n}\sum_{n,j} F_{jn}(-k'_n)(k'_n \cdot v_n - k'_n \cdot v_j - \omega)^{-1} \\ &\times \Big\{ \rho_{k'_n,-k'_n}(n, j| \ldots ; 0) \\ &+ (-e^2)^2 \sum_{a,b} F_{ja}(-k'_n)\,(k'_n \cdot v_n - k'_n \cdot v_a - \omega)^{-1} \\ &\times F_{ab}(-k'_n)\,(k'_n \cdot v_n - k'_n \cdot v_b - \omega)^{-1}\, \rho_{\,k'_n,k'_n}(b, n| \ldots ; 0) \\ &+ (-e^2)^3 \sum_{a,b,c} \ldots \Big\}, \end{aligned} \qquad (5)$$

where

$$F_{jn}(k_j) = -\frac{4\pi^2}{\Omega} V_{kj}\,(m^{-1} k_j \cdot \partial_{jn})$$

and we have used the notation of ref. [16].

We now differentiate Eq. (5) with respect to time and integrate over all velocities except v_a. The result is

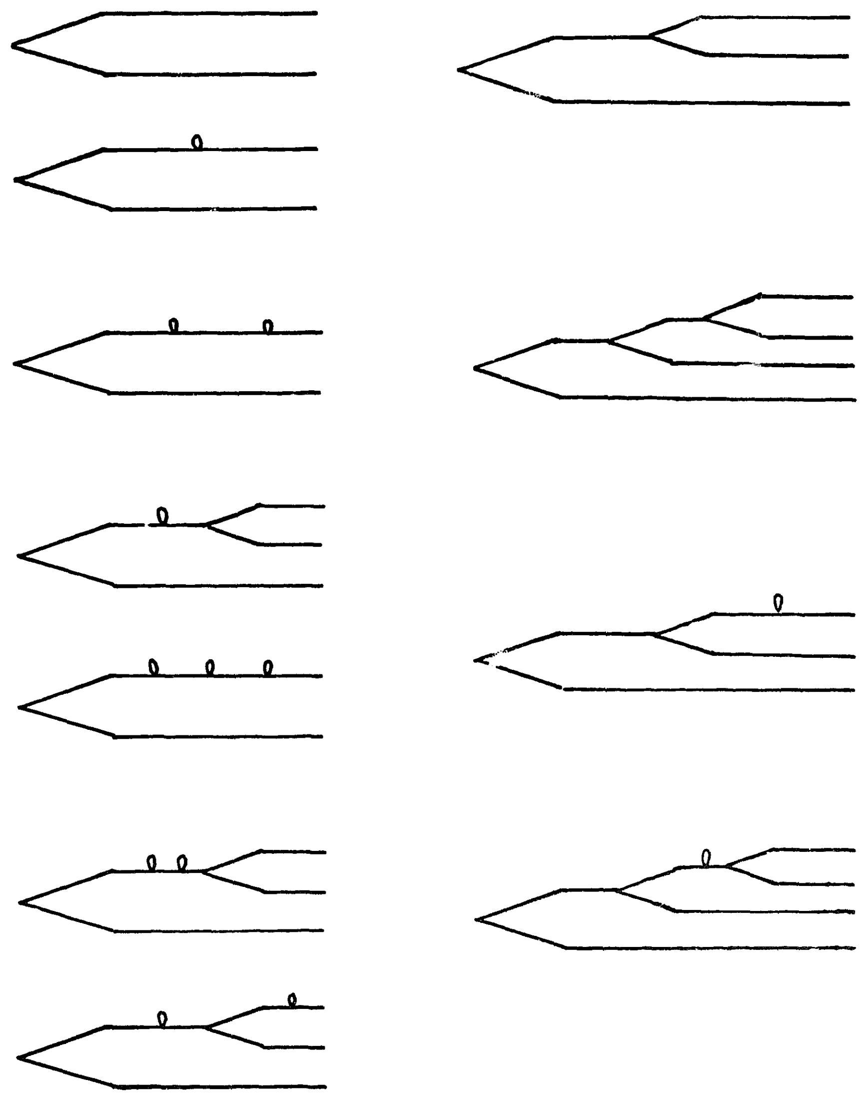

Fig. 2. Diagrams which give the relevant first and second order contributions to ρ_0.

$$\partial_t \rho_0 (|\boldsymbol{v}_\alpha ; t) = 0 , \tag{6}$$

i.e., to this order of approximation, ρ_0 remains constant in time.

3.2. First and second order contributions to ρ_k

The contributions to ρ_k are derived from diagrams ending at the left with one line. Then, the relevant diagrams are composed by A and C vertices. A general diagram in this case is represented in fig. 3 and its corresponding mathematical expression is

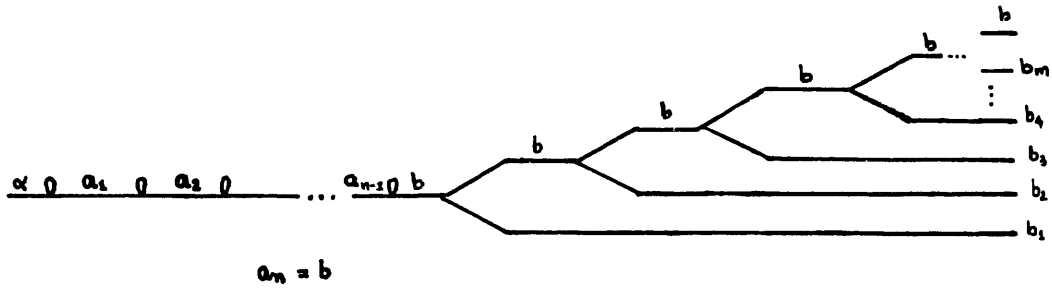

Fig. 3. A general diagram for first and second order contributions to ρ_k.

$$
\begin{aligned}
\rho^{(g)}_{k_\alpha} = {} & \frac{-(-e^2)^{n+m}}{2\pi\,\mathrm{i}} \int \mathrm{d}\omega\, \mathrm{e}^{-\mathrm{i}\omega t} \\
& \times \sum_{a_1,\ldots,a_{n-1},b} \; \sum_{b_1,\ldots,b_m} \; \sum_{k_{b_1},\ldots,k_{b_m}} (\boldsymbol{k}_\alpha \cdot \boldsymbol{v}_\alpha - \omega)^{-1} \\
& \times F_{\alpha,a_1}(\boldsymbol{k}_\alpha)\,(\boldsymbol{k}_\alpha \cdot \boldsymbol{v}_{a_1} - \omega)^{-1} F_{a_1 a_2}(\boldsymbol{k}_\alpha) \cdots \\
& \cdots F_{a_{n-1},b}(\boldsymbol{k}_\alpha)\,(\boldsymbol{k}_\alpha \cdot \boldsymbol{v}_b - \omega)^{-1} D_{b,b_1}(\boldsymbol{k}_\alpha - \boldsymbol{k}_{b_1}; \boldsymbol{k}_\alpha) \\
& \times [(\boldsymbol{k}_\alpha - \boldsymbol{k}_{b_1}) \cdot \boldsymbol{v}_b + \boldsymbol{k}_{b_1} \cdot \boldsymbol{v}_{b_1} - \omega]^{-1} D_{b,b_2}(\boldsymbol{k}_\alpha - \boldsymbol{k}_{b_1} - \boldsymbol{k}_{b_2}; \boldsymbol{k}_\alpha - \boldsymbol{k}_{b_1}) \cdots \\
& \cdots D_{b,b_m}(\boldsymbol{k}_\alpha - \boldsymbol{k}_{b_1} - \cdots - \boldsymbol{k}_{b_m}; \boldsymbol{k}_\alpha - \boldsymbol{k}_{b_1} - \cdots - \boldsymbol{k}_{b_{m-1}}) \\
& \times \left[\left(\boldsymbol{k}_\alpha - \sum_{j=1}^{m} \boldsymbol{k}_{b_j} \right) \cdot \boldsymbol{v}_b + \sum_{j=1}^{m-1} \boldsymbol{k}_{b_j} \cdot \boldsymbol{v}_{b_j} - \omega \right]^{-1} \\
& \times \rho_{k_\alpha - k_{b_1} - \cdots - k_{b_m}, k_{b_m}, \ldots, k_{b_1}} (\boldsymbol{v}_b, \boldsymbol{v}_{b_1}, \ldots, \boldsymbol{v}_{b_m} | \ldots, 0)\,,
\end{aligned}
\tag{7}
$$

where

$$
D_{jn}(\boldsymbol{k}'_j, \boldsymbol{k}_j) = -\frac{4\pi^2}{\Omega} V_{|\boldsymbol{k}'_j - \boldsymbol{k}_j|}\, m^{-1} (\boldsymbol{k}'_j - \boldsymbol{k}_j) \cdot \boldsymbol{\partial}_{jn}\,.
$$

The other relevant diagrams are obtained inserting A vertices between C vertices in fig. 3. Correspondingly we must introduce the mathematical expression associated with the A vertex in eq. (7). As an example, in fig. 4, some general diagrams in this case are shown. We can add all those diagrams and we obtain ρ_k given by

$$
\begin{aligned}
\rho_{k_\alpha}(\alpha | \ldots; t) = {} & -\frac{1}{2\pi\,\mathrm{i}} \int \mathrm{d}\omega\, \mathrm{e}^{-\mathrm{i}\omega t} (\boldsymbol{k}_\alpha \cdot \boldsymbol{v}_\alpha - \omega)^{-1} \rho_{k_\alpha}(\alpha | \ldots; 0) \\
& + \frac{(-e^2)}{2\pi\,\mathrm{i}} \int \mathrm{d}\omega\, \mathrm{e}^{-\mathrm{i}\omega t} (\boldsymbol{k}_\alpha \cdot \boldsymbol{v}_\alpha - \omega)^{-1} \sum_{a_1} F_{\alpha 1} a_1(\boldsymbol{k}_\alpha)
\end{aligned}
$$

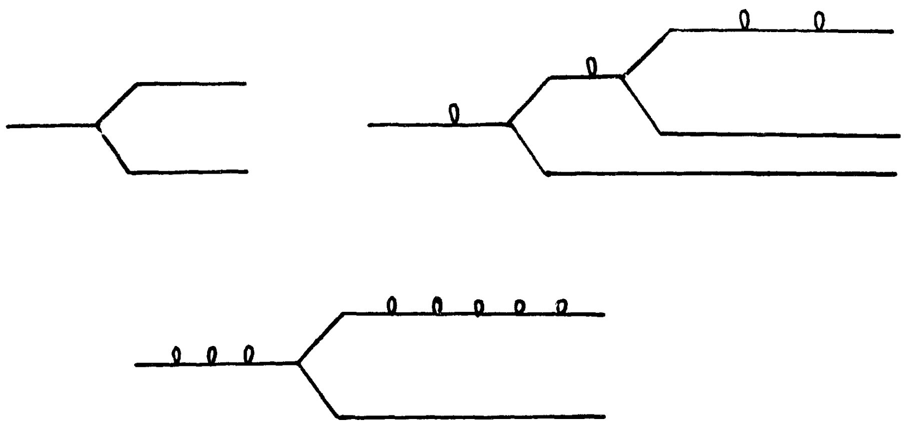

Fig. 4. Examples of diagrams obtained inserting A vertices between C vertices in the diagrams of fig. 3.

$$\times \Big\{ (\boldsymbol{k}_\alpha \cdot \boldsymbol{v}_{a_1} - \omega)^{-1} [\rho_{\boldsymbol{k}_\alpha}(a_1|\ldots;0) + \cdots$$

$$+ (-e^2) \sum_{a_2} F_{a_1 a_2}(\boldsymbol{k}_\alpha)(\boldsymbol{k}_\alpha \cdot \boldsymbol{v}_{a_2} - \omega)^{-1} \rho_{\boldsymbol{k}_\alpha}(a_2|\ldots;0) + \ldots$$

$$+ (-e^2) \sum_{\boldsymbol{k}'} \sum_{a_2} D_{a_1 a_2}(\boldsymbol{k}_\alpha, \boldsymbol{k}') \, [\boldsymbol{k}' \cdot \boldsymbol{v}_{a_2} + (\boldsymbol{k}_\alpha - \boldsymbol{k}') \cdot \boldsymbol{v}_{a_1} - \omega]^{-1}$$

$$\times \rho_{\boldsymbol{k}_\alpha - \boldsymbol{k}', \boldsymbol{k}_\alpha}(a_1, a_2|\ldots;0) + \cdots] \Big\}$$

$$- \frac{(-e^2)}{2\pi \mathrm{i}} \int \mathrm{d}\omega \, \mathrm{e}^{-\mathrm{i}\omega t} (\boldsymbol{k}_\alpha \cdot \boldsymbol{v}_\alpha - \omega)^{-1} \sum_{\boldsymbol{k}'} \sum_{a_1} D_{\alpha a_1}(\boldsymbol{k}_\alpha, \boldsymbol{k}')$$

$$\times \{ [\boldsymbol{k}' \cdot \boldsymbol{v}_\alpha + (\boldsymbol{k}_\alpha - \boldsymbol{k}') \cdot \boldsymbol{v}_{a_1}]^{-1}$$

$$\times \rho_{\boldsymbol{k}', \boldsymbol{k}_\alpha - \boldsymbol{k}'}(\alpha, a_1|\ldots;0) + \cdots \} . \tag{8}$$

We can now differentiate the two sides of (8) with respect to time, integrate over all velocities except $\boldsymbol{v}_\alpha$ and take the limit $N, \Omega \to \infty$; $(N/\Omega) \sim c < \infty$. As a result we obtain

$$\partial_t \rho_{\boldsymbol{k}}(\alpha; t) + \mathrm{i}\boldsymbol{k} \cdot \boldsymbol{v}_\alpha \, \rho_{\boldsymbol{k}}(\alpha; t)$$

$$- 4\pi^2 e^2 c m^{-1} \Big\{ V_k \, \mathrm{i} \, \boldsymbol{k} \cdot \boldsymbol{\partial}_\alpha \int \mathrm{d}\boldsymbol{v}_j \, \rho_{\boldsymbol{k}}(\boldsymbol{v}_j | \boldsymbol{v}_\alpha; t)$$

$$+ \boldsymbol{\partial}_\alpha \cdot \int \mathrm{d}\boldsymbol{k}' \, \mathrm{d}\boldsymbol{v}_j \, \mathrm{i}(\boldsymbol{k} - \boldsymbol{k}') V_{|\boldsymbol{k}-\boldsymbol{k}'|} \, \rho_{\boldsymbol{k}'}(\alpha; t) \, \rho_{\boldsymbol{k}-\boldsymbol{k}'}(j; t) \Big\}$$

$$= I_{\mathrm{cor}}(\boldsymbol{k}, \boldsymbol{v}_\alpha; t) , \tag{9}$$

with

$$I_{\rm cor}(\boldsymbol{k}, \boldsymbol{v}_\alpha; t) = 4\pi^2 e^2 c m^{-1}\, \boldsymbol{\partial}_\alpha \cdot \int \mathrm{d}\boldsymbol{k}'\, \mathrm{d}\boldsymbol{v}_j\, \mathrm{i}(\boldsymbol{k}-\boldsymbol{k}') \times V_{|\boldsymbol{k}-\boldsymbol{k}'|}\, \rho_{[\boldsymbol{k}', \boldsymbol{k}-\boldsymbol{k}']}(\boldsymbol{v}_\alpha, \boldsymbol{v}_j; t)\,, \tag{10}$$

where we have used eq. (4) to obtain eq. (9) and eq. (10).

Eq. (9) can be transformed to phase space, by multiplying both sides with $(\Omega/4\pi^2)\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}_\alpha)$ and integrating over $\boldsymbol{k}$. Hence, we obtain

$$\partial_t f_1(\boldsymbol{v}_\alpha, \boldsymbol{r}_\alpha; t) + \boldsymbol{v}_\alpha \cdot \boldsymbol{\partial}_\alpha f_1(\boldsymbol{v}_\alpha, \boldsymbol{r}_\alpha; t) - e^2 m^{-1}[\boldsymbol{\partial}_\alpha f_1(\boldsymbol{v}_\alpha, \boldsymbol{r}_\alpha; t)]\cdot\boldsymbol{\nabla}_\alpha \int \mathrm{d}\boldsymbol{r}_j\, \mathrm{d}\boldsymbol{v}_j\, V(|\boldsymbol{r}_\alpha - \boldsymbol{r}_j|) \times [f_1(\boldsymbol{r}_j, \boldsymbol{v}_j; t) - c\varphi(\boldsymbol{v}_j; t)] = I_{\rm cor}(\boldsymbol{r}_\alpha, \boldsymbol{v}_\alpha; t)\,, \tag{11}$$

where $\varphi(\boldsymbol{v}; t)$ is the one-particle velocity distribution function and

$$I_{\rm cor}(\boldsymbol{r}_\alpha, \boldsymbol{v}_\alpha; t) = \frac{\Omega}{4\pi^2}\int \mathrm{d}\boldsymbol{k}\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}_\alpha} I_{\rm cor}(\boldsymbol{k}, \boldsymbol{v}_\alpha; t)\,. \tag{12}$$

The left-hand side in eq. (11) is the equation first proposed by Vlasov [18]. The right-hand side represents a correction to the Vlasov equation. By our development this term is originated from $\rho_{[\boldsymbol{k}', \boldsymbol{k}-\boldsymbol{k}']}$, i.e, it is asssociated with true correlations. Eq. (11) we will call generalized Vlasov equation (GVE).

4. Relation between the GVE and the STLS approximation

Considering eq. (1), the first BBGKY equation [19] can be written as

$$\partial_t f_1(\boldsymbol{r}_\alpha, \boldsymbol{v}_\alpha; t) + \boldsymbol{v}_\alpha \cdot \boldsymbol{\nabla}_\alpha f_1(\boldsymbol{r}_\alpha, \boldsymbol{v}_\alpha; t) - e^2 m^{-1}\, \boldsymbol{\partial}_\alpha \cdot \int \mathrm{d}\boldsymbol{r}\, \mathrm{d}\boldsymbol{v}_j\, [\boldsymbol{\nabla}_\alpha V(|\boldsymbol{r}_\alpha - \boldsymbol{r}_j|)] f_1(\boldsymbol{r}_\alpha, \boldsymbol{v}_\alpha; t) \times [f_1(\boldsymbol{r}_j, \boldsymbol{v}_j; t) - c\varphi(\boldsymbol{v}_j; t)] = I'_{\rm cor}(\boldsymbol{r}_\alpha, \boldsymbol{v}_\alpha; t)\,, \tag{13}$$

where

$$I'_{\mathrm{cor}}(\boldsymbol{r}_\alpha, \boldsymbol{v}_\alpha; t) = \tfrac{1}{2}e^2cm^{-1}\,\boldsymbol{\partial}_\alpha \cdot \int \mathrm{d}\boldsymbol{r}_j\,\mathrm{d}\boldsymbol{v}_j\,[\boldsymbol{\nabla}_\alpha V(|\boldsymbol{r}_\alpha - \boldsymbol{r}_j|)]\,G(\boldsymbol{r}_\alpha - \boldsymbol{r}_j; \boldsymbol{v}_\alpha, \boldsymbol{v}_j; t) \tag{14}$$

and

$$G(\boldsymbol{r}_\alpha - \boldsymbol{r}_j, \boldsymbol{v}_\alpha, \boldsymbol{v}_j; t) = \frac{2}{c}\,[g(\boldsymbol{r}_\alpha - \boldsymbol{r}_j) - 1][f_1(\boldsymbol{r}_\alpha, \boldsymbol{v}_\alpha; t)f_1(\boldsymbol{r}_j, \boldsymbol{v}_j; t) - cf_1(\boldsymbol{r}_\alpha, \boldsymbol{v}_\alpha; t)\varphi(\boldsymbol{v}_j; t)]\,. \tag{15}$$

The function $G(\boldsymbol{r}_\alpha - \boldsymbol{r}_j, \boldsymbol{v}_\alpha, \boldsymbol{v}_j; t)$ is the correlation function [16] in the approximation (1).

We can write eq. (14) in terms of a r-Fourier transform, i.e.

$$I'_{\mathrm{cor}}(\boldsymbol{r}_\alpha, \boldsymbol{v}_\alpha; t) = \frac{\Omega}{4\pi^2}\int \mathrm{d}\boldsymbol{k}\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}_\alpha} I'_{\mathrm{cor}}(\boldsymbol{k}, \boldsymbol{v}_\alpha; t)\,, \tag{16}$$

where

$$I'_{\mathrm{cor}}(\boldsymbol{k}, \boldsymbol{v}_\alpha; t) = \frac{4\pi^2}{\Omega}\,e^2cm^{-1}\,\boldsymbol{\partial}_\alpha \cdot \int \mathrm{d}\boldsymbol{k}'\,\mathrm{d}\boldsymbol{v}_j\,\boldsymbol{F}_{\boldsymbol{k}-\boldsymbol{k}'}\,\rho_{\boldsymbol{k}+\boldsymbol{k}',-\boldsymbol{k}-\boldsymbol{k}'}\,\delta(\boldsymbol{k}) \tag{17}$$

and $F_{k-k'}$ is the Fourier transform of the function $\boldsymbol{\nabla}_\alpha V(|\boldsymbol{r}_\alpha - \boldsymbol{r}_\beta|)$.

Comparing eq. (12) with eq. (14), we obtain that the equation proposed by STLS is derived from GVE if

$$\rho_{[\boldsymbol{k}',\boldsymbol{k}-\boldsymbol{k}']} \equiv \rho_{\boldsymbol{k}+\boldsymbol{k}',-\boldsymbol{k}-\boldsymbol{k}'}\,\delta(\boldsymbol{k})\,, \tag{18}$$

where $\delta(\boldsymbol{k})$ is the δ-Dirac function.

Eq. (18) shows that in the STLS approximation the true correlation coefficient $\rho_{[\boldsymbol{k}',\boldsymbol{k}-\boldsymbol{k}']}$, as a function of $\boldsymbol{k}$, is sharply peaked at $\boldsymbol{k} = \boldsymbol{0}$. This fact is in agreement with the general analysis of the Fourier components in DC theory. Indeed, in this analysis, the two-particle correlation functions $G(\boldsymbol{r}_\alpha, \boldsymbol{r}_\beta, \boldsymbol{v}_\alpha, \boldsymbol{v}_\beta; t)$ is written in terms of the center of mass ($\boldsymbol{R}$) and relative ($\boldsymbol{r}$) coordinates by [16]

$$G(\boldsymbol{R}, \boldsymbol{r}, \boldsymbol{v}_\alpha, \boldsymbol{v}_\beta; t) = \int \mathrm{d}\boldsymbol{k}_1\,\mathrm{d}\boldsymbol{k}_2\,\exp\{\mathrm{i}(\boldsymbol{k}_1 + \boldsymbol{k}_2)\cdot\boldsymbol{R} + \mathrm{i}(\boldsymbol{k}_1 - \boldsymbol{k}_2)\cdot\boldsymbol{r}\} \times \rho_{[\boldsymbol{k}_1,\boldsymbol{k}_2]}(\boldsymbol{v}_\alpha, \boldsymbol{v}_\beta; t)\,. \tag{19}$$

The variation of G with the coordinate $\boldsymbol{R}$ of the center of mass of the couple of particles (α, β) is slow in molecular scale, L_{m}, if $L_{\mathrm{h}} \gg L_{\mathrm{m}}$ (L_{h} is the hydrodynamic length scale and L_{m} the largest characteristic molecular length scale).

Thus, $\rho_{[k_1,k_2]}(v_\alpha, v_\beta)$ as a function of $k_1 + k_2$ is sharply peaked around $k_1 + k_2 = 0$, with a width of order L_h^{-1} [16]. In our case, $k_1 = k'$ and $k_2 = k - k'$. Then, we have $k_1 + k_2 = k$, i.e. $\rho_{[k',k-k']}$ must be sharply peaked at $k = 0$, as indicated in eq. (18).

On the other hand, $\rho_{[k',k-k']}$, as function of k', has for $k = 0$ the behaviour of $\rho_{k',-k'}$, i.e., $\rho_{[k',k-k']}$ has for $k = 0$ the same physical nature as the Fourier components, which describe correlations in a homogeneous system. It follows from our development, by using DC theory, that the STLS approach corresponds to the study of a non-homogeneous system by considering the correlations as given in a homogeneous system.

5. Concluding remarks

We have derived a generalized Vlasov equation (GVE) by using the DC theory developed mainly by Prigogine and Balescu. From GVE we have obtained the condition which the true correlation coefficient $\rho_{[k',k-k']}$ must satisfy in STLS approximation, i.e.,

$$\rho_{[k',k-k']} \equiv \rho_{k+k',-k-k'}\delta(k) .$$

This relation shows that in STLS approximation $\rho_{[k',k-k']}$ is sharply peaked around $k = 0$ and that, for $k = 0$, $\rho_{[k',k-k']} = \rho_{k',-k'}$. Hence, we can say that STLS approximation describes a non-homogeneous system by considering the correlations as a homogeneous system.

Our development shows also by analyzing the term $I_{\text{cor}}(r, v; t)$, in eq. (11), that it is possible to use other approximations. For example, we can consider a velocity-depending true correlation coefficient. Such an approximation will be an improvement to the STLS approach.

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